Multi-Component Dark Matter: Identifying at Collider

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[Dark Matter\(DM\)](#page-1-0)

What we know about DM ?

- \checkmark Non-luminous and non-baryonic.
- \checkmark ~ 24% of our Universe made of DM.
- $\sqrt{\ }$ Massive and interact gravitationally.
- \checkmark Stable on cosmological time scale. SM fails to accomodate DM.

Thermal DM : WIMP

- Kinetic Eqlbm. $(T_{DM} = T_{SM})$ $DM SM \leftrightarrow DM SM$
- Chemical Eqlbm. $(n_{DM}^{eq.} = n_{SM}^{eq.})$ DM $DM \leftrightarrow SM SM$

when $\Gamma << H$: DM DM \rightarrow SM SM DM becomes relic.

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	- larger allowed parameter space \ldots (sB, PG.., JHEP 03 (2020) 090)

Single Comp. DM: scalar singlet ϕ and inert $\Phi = \left(H^+ - \frac{H^0 + iA^0}{\sqrt{2}}\right)^T$

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Two Comp. DM: $DM1(\phi) + DM1(\phi) \rightarrow DM2(\Phi) + DM2(\Phi)$

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Ref: JHEP12(2022)049 (arXiv: 2202.12097) Subhaditya Bhattacharya, P Ghosh, Jayita Lahiri and Biswarup Mukhopadhyaya

[OSDL+ME @ ILC](#page-18-0) $e^- e^+ \rightarrow X + \ell^+ \ell^-$

• Dark sector with $SU(2)_L$ multiplet: $\{X^0(m_{\text{DM}}), X^{\pm}(m_{\text{DM}} + \Delta m), \ldots\}$

 $e^- e^+ \to X^+ X^-; (X^- \to \ell^- \ \overline{\nu_\ell} \ X^0), (X^+ \to \ell^+ \ \nu_\ell \ X^0)$ (with $\ell = e, \mu$)

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- The peak of the distribution shifts to the right with an increase in splitting (Δm) for a fixed mass m_{DM} .
- Increasing mass (m_{DM}) doesn't shift the peak position; it shifts the distribution endpoint.

- • Two component DM with:
	- DM1: $\{X_1^0(m_{\text{DM1}}), X_1^{\pm}(m_{\text{DM1}} + \Delta m_1)\} : X_1^- \to \ell^- \overline{\nu_{\ell}} X_1^0$
	- DM2: $\{X_2^0(m_{\text{DM2}}), X_2^{\pm}(m_{\text{DM2}} + \Delta m_2)\} : X_2^- \to \ell^- \overline{\nu_{\ell}} X_2^0$
	- Both having identical collider signal: $\ell^+ \ell^- + 0j + \text{ME}$.

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\mathcal{G} = \mathrm{SM} {\otimes} (\mathcal{Z}_2 \otimes \mathcal{Z}_2')
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 $'_{2})$

Scalar DM (ϕ^0): Inert doublet $\Phi = \begin{pmatrix} \phi^+ & \frac{\phi^0 + iA^0}{\sqrt{2}} \end{pmatrix}$ $\big)^T$; $\Phi \stackrel{\mathcal{Z}_2}{\longrightarrow} -\Phi;$ with $m_{\phi^0} < m_{\phi^{\pm}} < m_{A^0}$.

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Fermion DM (ψ^0): Lepton doublet $\Psi = (\psi \quad \psi^-)^T +$ Lepton Singlet χ_R ; $(\Psi, \chi) \stackrel{\mathcal{Z}'_2}{\longrightarrow} (-\Psi, -\chi);$ with $m_{\psi^0} < m_{\psi^\pm} < m_{\psi_2} < m_{\psi_3}$.

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A Model with two component DM: $'_{2})$ **Scalar DM** (ϕ^0): Inert doublet $\Phi = \begin{pmatrix} \phi^+ & \frac{\phi^0 + iA^0}{\sqrt{2}} \end{pmatrix}$ $\big)^T$; $\Phi \stackrel{\mathcal{Z}_2}{\longrightarrow} -\Phi;$ with $m_{\phi^0} < m_{\phi^{\pm}} < m_{A^0}$. **Fermion DM** (ψ^0): Lepton doublet $\Psi = (\psi \quad \psi^-)^T +$ Lepton Singlet χ_R ; $(\Psi, \chi) \stackrel{\mathcal{Z}'_2}{\longrightarrow} (-\Psi, -\chi);$ with $m_{\psi^0} < m_{\psi^\pm} < m_{\psi_2} < m_{\psi_3}$.

Signal and Background: $\ell^+\ell^- + 0j+$ ME

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Subhaditya Bhattacharya, PG, Jayita Lahiri, Biswarup Mukhopadhyaya JHEP12(2022)049

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• Lepton Energy Cut (Leading lepton energy distributions):

- The separation of the peaks depends on Δm ; while height depnd on production crosssection.
- Two peak Gaussian Fitting :

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\begin{split} y_H&=G(t)=A_1e^{-\frac{(t-\mu_1)^2}{2\sigma_1^2}}+A_2e^{-\frac{(t-\mu_2)^2}{2\sigma_2^2}}+B\\ \chi^2(\mu_1,\sigma_1;\mu_2,\sigma_2)&=\sum_{i=1}^n\frac{\left(G(\mu_1,\sigma_1;\mu_2,\sigma_2)[t_H^i]-y_H^i\right)^2}{y_H^i}. \end{split}
$$

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• Conditions for segregating the peaks

C1:
$$
\Delta N_1 = \int_t^{t_1} y dt, \quad \Delta N_2 = \int_{t_1}^{t'} y dt
$$

$$
R_{C1} = \frac{|\Delta N_2 - \Delta N_1|}{\sqrt{\Delta N_1}} > 2.
$$

$$
R_{C2} = \frac{y(t'') - y'(t'')}{\sqrt{y'(t'')}} > 2.
$$

C4:
$$
R_{C4} = \frac{y(t_2) - y(t_{\min})}{\sqrt{y(t_{\min})}} > 2.
$$

Purusottam Ghosh [IACS](#page-0-0) December 15, 2023 11 / 16

Can we observe two peak distribution at LHC ? Signal: $\ell^- \ell^+ + 0j + X$

- Two peaks can be observed in the signal.
- The signal encounter a huge QCD background.

Work in Progress

- • The key variables that play a role in producing distinguishable peaks are
	- \bullet both of the DM masses (m_{DM1}, m_{DM2}) ; their mass-splitting with the corresponding HDSPs (Δm_1 , Δm_2); and their production cross-sections $(\sigma_{X_1^+ X_1^-}, \, \sigma_{X_2^+ X_2^-})$.

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Ref: $JHEP12(2022)049(\text{arXiv}:2202.12097)$ pghoshiitg@gmail.com

The minimal renormalizable Lagrangian for this model then reads,

$$
\mathcal{L} \supset \mathcal{L}^{\text{SDM}} + \mathcal{L}^{\text{FDM}}.\tag{3}
$$

The Lagrangian for the SDM sector, having inert scalar doublet Φ can be written as :

$$
\mathcal{L}^{\text{SDM}} = \left| \left(\partial^{\mu} - ig_2 \frac{\sigma^a}{2} W^{a\mu} - ig_1 \frac{Y}{2} B^{\mu} \right) \Phi \right|^2 - V(\Phi, H);
$$

$$
V(\Phi, H) = \mu_{\Phi}^2 (\Phi^{\dagger} \Phi) + \lambda_{\Phi} (\Phi^{\dagger} \Phi)^2 + \lambda_1 (H^{\dagger} H) (\Phi^{\dagger} \Phi) + \lambda_2 (H^{\dagger} \Phi) (\Phi^{\dagger} H) + \frac{\lambda_3}{2} [(H^{\dagger} \Phi)^2 + h.c.]
$$

The minimal renormalizable Lagrangian for FDM having one vector-like doublet (Ψ) and one right-handed singlet (χ_R) reads:

$$
\mathcal{L}^{\text{FDM}} = \overline{\Psi}_{L(R)} \left[i \gamma^{\mu} (\partial_{\mu} - i g_2 \frac{\sigma^a}{2} W_{\mu}^a - i g_1 \frac{Y'}{2} B_{\mu}) \right] \Psi_{L(R)} + \overline{\chi}_{R} \left(i \gamma^{\mu} \partial_{\mu} \right) \chi_{R} - m_{\psi} \overline{\Psi} \Psi - \left(\frac{1}{2} m_{\chi} \overline{\chi}_{R} (\chi_{R})^c + h.c \right) - \frac{Y}{\sqrt{2}} \left(\overline{\Psi}_{L} \widetilde{H} \chi_{R} + \overline{\Psi}_{R} \widetilde{H} \chi_{R}^c \right)
$$

where $\Psi_{L(R)} = P_{L(R)} \Psi; P_{L/R} = \frac{1}{2}$ $\frac{1}{2}(1 \mp \gamma_5).$

| BPs | SDM sector $\{m_{\alpha 0}, \Delta m_1, \lambda_L\}$ | FDM sector ${m_{\psi_1}, \Delta m_2, \sin\theta}$ | Ω_{A} ² | $\Omega_{ab}h^2$ | $\sigma_{\geq 0}^{\text{eff}}$ (cm ²) | $\sigma_{\rm abs}^{\rm eff}$ (cm ²) | $BR(H_{inv})\%$ |
|-----------------|---|--|---------------------------|------------------|---|---|-----------------|
| BP1 | 100, 10, 0.01 | 60.5, 370, 0.022 | 0.00221 | 0.1195 | 3.45×10^{-46} | 2.03×10^{-47} | 0.25 |
| BP ₂ | 100, 10, 0.01 | 58.91, 285, 0.032 | 0.00221 | 0.10962 | 3.45×10^{-46} | 5.38×10^{-47} | 1.60 |
| BP3 | 100, 10, 0.01 | 58.87, 176, 0.04 | 0.00221 | 0.11941 | 3.45×10^{-46} | 5.00×10^{-47} | 1.50 |
| BP4 | 100, 10, 0.01 | 58.48, 190, 0.042 | 0.00221 | 0.1114 | 3.45×10^{-46} | 7.01×10^{-47} | 2.4 |

Table 2. Benchmark points of the model; contribution to relic density, spin-independent direct detection cross-section as well as that of invisible Higgs decay branching ratios of the DM components ϕ^0 and ψ_1 are mentioned.

| Benchmarks | | Collider cross-section (fb) | | | | | | | | | |
|-------------------|-----------------|-------------------------------------|-----------|------------|-------------------------------|----------------|------------|-------------------------------|----------------|----------------|--|
| | | $\sigma_{\text{total}}(\text{OSD})$ | | | $\sigma_{\phi^+\phi^-}$ (OSD) | | | $\sigma_{\psi^+\psi^-}$ (OSD) | | | |
| \sqrt{s} | Points | P1 | P2 | P3 | P1 | P ₂ | P3 | P1 | P ₂ | P ₃ | |
| 1000 | BP1 | 232(10.8) | 115(5.5) | 58.5(2.75) | 57.4(2.9) | 28.9(1.5) | 14.5(0.75) | 173(8.4) | 83.0(4.0) | 44.0(2.0) | |
| | BP ₂ | 276(13.4) | 141(6.6) | 70.0(3.3) | 57.4(2.9) | 28.9(1.5) | 14.5(0.75) | 218(10.4) | 111(5.3) | 55.5(2.7) | |
| 500 | BP3 | 686(33.0) | 339(15.9) | 168.1(7.8) | 180(8.9) | 90.3(4.5) | 44.3(2.3) | 494(22.2) | 253(11.3) | 123.8(5.5) | |
| | BP4 | 345(16.7) | 170(8.4) | 83.5(3.9) | 180(8.9) | 90.3(4.5) | 44.3(2.3) | 171.4(7.4) | 82.4(3.9) | 39.2(1.9) | |

Table 3. Signal cross-sections for HDSP pair production (OSD final state) at ILC. Total crosssection (σ_{total}), as well as individual contributions from SDM ($\sigma_{\phi+\phi-}$) and FDM ($\sigma_{\psi+\psi-}$) are mentioned. Three choices of beam polarisation are used: P1 \equiv {P_e-: -0.8, P_e+: +0.3}, P2 $\equiv \{P_{e^-}: 0, P_{e^+}: 0\}$ and $P3 \equiv \{P_{e^-}: +0.8, P_{e^+}: -0.3\}$. CM energy (\sqrt{s}) is in the units of GeV.

| | Backgrounds | $Cross-section(fb)$ | | | |
|-------------------|-------------|---------------------|----------------|----------------|--|
| /s | Processes | P1 | P ₂ | P ₃ | |
| 1 TeV | WW | 296 | 128 | 18.3 | |
| | ZZ | 7.5 | 4.4 | 3.5 | |
| | WW Z | 1.2 | 0.5 | 0.08 | |
| 500 GeV | WW | 802 | 342 | 51 | |
| | ZZ | 21 | 12 | 9.6 | |
| | WW Z | 0.8 | 0.37 | 0.06 | |

Table 4. Production cross-sections for $W^+(\ell^+\nu)W^-(\ell^-\bar{\nu})$, $Z(\ell^+\ell^-)Z(\nu\bar{\nu})$ and $W^+(\ell^+\nu)W^-(\ell^-\bar{\nu})Z(\nu\bar{\nu})$ background at $\sqrt{s} = 1$ TeV and 500 GeV for various polarization combinations P1, P2 and P3 (see caption of Table 3).