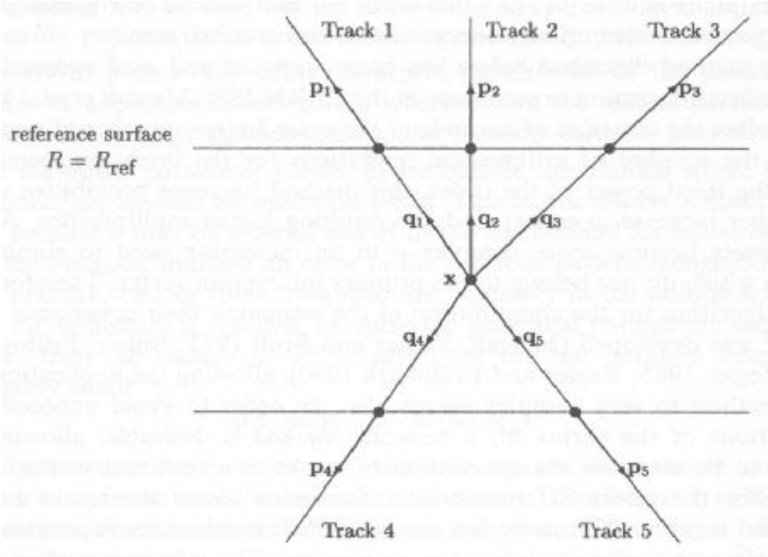
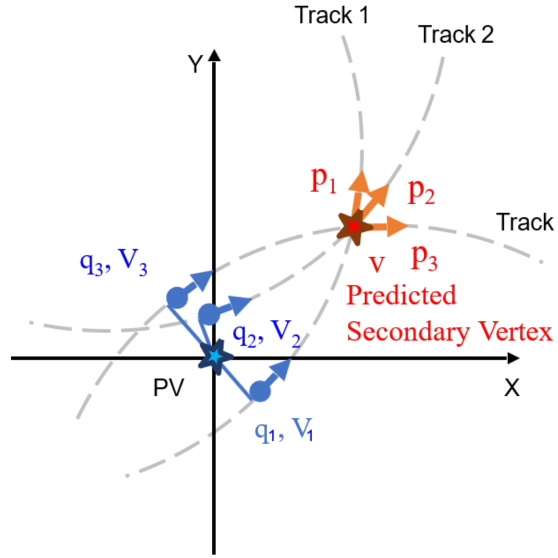
Kinematic constraints

- Let us assume a hypothetical case
  - Measured energy and momentum of a particle, whose mass is known
  - How to improve the momentum/energy ?
  - Use the constraint equation, $\sqrt{E^2 - P^2} - M = 0 \equiv f(m)$
- The effective number of unknowns in the fit is reduced by the number of constraints.
  - $f(m, p) = f(m_0, p) + (m - m_0)\frac{\partial f(m,p)}{\partial m} \equiv D(m - m_0) + f(m_0, p) = 0$
- This result suggests that the solution can be “factored" into two pieces:

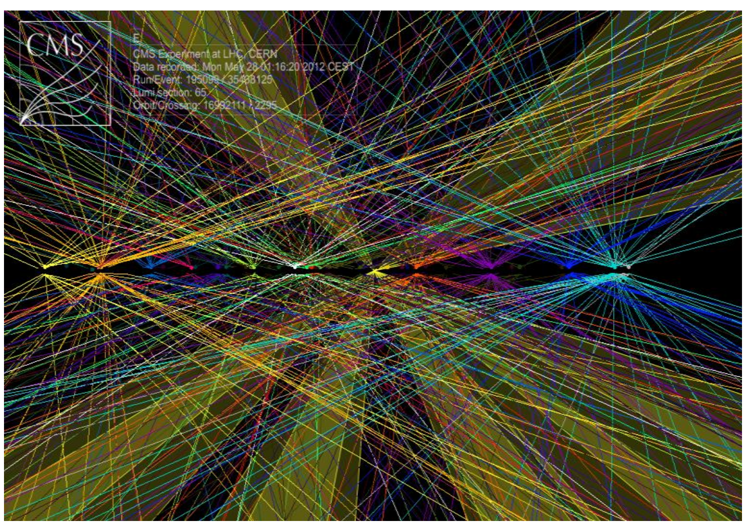
1) solving the unconstraint equations for m_0 and
2) applying the constraints to solve for m in terms of m_0.
   1) Like perturbation
   2) You are familiar with Lagrange undetermined multupliers

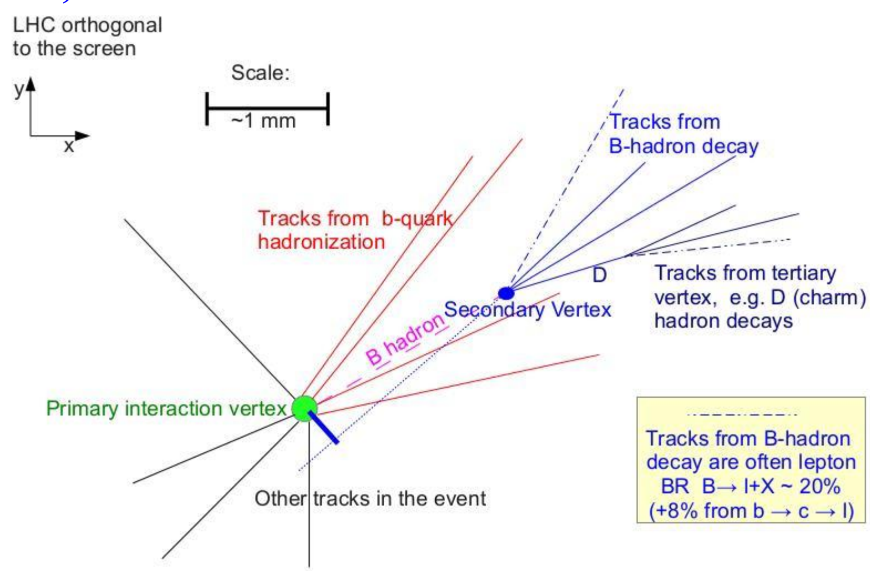
Vertex constraint





- **Extrapolate tracks to a point, where all passing through the same point.**
- **Primary, detached/secondary/tertiary vertex, lifetime………**



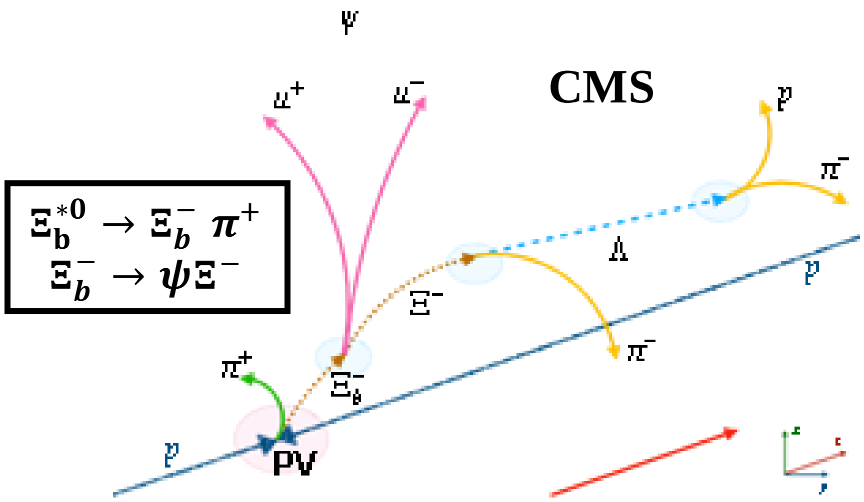


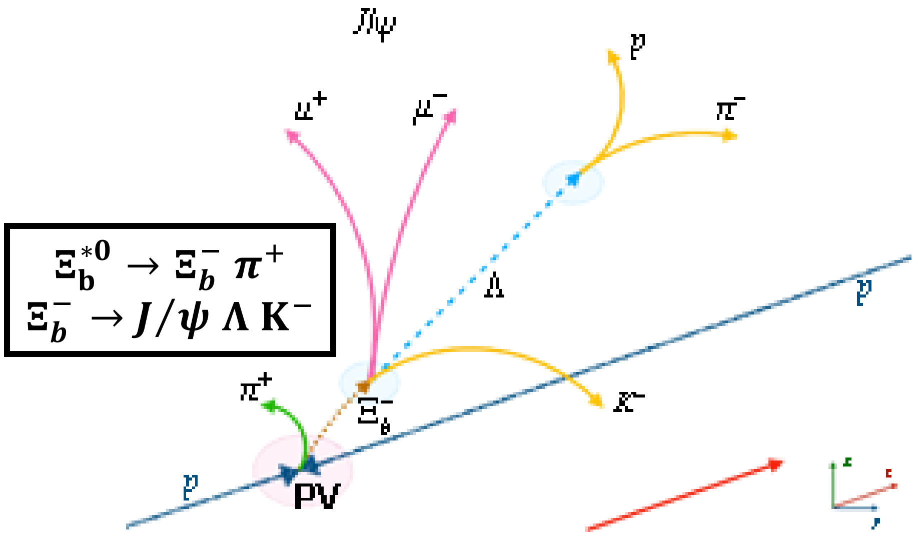
Kinematic constraints

- Assume the following decay chains of $\overline{B}^0$ meson
  - $\overline{B}^0 \rightarrow D^{*+}\pi^-$
    - $D^{*+} \rightarrow D^0\pi^+$
      - $D^0 \rightarrow K^-\pi^+\pi^0$
        - $\pi^0 \rightarrow \gamma\gamma$
- Several kinematic constraints may be applied to improve mass resolution of B , e.g. and an example from B-factory,

1. Mass of $\gamma\gamma$ to M_{π^0} (Mass constraint)
2. K^- and π^+ from D^0 decay intersect single space point (vertex constraint)
3. The $K^-\pi^+\pi^0$ mass is equal to M_D
4. ‘0 (Mass constraint)
5. Inv mass of $D^0\pi^+$ is equal to $M_{D^{*+}}$
6. Slow π^+ from D^+ decay and fast π^- from $\overline{B}^0$ decay come from same space Point
7. Sum of energies of final state particles (in each level) is the energy of beam (in CM frame)

Importance of the kinematic fit

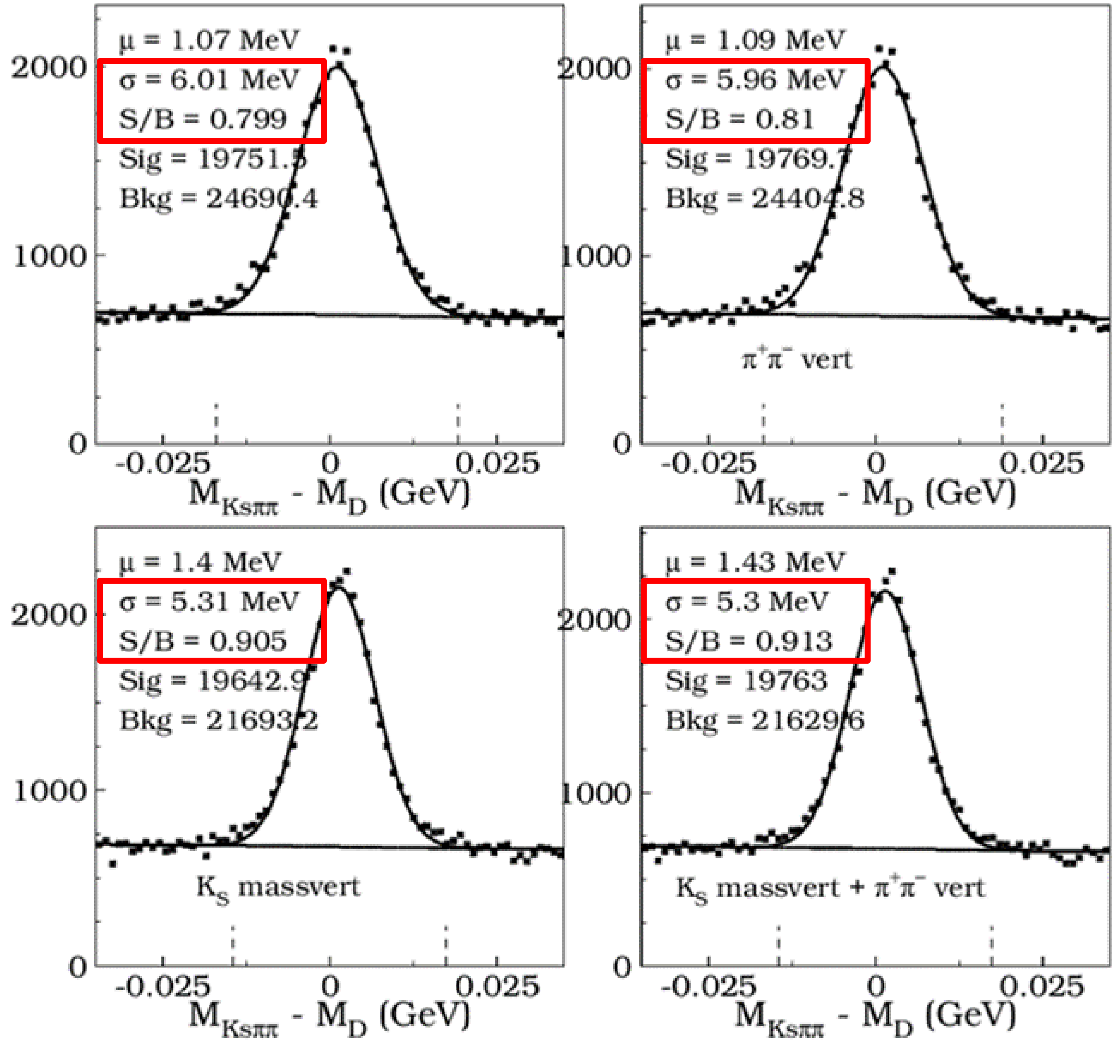




- **Many discoveries were based on these constraint fit, by**
  - **Reducing the background level**
  - **Improving the mass resolution**

An example of Kinematic constraint fit

- **Effect of mass and vertex constraint fit to improve mass resolution in $D^0 \rightarrow K_S \pi^+ \pi^-$ decay chain.**



Generalised form of kinematic constraints

- **Measured variables**
  - **N : number of measured variables (input)**
  - **m_0 : measurements of these variable, N-dimensional (input)**
  - **V : covariance matrix, N × N (input)**
  - **m : vector of fit values of measured variables, N-dimensional**
  - **The χ^2 ansatz, $\chi^2 = (m - m_0)^T V^{-1} (m - m_0)$**
- **Unmeasured variables**
  - **J : number of unmeasured variables (input)**
  - **p : vector of unmeasured variable values, J-dimensional**
- **Constraints**
  - **K : number of constraints (input)**
  - **f : vector of constraint functions, K dimensional (input)**
- **Each constraint is a function of measured and unmeasured variables. When constraint is satisfied,**

$$f_k\left(m_1, m_2, \dots \dots m_N, p_1, p_2, \dots . p_j\right) = 0 \; for \; k = 1, \dots, K$$

- **Lagrange multipliers : λ vector of multipliers, K-dimensional**

Generalised form of kinematic constraints

- The constraints equation can be linearised as
  - $\boldsymbol{f(m,p) = f(m_0,p_0) + A\cdot\delta m + B\cdot\delta p = 0}$ **where** $\boldsymbol{A = \partial f/\partial m|_{m=m_0,p=p_0}}$,
    $\boldsymbol{B = \partial f/\partial p\big|_{m=m_0,p=p_0}}$, $\boldsymbol{\delta m = m - m_0}$ ***and*** $\boldsymbol{\delta p = p - p_0}$
- The best estimates of measured and unknown quantities are obtained by minimising $\chi^2(m,p,\lambda) = (m-m_0)^T V^{-1}(m-m_0) + 2\,\lambda^T f(m,p)$
- Set all partial derivatives to zero:

1) $\boldsymbol{\frac{\partial\chi^2}{\partial m} = \left[2(m-m_0)^T V^{-1} + 2\lambda^T A\right] = 0 \Longrightarrow V^{-1}\delta m = -A^T\lambda \rightarrow \delta m = -VA^T\lambda}$
(⟸ step 3)

2) $\boldsymbol{\frac{\partial\chi^2}{\partial p} = 2\lambda^T B = 0}$

3) $\boldsymbol{\frac{\partial\chi^2}{\partial\lambda} = f(m,p) = 0}$

In general, a system of nonlinear equations, N + J + K equations with N + J + K unknowns, i.e., $\delta m, \delta p, \delta\lambda$

Generalised form of kinematic constraints

- From the third constraint, $\boldsymbol{f(m,p) = 0 = f(m_0, p_0) + A\cdot\delta m + B\cdot\delta p \implies}$
  $\boldsymbol{const\ - A\,V\,A^T\lambda + B\delta p = 0 \implies \lambda = \left(AVA^T\right)^{-1}[const\ + B\cdot\delta p]} \Longleftarrow$ (***Step* 2**)
  $$\delta m = -VA^T(AVA^T)^{-1}[const + B\cdot\delta p]$$
- From second constraints, $\lambda^T B = 0 \to B^T\lambda = 0;$
  - $\boldsymbol{B^T\left(AVA^T\right)^{-1}[const] + B^T\left(AVA^T\right)^{-1} B\,\delta p = 0}$
  - $\boldsymbol{\delta p = -\left[B^T\left(AVA^T\right)^{-1}B\right]^{-1} B^T\left(AVA^T\right)^{-1}[const]} \Longleftarrow$ (***Step* 1**)
- These give the solution for N + J + K unknowns, i.e., i.e., $\delta m, \delta p, \lambda$

- Unique solution!
- But an approximation was made: need to iterate
- Choose new m_0, p_0 which are based on m & p
- After a few iterations, size of δm & δp get small, change in χ^2 gets small.

Covariance matrices

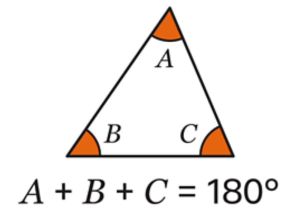
- For the estimation of errors, modify the constraints equation by incorporating the measured values in any steps, i,.e., m_s,
  - $\boldsymbol{f(m,p) = f(m_0,p_0) + A\cdot(m - m_s + m_s - m_0) + B\cdot(p - p_0) = 0}$
  - $\boldsymbol{f(m,p) = const + A\cdot\delta m + A\cdot(m_s - m_0) + B\cdot(p - p_0) = 0}$
- In that scenario and with $W = (AVA^T)^{-1}$
  - $\boldsymbol{\delta p = -[B^T W B]^{-1} B^T W[const + A\cdot(m_s - m_0)]}$
  - $\boldsymbol{\lambda = W[const + A\cdot(m_s - m_0) + B\cdot\delta p]}$
  - $\boldsymbol{\delta m = -VA^T\lambda = -VA^T W[const + A\cdot(m_s - m_0) + B\cdot\delta p]}$
    $$\boldsymbol{= -VA^T W\left[1 - B\left(B^T W\, B\right)^{-1} B^T W\right][const + A\cdot(m_s - m_0)]}$$
  - $\boldsymbol{\chi^2 = (\delta m)^T V^{-1}(\delta m) = \left(-VA^T\lambda\right)^T V^{-1}\left(-VA^T\lambda\right) = \lambda^T A V^T V^{-1} V A^T \lambda = \lambda^T A V^T A^T \lambda = \lambda^T\left(AVA^T\right)\lambda = \lambda^T[f(m_0,p_0) + A\cdot\delta m + B\cdot\delta p] = \lambda^T(A\cdot\delta m + B\cdot\delta p + const)}$

Covariance matrices

- With the approximation that the during the last stages, const = f(m_0; p_0) ≈ f(m; p) = 0, the covariance of
  - Unmeasured variables : $C_p = <(\delta p)(\delta p)^T> = \left(B^T W\, B\right)^{-1}$
  - Measured variables : $C_m = <(\delta m + (m_s - m_0))(\delta m + (m_s - m_0)^T> \approx V\; - VA^T WAV + VA^T W\left[B\left(B^T WB\right)^{-1} B^T W\right] AV$
  - Covariances between fit and unmeasured variables : $C_{p,m} = <(\delta p)(\delta m + (m_s - m_0)^T> = -VA^T WB\left(B^T WB\right)^{-1}$

Derive all these covariance terms

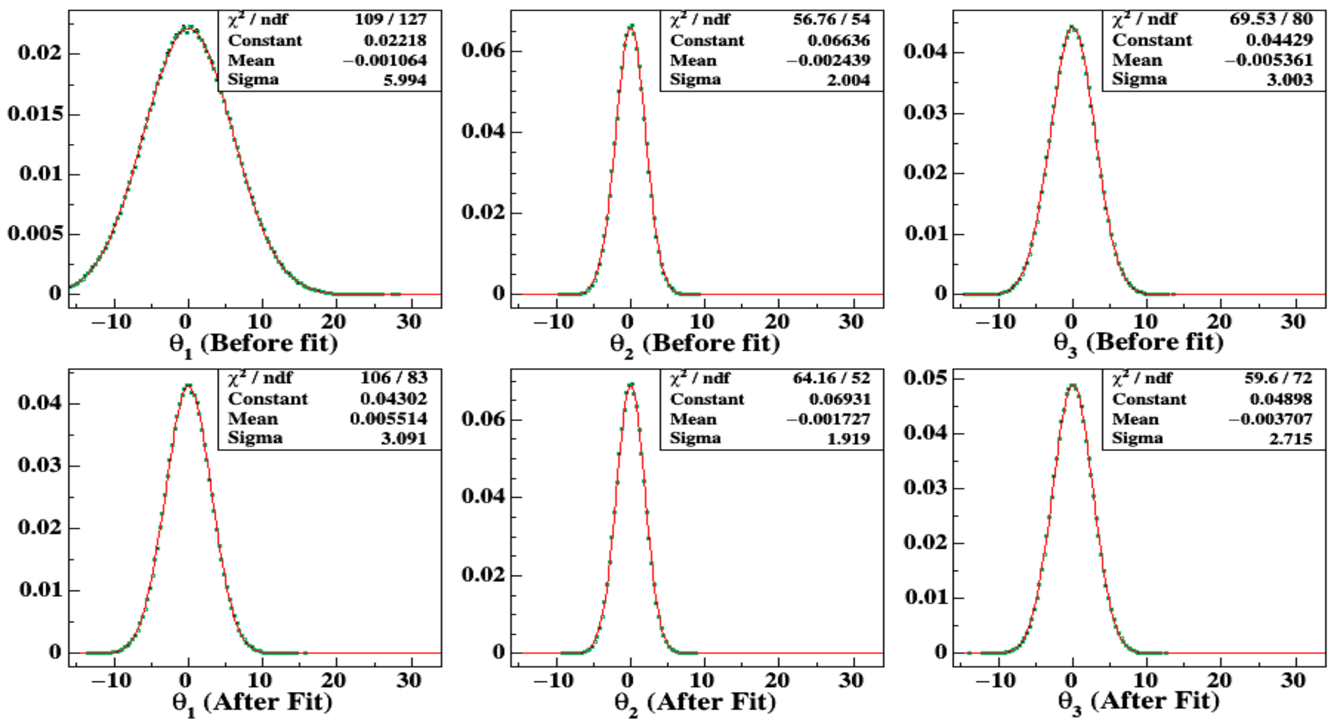
Measurement of three angles of a triangle



- There is no unmeasured variables, J=0
- Measured variables are three angle of triangle, N=3, corresponding $m = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$
- The error Matrix, $V = \begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix}$
- Number of constraint is one, K=1 and $f(m) = \theta_1 + \theta_2 + \theta_3 - 180^\circ = 0$
- Derivatives, $A = df/dm = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$
- $W = \left(AVA^T\right)^{-1} = \left[\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right]^{-1} = 1/\left(\sum_i \sigma_i^2\right)$
- $\lambda = W \cdot f(m_0) = (\theta_{10} + \theta_{20} + \theta_{30} - 180^\circ)/\left(\sum_i \sigma_i^2\right)$
- $\delta m = -VA^T\lambda = -\begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \lambda = -\lambda \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \end{pmatrix}$

Measurement of three angles of a triangle

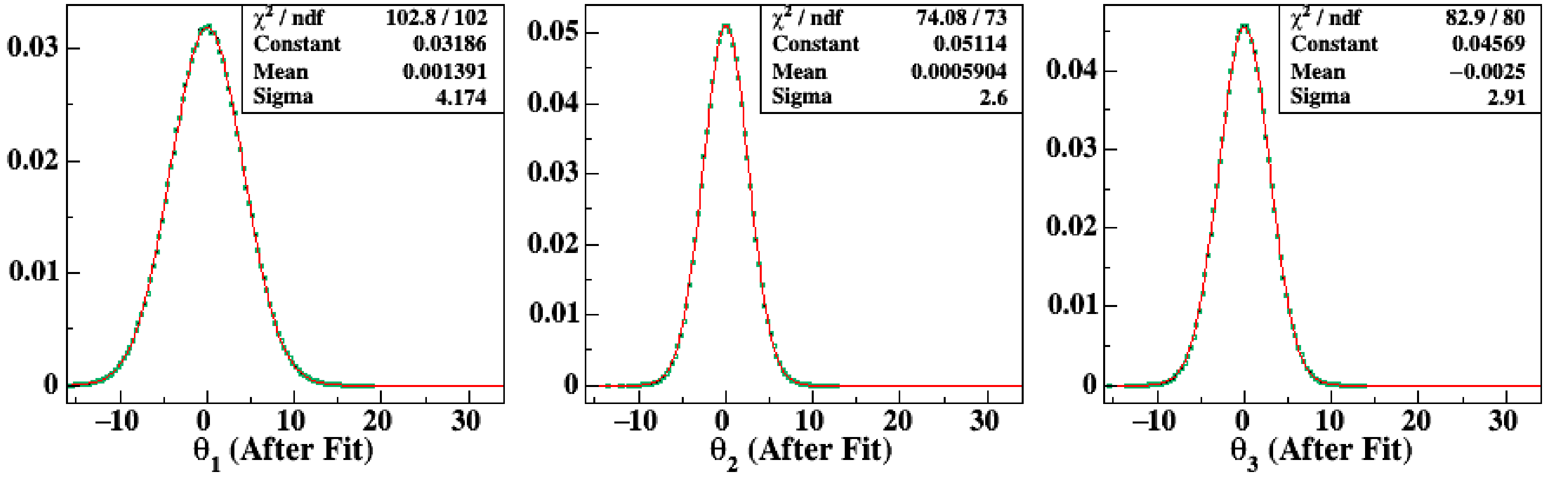
- Updated error matrix, $V_{up} = V - VA^T WAV$
- For an example of $\sigma_1^2 = 6°, \sigma_2^2 = 2°, \sigma_3^2 = 3°$, $V_{up} = \begin{pmatrix} 9.551 & -2.939 & -6.612 \\ -2.939 & 3.675 & -0.735 \\ -6.612 & -0.735 & 7.347 \end{pmatrix}$
- Corresponding to the error in individual measurements are (3.091, 1.917, 2.711)°
- Note that error is independent of input m_i, because the derivatives of constraint eqn, $df/dm_i = 1$
- Solution converges in the 1st iteration
- In general, this is not a constant and it requires an iterative approaches



After the fit errors are reduced, which were expected and also the error on the fitted values are the same as the calculated value, those are also expected.

Measurement of three angles of a triangle

- But, be sure to use the proper errors for all three triangles,
  - e.g., if one uses the average error for all three sides, the same observed data gives worse results.

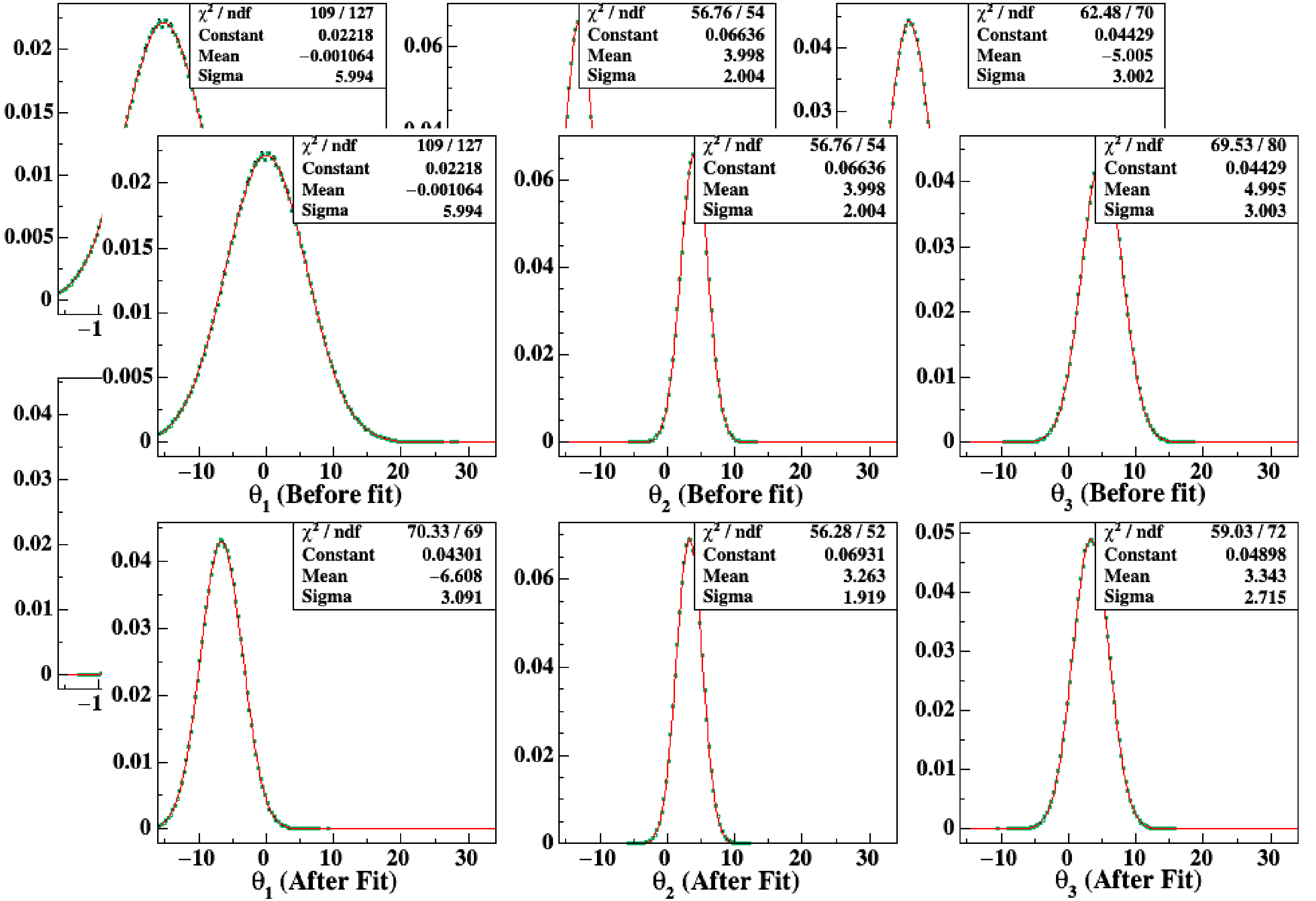


Do we need a complicated matrix solution for this problem ?

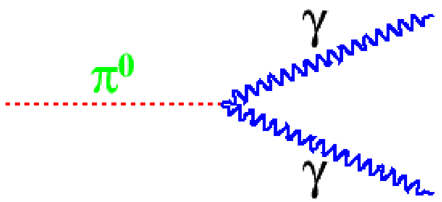
- **Let us use the simple definition of modified χ^2 of this problem,**
  - $\chi^2 = \frac{(\theta_1-\theta_{10})^2}{\sigma_1^2} + \frac{(2-\theta_{20})^2}{\sigma_2^2} + \frac{(\theta_3-\theta_{30})^2}{\sigma_3^2} + 2\lambda(\theta_1+\theta_2+\theta_3-180^\circ)$
  - $\frac{1}{2}\frac{\partial\chi^2}{\partial\theta_i} = \frac{\theta_i-\theta_{i0}}{\sigma_i^2} + \lambda = 0 \Longrightarrow \theta_i = \theta_{i0} + \lambda\cdot\sigma_i^2$
  - $\frac{1}{2}\frac{\partial\chi^2}{\partial\lambda} = (\theta_1+\theta_2+\theta_3-180^\circ) = 0$
  - $\lambda = \frac{180^\circ-(\theta_{10}+\theta_{20}+\theta_{30})}{(\sigma_1^2+\sigma_2^2+\sigma_3^2)}$
  - $\theta_i = \theta_{i0} + \lambda\cdot\sigma_i^2 = \theta_{io} + \left[\sigma_i^2/\left(\sum_i\sigma_i^2\right)\right][180^\circ - (\theta_{10}+\theta_{20}+\theta_{30})]$

- **The result returns back to the exact solution of the Matrix solution $\Rightarrow$ Gives a confidence that the matrix solution is correct and can be used for complicated non-linear problems.**

Warning : It does not take care of any shift in the measurements.



Improvement of P_{π^0} in the reconstruction of the mass of π^0 from two photons

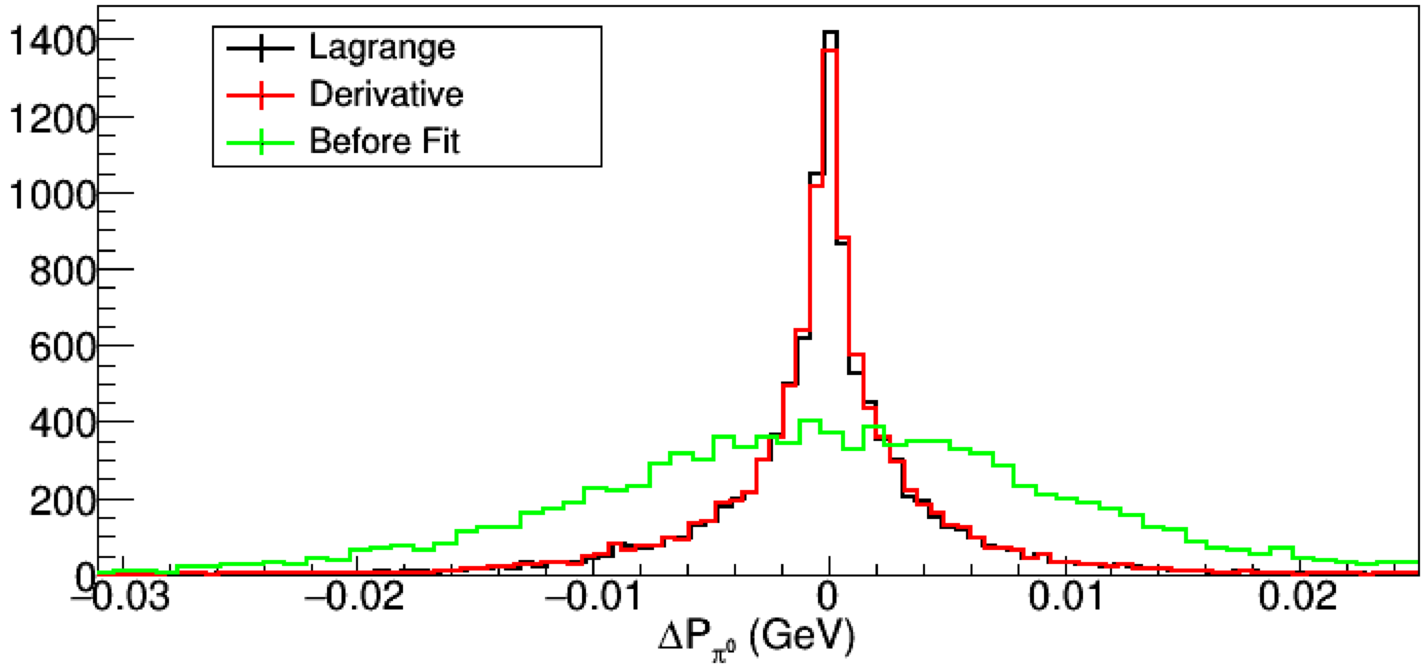


- Measured parameters (y) : $E_1, \theta_1, \phi_1, E_2, \theta_2$ *and* ϕ_2
- **Constraint eqn :** $\boldsymbol{2E_1E_2\,(1\ - cos\theta_{12}) - m^2_{\pi^0} = f(y) - m^2_{\pi^0} = 0}$, **with** $\boldsymbol{cos\theta_{12} = sin\theta_1 \sin\theta_2\ cos(\phi_1 - \phi_2) + cos\theta_1\ cos\theta_2}$
- Using Lagrange multipliers
  - $\boldsymbol{F(y_0) = 2E_1E_2(1 - cos\theta_{12})}$
  - $\boldsymbol{\lambda = \left(AVA^T\right)^{-1} F(y_0)}$, $\boldsymbol{\delta y = -VA^T\lambda}$ $\boldsymbol{\chi^2 = \lambda^T\,[F(y_0) + A\delta y]}$, **where** $\boldsymbol{A = \partial F/\partial y}$, $\boldsymbol{V}$ **is the covariant error matrix, of** $\boldsymbol{y}$ $(E_1, \theta_1, \phi_1, E_2, \theta_2, \phi_2)$

Improvement of P_{π^0} in the reconstruction of the mass of π^0 from two photons

- An alternative way to use the constraint eqn is
  - $\chi^2 = \sum_{i=1}^{6} \frac{(y_i - y_{i0})^2}{\sigma_{y_i}^2} + \frac{\left(f(y) - m_{\pi^0}^2\right)^2}{\sigma_{\pi^0}^2}$
  - $\frac{\partial \chi^2}{\partial y_i} = 2\frac{y_i - y_{i0}}{\sigma_{y_i}^2} + 2\frac{\partial f}{\partial y_i}\frac{\left(f(y) - m_{\pi^0}^2\right)}{\sigma_{\pi^0}^2} = 0$
  - $\frac{Dy_i}{\sigma_{y_i}^2} = -\frac{\partial f}{\partial y_i}\frac{\left[f(y_0) + \frac{\partial f}{\partial y_i} Dy_i - m_{\pi^0}^2\right]}{\sigma_{\pi^0}^2}$, where $Dy_i = y_i - y_{i0}$
- In matrix notation,
  - $V^{-1} \cdot Dy = A^T \cdot (m_{\pi^0}^2 - f(y_0) - A \cdot Dy)/\sigma_{\pi^0}^2$
  - $(V^{-1} + A^T \cdot A/\sigma_{\pi^0}^2) \cdot Dy = A^T \cdot (m_{\pi^0}^2 - f(y_0))/\sigma_{\pi^0}^2$
  - $Dy = \left(V^{-1} + A^T \cdot A/\sigma_{\pi^0}^2\right)^{-1} \cdot A^T \cdot (m_{\pi^0}^2 - f(y_0))/\sigma_{\pi^0}^2$
- With an iterative procedure, recalculate parameters, until the change in m_π and χ^2 is lower than a certain value (e.g., 10^{-5}).
- Updated Error matrix of measured variables, $V_{UP} = V - VA^T(AVA^T)^{-1}AV$
- Error on the fitted pion momentum, $\sigma_{P_{\pi^0}} = A' V_{UP} A'^T/(2 P_{\pi^0})$, where $A' = \partial P_{\pi^0}^2/\partial y$
- The calculated values of error on the momentum can be compared with the resolution of fitted pion momentum.

Improvement of P_{π^0} in the reconstruction of the mass of π^0 from two photons



- There is a difference in two procedures (in the triangle case, they were exact), primarily due to the steps in iterative processes.
- Any two different numerical procedures have different results (expected within the estimated error)
- Input errors vary with events, thus σ_{π^0} also varies i.e., superposition of many Gaussian functions with different widths $\Rightarrow$ Non Gaussian distributions of ΔP_{π^0}

Extension to three body decays

- $B \to J/\psi\ (\to \mu\mu)K$ channel and extension for the searches of new particles (e.g., X/Y) using $B \to X(\mu\mu)K$ or $Y \to J/\psi\ (\to \mu\mu)K$
- Using mass constraint fit of B meson, you can improve the mass resolution of X(J/ψ)
  - Exactly same analogy for $t \to H^+\ (c\bar{s})b\ of\ unknown\ mass\ of\ H^+$
- Similarly mass constraint fit of J/ψ meson can improve the mass resolution of Y (B).
  - Measurement of top mass using W-mass constraint of $t \to W(q\bar{q}')b$ decay

$$\boldsymbol{B \to J/\psi\ (\to \mu\mu)K}$$

- The masses of B, J/ψ , K and μ are 5.279, 3.094, 0.494 and 0.1067 GeV respectively, where momentum vector of B meson is (2.0x, 0.1x, 0.1x) GeV, x=[0,1].
- Resolutions : $\sigma_p/p = (0.0032 \oplus 0.0019p)$ and $\sigma_\theta/\sigma_\phi$ = 2/1 mrad for all stable particles.

$$\mathbf{t} \to \boldsymbol{W(\to q\bar{q})b}$$

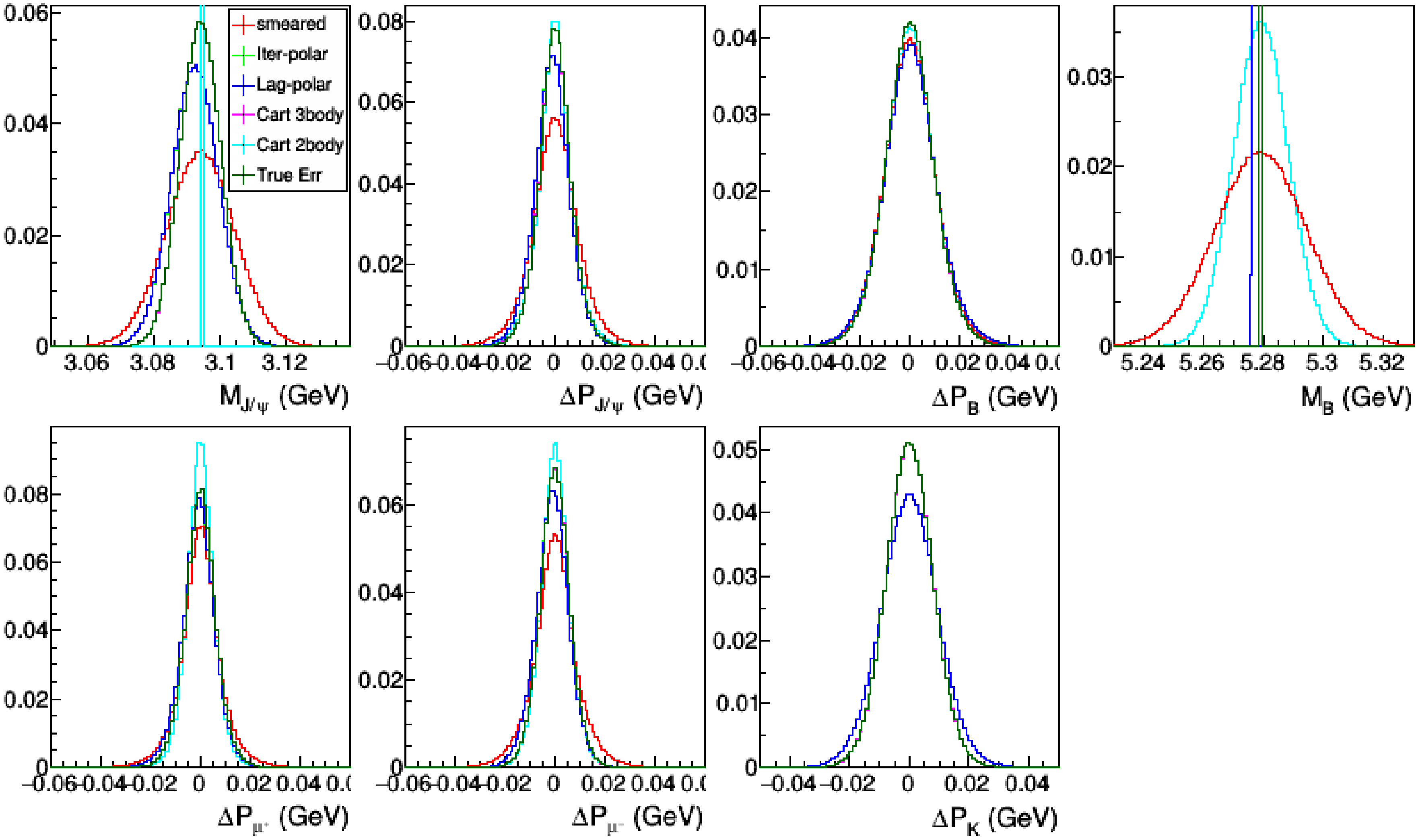
- The masses of t, W, b and $q/\bar{q}$ are 172, 80.44, 5.28 and 0 GeV respectively, where momentum vector of top quark is (500x, 500x, 500x) GeV, x=[0,1].
- Resolutions : $\sigma_E/E = \left(\ 2.0 \oplus 125/\sqrt{E} \oplus 56/E\right)\%$ and $\sigma_\theta/\sigma_\phi$ = 10/10 mrad for all stable particles (resolution of jets)

Different constraints

1. Iterative Polar (three body mass)
2. Lagrange Polar (three body mass)
3. Lagrange Cartesian (three body mass)
4. Lagrange Cartesian (three body mass)

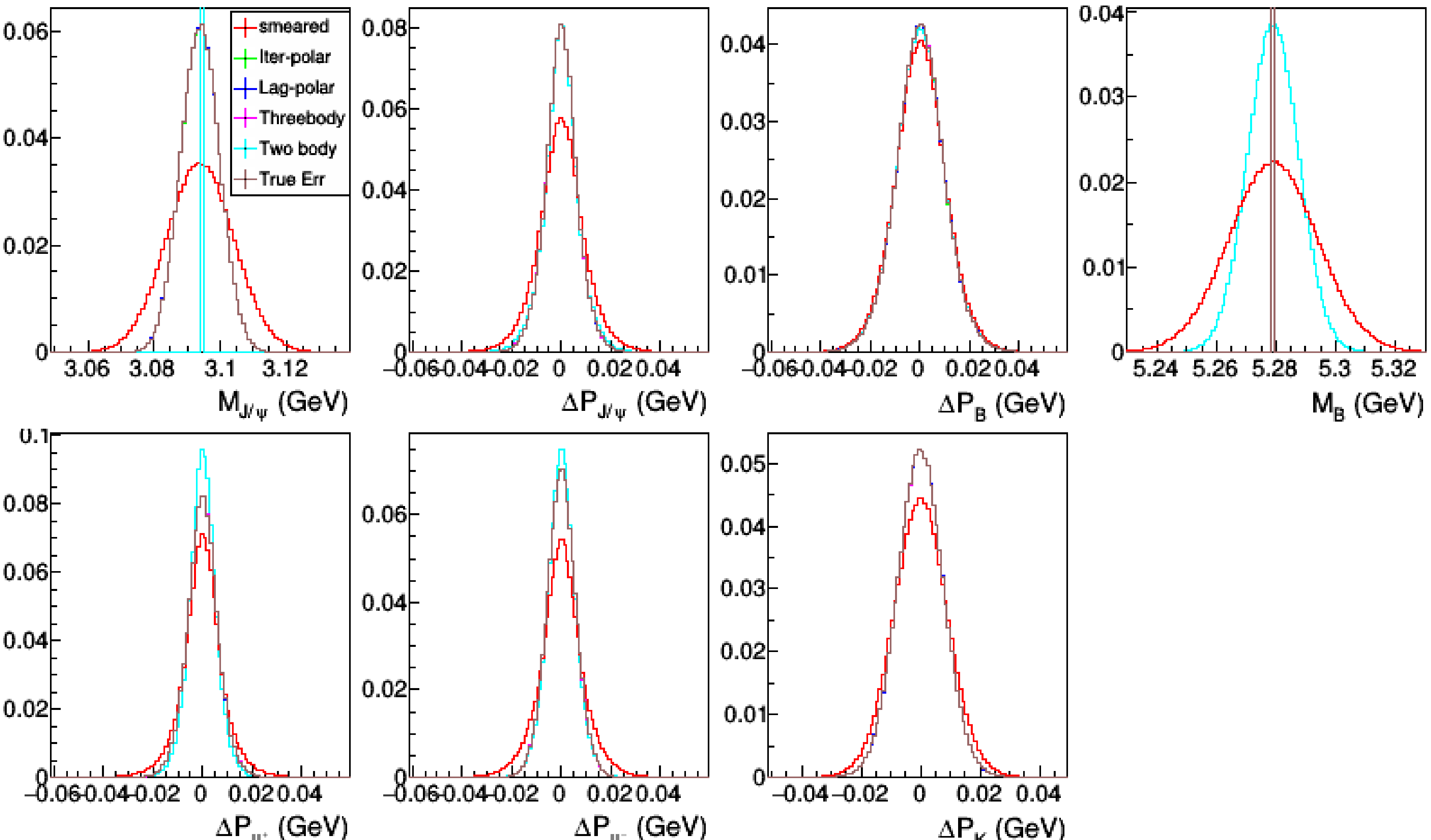
5. Lagrange Cartesian (two & three body masses)
6. Lagrange Cartesian (Energy of three body system)
7. Lagrange Cartesian (Energy and momentum of three body)
8. Lagrange Cartesian (Energy and momentum of three body + mass of two body)
9. Lagrange Cartesian (Energy of three body and mass of two body)
10. Lagrange Cartesian (Energy and mass of two body)
11. Lagrange Cartesian (three body mass, but error using true momentum

Extension to three body decays



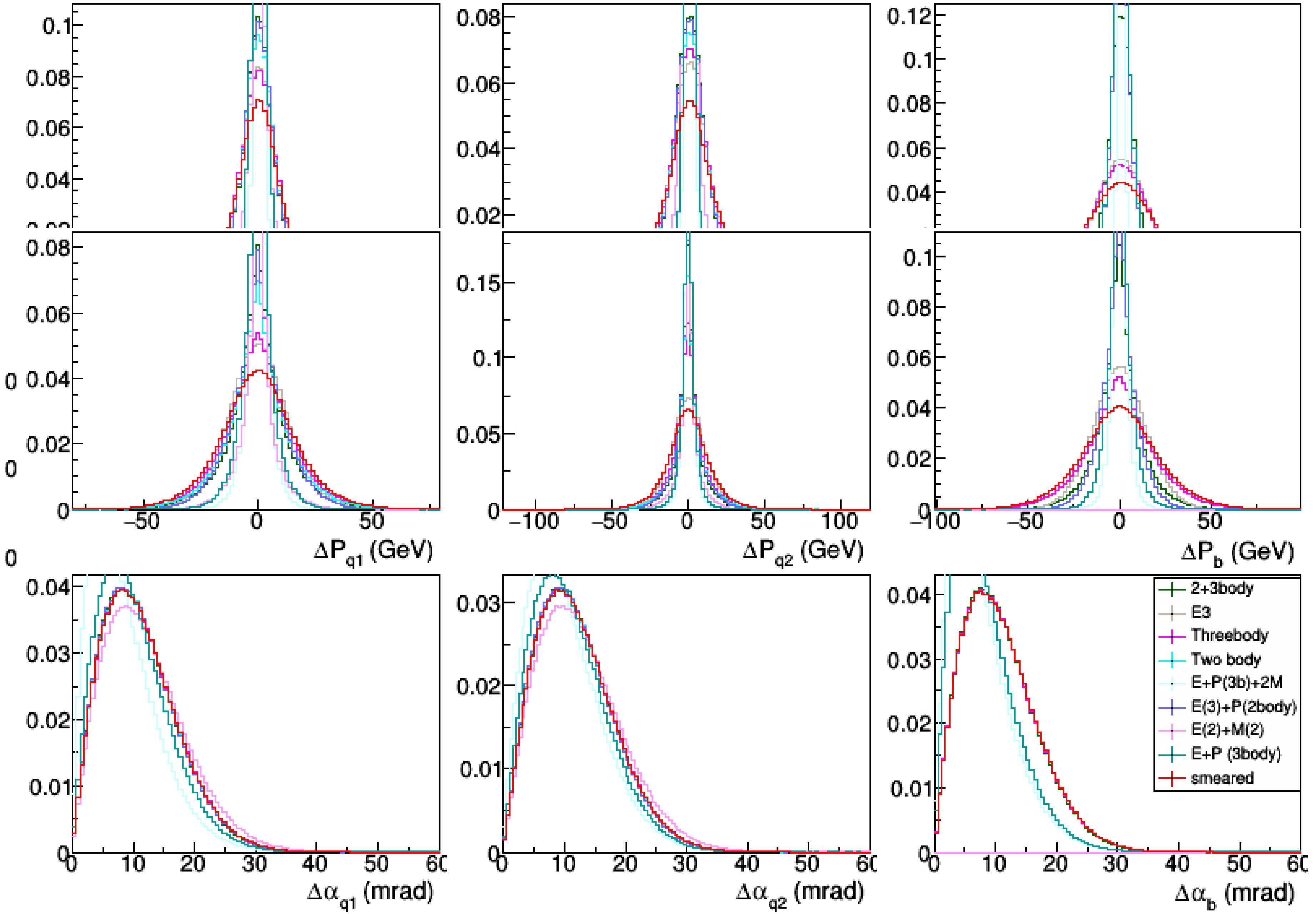
- Differences with cartesian and polar angle constraints are using fit parameters as momentum and energy respectively, but the resolution parameters were taken the same.

Changing momentum as free parameters in all cases



All are nearly the same, but as expected any two independent numerical analyses need not give exactly the same results.

Changing momentum as free parameters in all cases



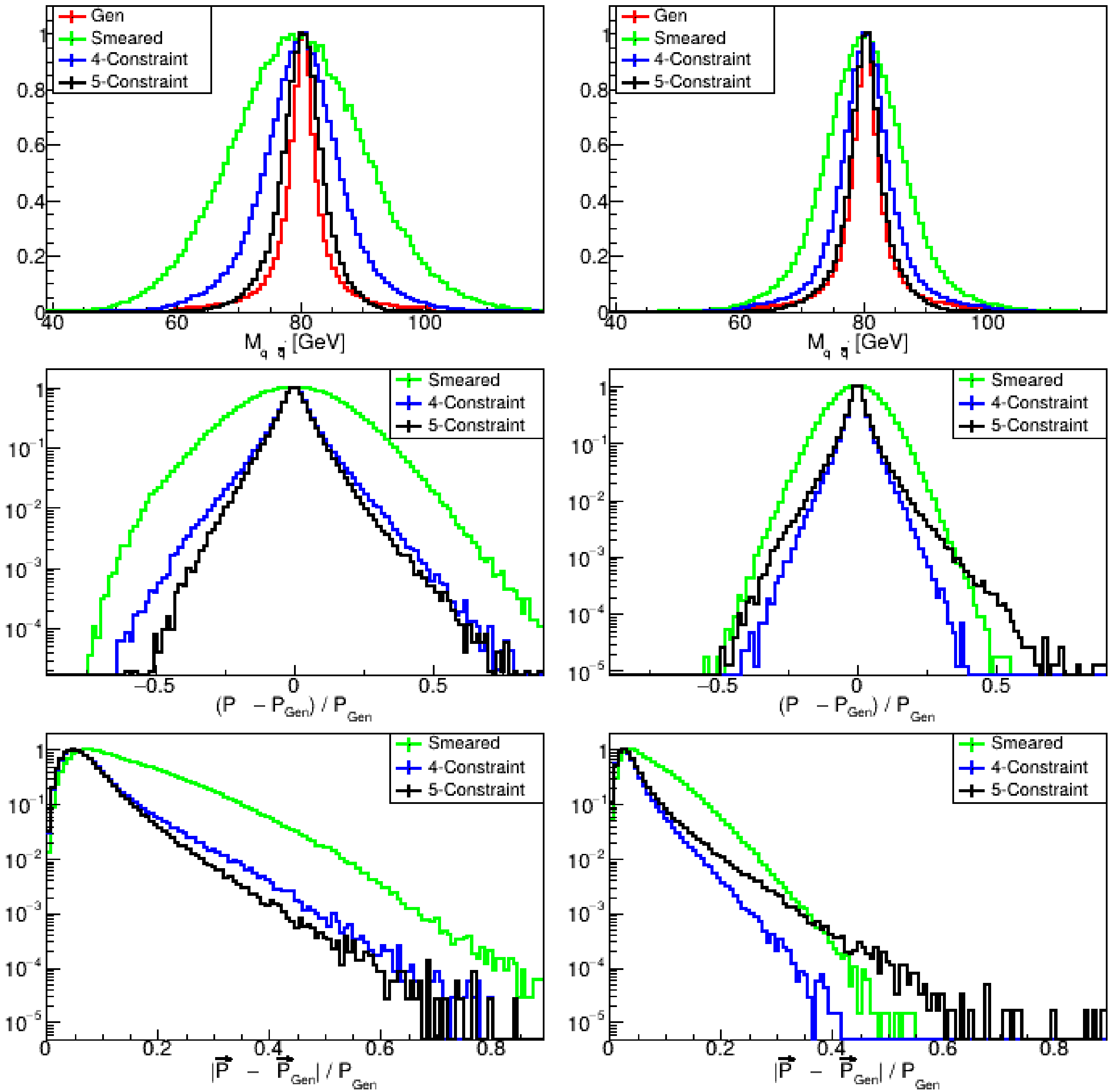
Another application of constraint fit

- An experiment wants to measure the mass of one particle which is produced as a pair in electron-positron collisions and then each of them decays into a pair of objects $e^+e^- \rightarrow W^+(\rightarrow q\overline{q'})W^-(\rightarrow q''\bar{q}''')$
- Measure the energies of the 4 jets coming from the 4 quarks and their angles. Jet directions are well measured but the energies are not so well measured. So try to utilise some of the known phenomena and see if this can help the measurements,
  - Energy and momentum are conserved in the particle production and decay processes
  - Since electrons and positrons collide head-on, the initial state momentum = 0 and initial state energy = $2E_{beam}$
  - The two W's might have the same mass
- Again all quantities are measured with some precision has unmeasured ($\vec{a}$) entries
- Variables ($\vec{m}$) are $E_i, \theta_i, \phi_i, \beta_i$, where $\beta_i = \sqrt{E_i^2 - m_i^2}/E_i$ for i=1,..4.

Another application of constraint fit

- Constraint equations are
- $f_1 : \sum_i E_i \ - 2\ E_{beam} = 0$
- $f_2 : \sum_i \beta_i E_i\ sin\theta_i\ cos\phi_i = 0$
- $f_3 : \sum_i \beta_i E_i\ sin\theta_i\ sin\phi_i = 0$
- $f_4 : \sum_i \beta_i E_i\ \ cos\theta_i = 0$
- $f_5 : (E_1 + E_2)^2 - (E_3 + E_4)^2\ - (\beta_1 E_1 sin\theta_1 cos\phi_1 + \beta_2 E_2 sin\theta_2 cos\phi_2)^2 + (\beta_3 E_3 sin\theta_3 cos\phi_3 + \beta_4 E_4 sin\theta_4 cos\phi_4)^2\ - (\beta_1 E_1 sin\theta_1 sin\phi_1 + \beta_2 E_2 sin\theta_2 sin\phi_2)^2 + (\beta_3 E_3 sin\theta_3 sin\phi_3 + \beta_4 E_4 sin\theta_4 sin\phi_4)^2\ - (\beta_1 E_1 cos\theta_1 + \beta_2 E_2 cos\theta_2)^2 + (\beta_3 E_3 cos\theta_3 + \beta_4 E_4 cos\theta_4)^2 = 0$
- Derivative matrix, $\partial f/\partial m$ (four constraint fit) is
- $$\begin{pmatrix} 1 & \beta_i sin\theta_i cos\phi_i & \beta_i sin\theta_i sin\phi_i & \beta_i cos\theta_i \\ 0 & \beta_i E_i cos\theta_i cos\phi_i & \beta_i E_i cos\theta_i sin\phi_i & -\beta_i E_i sin\ \theta_i \\ 0 & -\beta_i E_i sin\theta_i sin\phi_i & \beta_i E_i sin\theta_i cos\phi_i & 0 \\ 0 & E_i sin\theta_i cos\phi_i & E_i sin\theta_i sin\phi_i & E_i cos\theta_i \end{pmatrix}$$
- Measurement was done with data using $\sqrt{s} = 210\ GeV$ for σ_E and $\sigma_\theta/\sigma_\phi$ of $[(1.0/\sqrt{E} \oplus 0.08), 40\ mrad]$ and $[(0.5/\sqrt{E} \oplus 0.04), 20\ mrad]$ respectively.
- Uses events with number of iteration $\leq$10, where the termination condition is $\Delta\chi^2 \leq 10^{-5}$

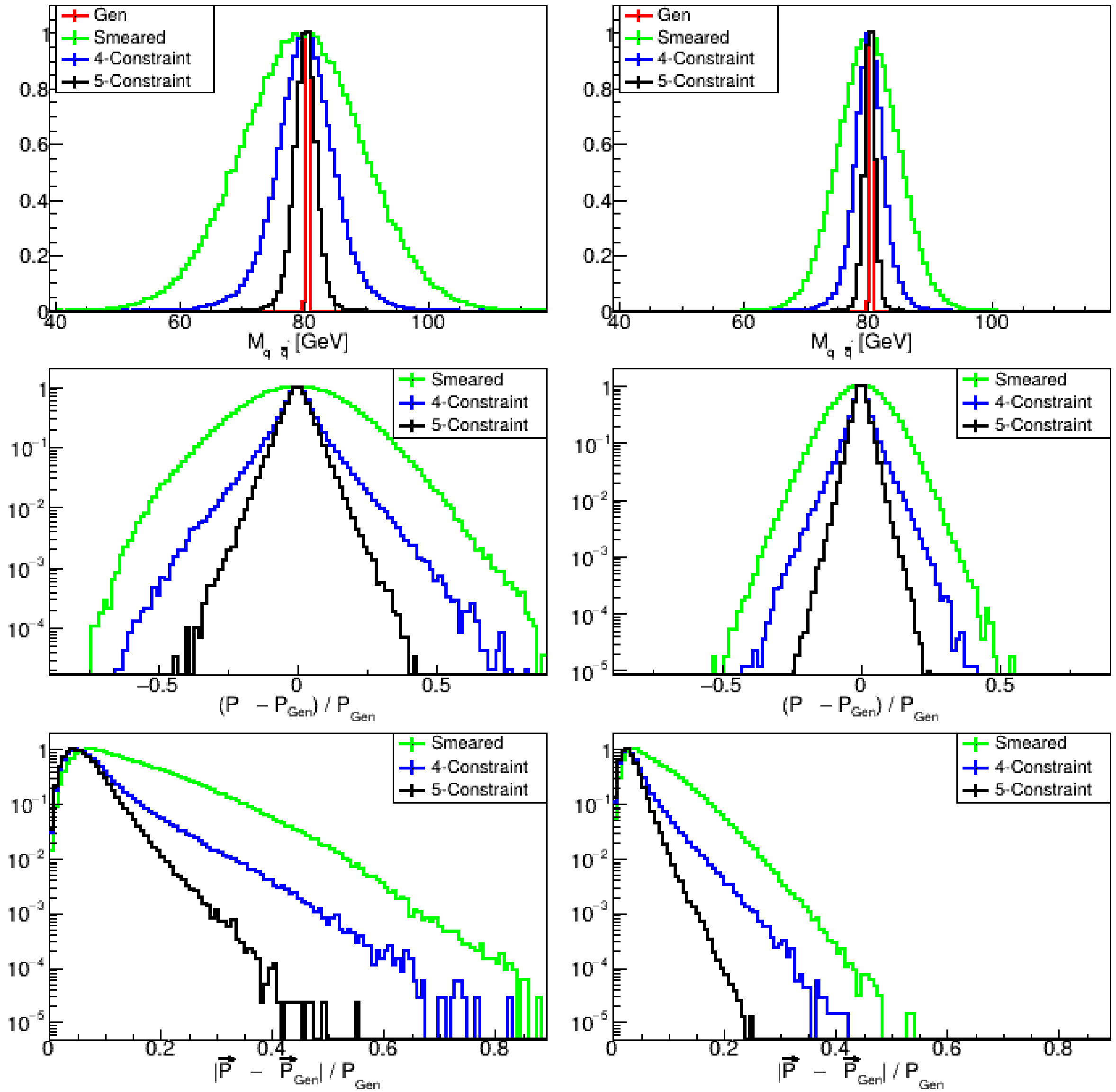
$e^+e^- \to W^+(\to q\bar{q}')W^-(\to q''\bar{q}''')$



- The 5-constraint fit
- Drastically improved the W-mass measurement
- But, biased the both massed towards the central value
  - Underestimate the width of W-boson.
- Also, wrong measurement of the 3-vector of individual jets due to mass constraint.

Pair production of a narrow width particle

- With the assumption that width of W is negligible (e.g., 10 MeV)

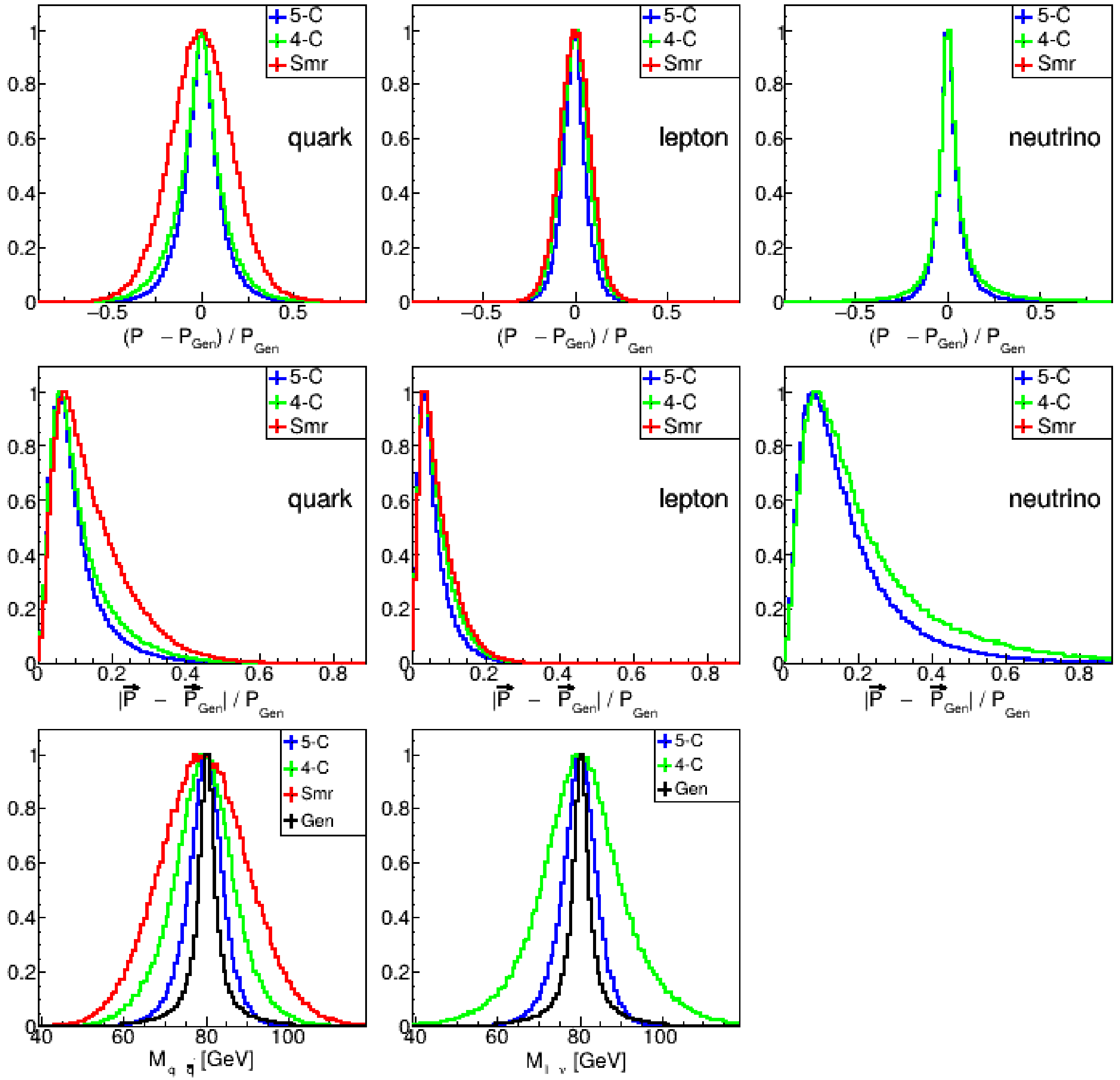


- Constraint mass is improved very much wrt measured mass.
- For the proper concept (masses are equal) the 3-vector of individual jets also improved.
- Reminder : For kinematic fit needs the correct constraint and proper estimation of error.

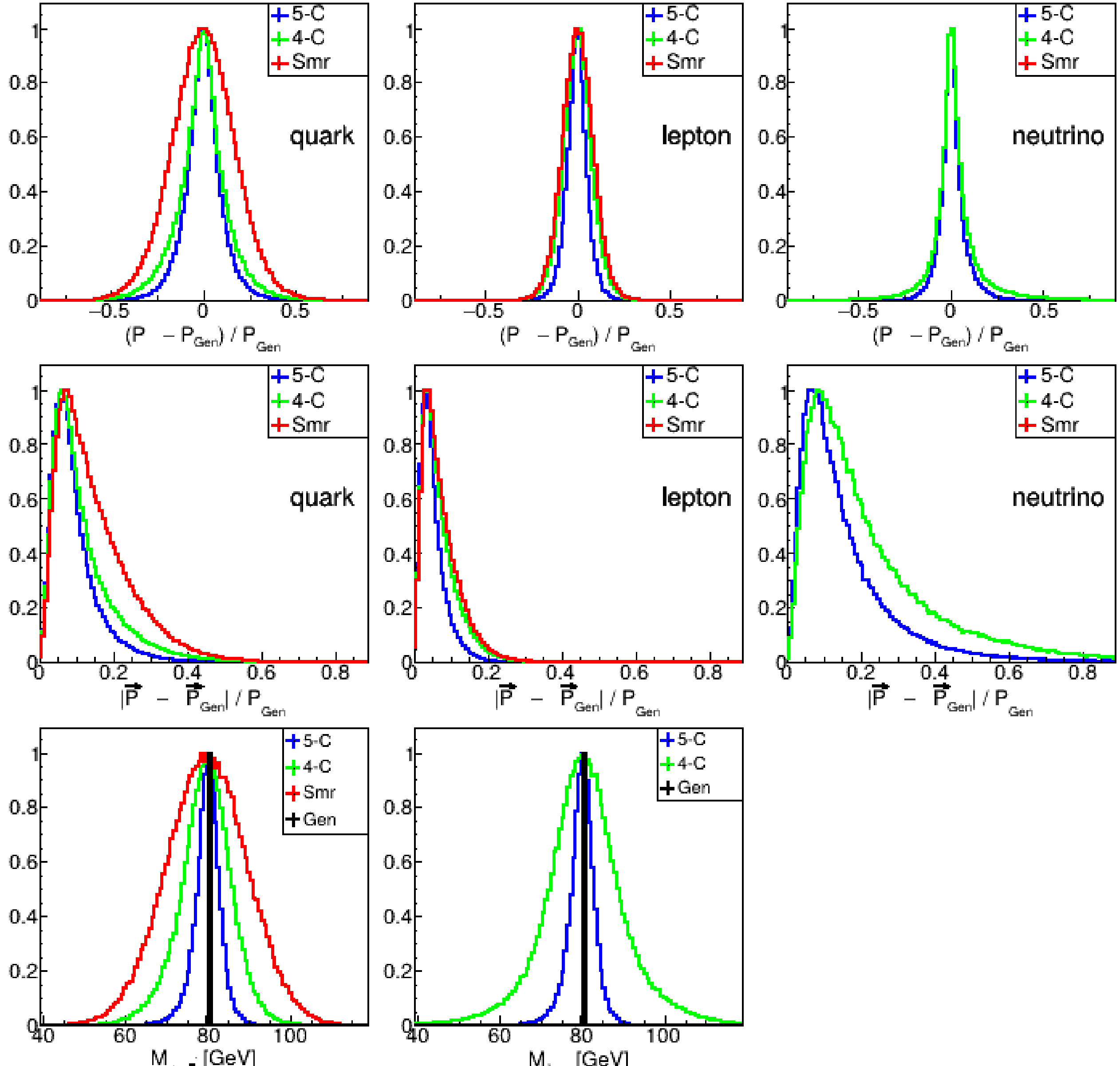
Constraint fit with unmeasured variables

- $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}' + \ell\nu$, where unmeasured 3-Vector of neutrino is use in constraint equation.
  - Unmeasured variables are three momentum components of neutrino, J = 3
  - Measured variables are 4-vectors of two jets and 3-Vector of lepton, N = 11
  - Number of constraint equations for the option of equal/oat W-masses in fit, i.e., K = 5/4

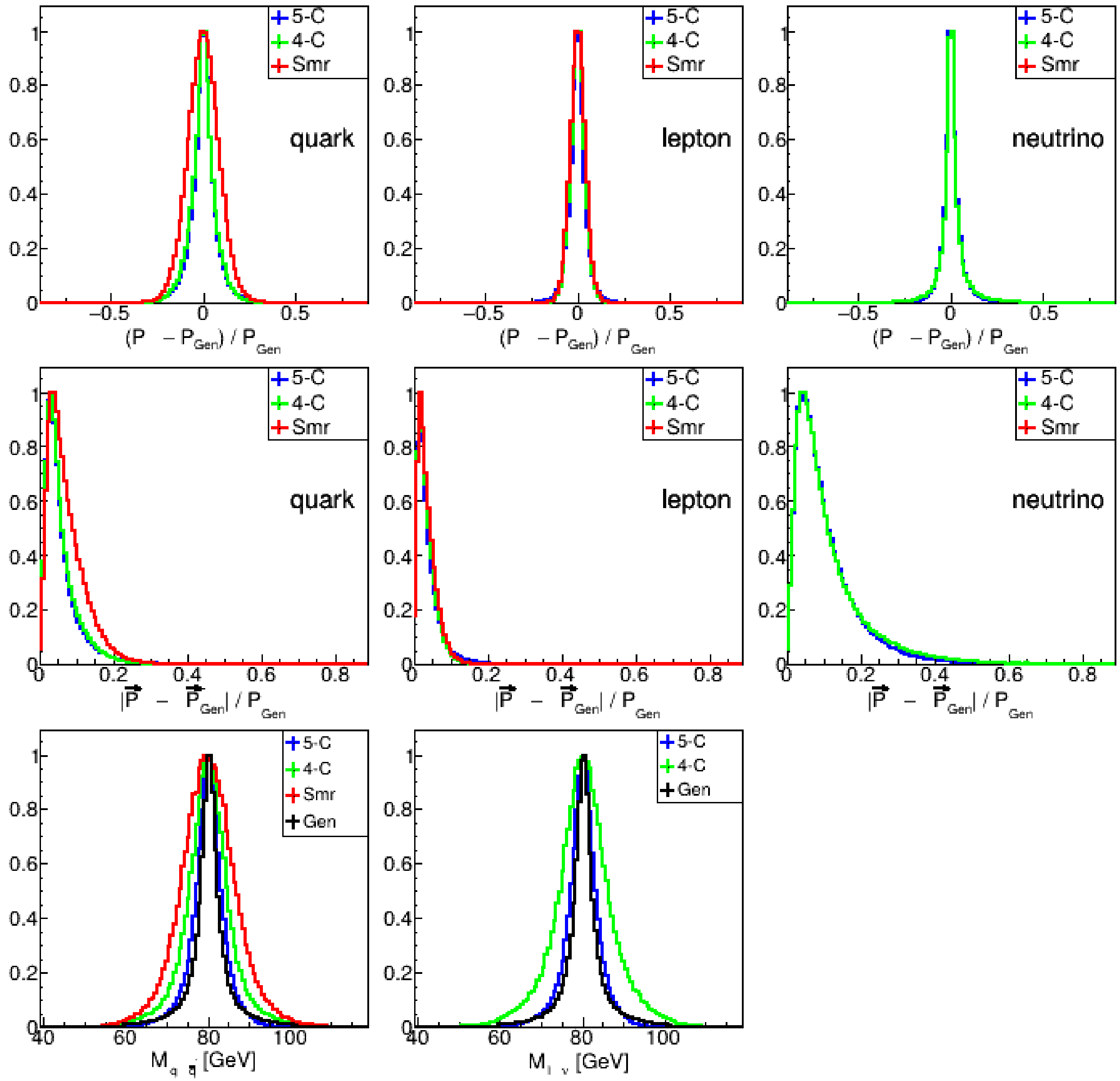
$[\sigma_E/E = (1.0/\sqrt{E} \oplus 0.08), \sigma_\theta/\sigma_\phi = 40mrad] + \Gamma_W = 2.01\,GeV$



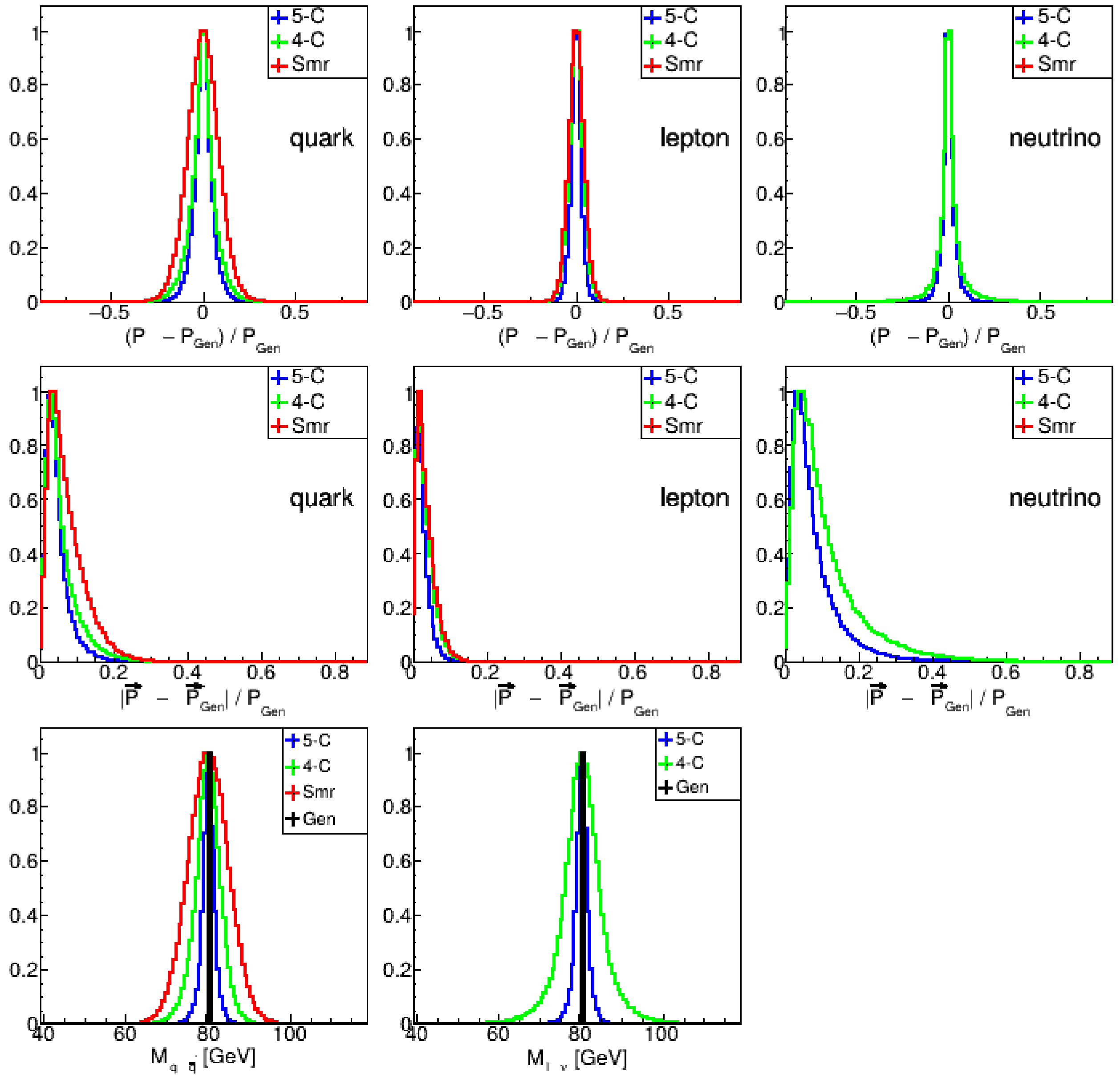
$$[\sigma_E/E = (1.0/\sqrt{E} \oplus 0.088), \sigma_\theta/\sigma_\phi = 40mrad] + \Gamma_W = 10\ MeV$$



$$[\sigma_E/E = (0.5/\sqrt{E} \oplus 0.04), \sigma_\theta/\sigma_\phi = 20mrad] + \Gamma_W = 2.01\ GeV$$



$$[\sigma_E/E = (0.5/\sqrt{E} \oplus 0.04), \sigma_\theta/\sigma_\phi = 20mrad] + \Gamma_W = 10\ MeV$$



Hands on Session

- Use the simple triangle problem,
  - Put resolution of three angles as 3, 5 and 7°.

- Improvement of P_{π^0} in the reconstruction of two photons. Verify that you have smeared all parameters properly by comparing the smeared mass and momentum distribution of pions.
  - Generate a set of 4-momenta for a pair of photons from decays of 5 GeV π^0 going in a direction given by θ = 60° and ϕ = 45°,
  - Assume the photons are measured in a calorimeter with an energy resolution of 1%, theta and phi resolutions of 0.5°,
  - Reconstruct back the π^0 momenta using the constraint equation that the effective mass of the two-photon system will be π^0 mass (0.1349739 GeV),
  - Use a sample of 1000 photon pairs and plot the measured and fitted mass of the two-photon system and
  - Also show the resolution of P_{π^0} for the measured and fitted 4-vectors of photons

  - **Three body problem,** $B \rightarrow J/\psi\ (\rightarrow \mu\mu)K$
  - $\boldsymbol{e^+e^- \rightarrow W^+(\rightarrow q\bar{q}')W^-(\rightarrow q''\bar{q}''')}$