Particle Dark Matter In The Early Universe

Satyanarayan Mukhopadhyay
IACS, Kolkata

ICHEPAP2023, SINP
December, 2023

Particle Dark Matter In Standard Cosmology

- I shall ***assume*** general theory of relativity to be the theory of gravity at all scales.

- Model for the ***Early*** Universe: a homogeneous & isotropic background + small perturbations of 1 part in 10^5 (Evidence: the Cosmic Microwave Background)

$$ds^2 = (-1 - 2\Psi(\vec{x},t))\,dt^2 + a^2(t)\,\delta_{ij}\,(1 + 2\Phi(\vec{x},t))\,dx^i dx^j$$

- **Model for Structure Formation:** Starting from the seed perturbations of $\boldsymbol{\delta \sim 10^{-5}}$, gravitational instability leads to growth of perturbations — first described by linear perturbation theory until **δ ~ O(1)**, subsequently by a theory/N-body simulation in the non-linear regime, when structures form by gravitational collapse. *Seed perturbations: nearly scale-invariant power spectrum for the gravitational potential*

- **Result:** With photons, baryons, and neutrinos, it is not possible to reach **δ ~ O(1)** by a redshift **z ~ O(10)**. **Reason:** baryons are tightly coupled to photons by Thompson scattering of **e-**γ and Coulomb scattering of **e-p**, until photon decoupling at **z ~ 1100**. This ensures their perturbations oscillate until **z ~ 1100**, and grow linearly with the scale factor afterwards, when they become free.

- **Resolution in standard cosmology:** a form of non-relativistic matter with ***no necessary interaction*** with photons, baryons, electrons or neutrinos, except through gravity.

Description of Particle Dark Matter In Cosmology

- As for any large collection of particles, in cosmology, dark matter is described by a phase-space distribution function ***f(x,p,t)***.

- If there are no interactions, then this distribution function evolves by the Liouville's theorem:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial E}\frac{dE}{dt} + \frac{\partial f}{\partial \hat{p}^i}\frac{d\hat{p}^i}{dt} = 0$$

- Just like the metric, the distribution function also has a homogeneous and isotropic part with small perturbations $f = f_0(|p|,t) + f_1(x,p,t)$

- The background distribution f_0 is not known a priori (unlike for thermalized photons). We work with the moments of the kinetic equations.

$$n = \int \frac{d^3p}{(2\pi)^3} f$$ **Number density**

$$v^i = \frac{1}{n}\int \frac{d^3p}{(2\pi)^3} f \frac{p\hat{p}^i}{E}$$ **Bulk velocity**

-

Description of Particle Dark Matter In Cosmology

- The **zeroth moment** satisfies (in conformal-Newtonian gauge):

$$\frac{\partial n}{\partial t} + \frac{1}{a}\frac{\partial (n v^i)}{\partial x^i} + 3\left(H + \frac{\partial \Phi}{\partial t}\right) n = 0$$

Continuity equation in expanding +fluctuating space-time

- The **zeroth moment at zeroth order** in perturbation gives

$$n = n^{(0)}\left[1 + \delta(\vec{x}, t)\right]$$

Average density+relative perturbations

$$\frac{d}{dt}\left(n^{(0)} a^3\right) = 0 \Rightarrow n^{(0)} \propto a^{-3}$$

Dilution from expansion

- The **zeroth and first moments in first order** give the key perturbation equations for CDM

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\frac{\partial v^i}{\partial x^i} + 3\frac{\partial \Phi}{\partial t} = 0$$

Continuity equation

$$\frac{\partial v^j}{\partial t} + H v^j + \frac{1}{a}\frac{\partial \Psi}{\partial x^j} = 0$$

Euler equation

- Here, we ignored terms of order ***$v^2 \sim (p/m)^2$*** : this is the definition of cold dark matter — which suffices to describe the CMB anisotropies and the large scale structure of the Universe.

- If ***(p/m)*** is larger, we need to consider higher moments — e.g., for warm dark matter. Too much ***(p/m)*** leads to too much free-streaming, and wash-out of smaller scale density correlations — the free-streaming should be less than about **100 kpc**.

Evolution of the density perturbations with CDM

CDM linear perturbation equations

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\frac{\partial v^i}{\partial x^i} + 3\frac{\partial \Phi}{\partial t} = 0$$

$$\frac{\partial v^j}{\partial t} + H v^j + \frac{1}{a}\frac{\partial \Psi}{\partial x^j} = 0$$

Einstein equations

$$-\nabla^2 \Phi + \frac{3a'}{a}\left(\Phi' - \Psi\frac{a'}{a}\right) = 4\pi G a^2\left[\bar{\rho}_m \delta_m + 4\bar{\rho}_r \Theta_{r,0}\right]$$

$$\nabla^2(\Phi + \Psi) = -32\pi G a^2 \bar{\rho}_r \Theta_{r,2}$$

With similar equations for baryons, photons (that also include the collision terms due to Thompson scattering of e-γ and Coulomb scattering of e-p) and neutrinos

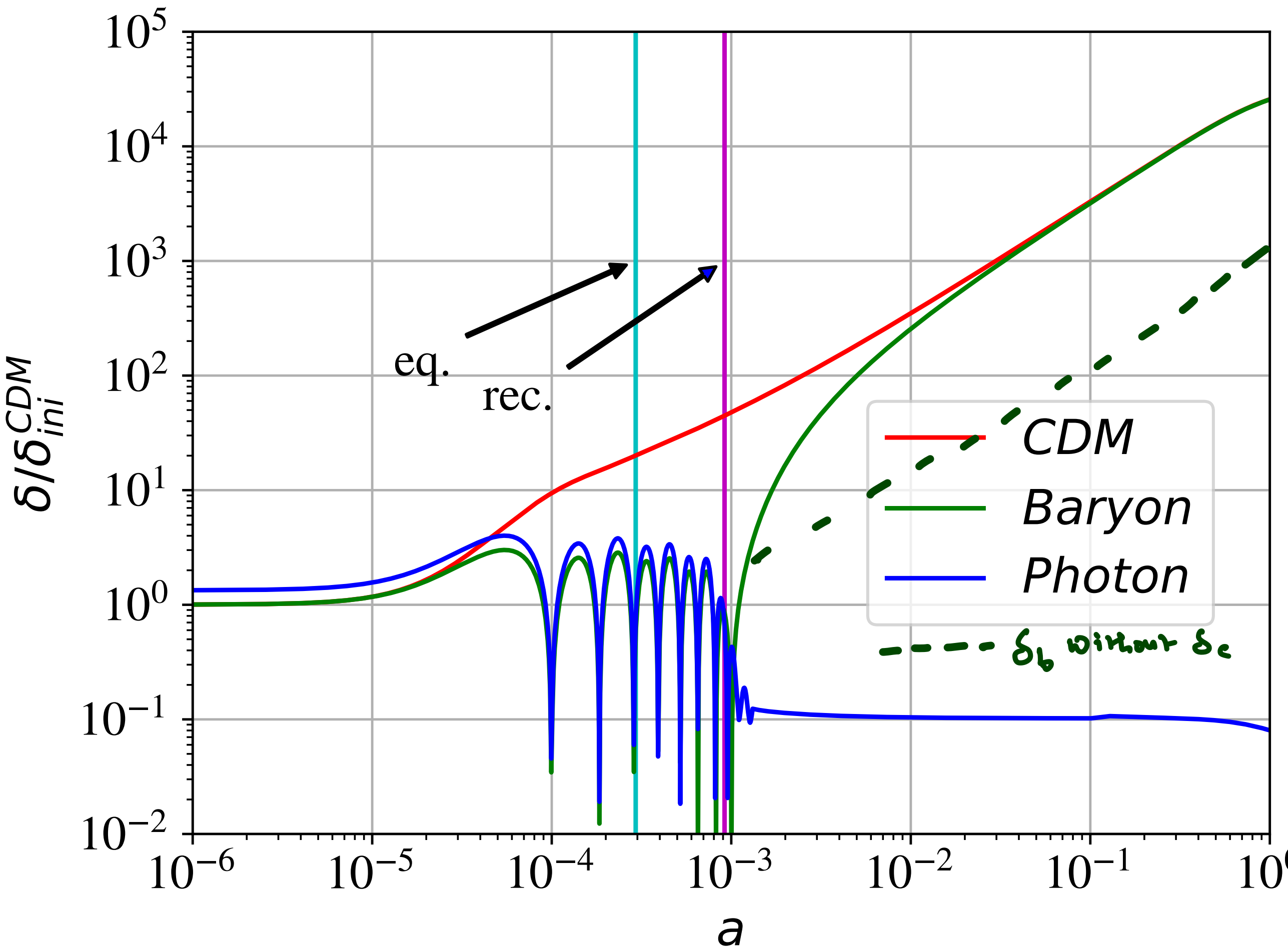


Fig. made by Arnab Pal, with CLASS, for a representative Fourier mode with $k = 0.25\ h$ Mpc^{-1}

Before decoupling, baryons are tightly coupled to the photons and oscillate after horizon crossing.

DM grows logarithmically during RD, and linearly in MD, thus $\delta_c \gg \delta_b$ at decoupling

After decoupling, baryons lose pressure support of photons and fall into the gravitational potential wells created by DM. At late times, $\delta_c/\delta_b \to 1$.

Without the DM assisted growth, baryons don't cluster fast enough to explain the formation of galaxies.

Summary: non-gravitational couplings of DM are not *necessary* for explaining CMB anisotropies and LSS; cosmology constrains the total DM density, but not the particle mass

DM Production in the Early Universe: where mass and couplings become relevant

- **Scenario without Inflation:** There is no theory of genesis of the radiation bath. The highest temperature of the radiation era is unknown. Given the required proton density, we can compute the abundance of light elements, and *successful BBN requires a temperature of around 1 MeV.* (A proton anti-proton asymmetry is also required, otherwise, the proton density becomes negligible due to strong annihilations.) Hence, DM density may be (1) an initial condition, (2) produced gravitationally, or (3) through interactions with the SM bath.

- **Scenario with Inflation:** The radiation and DM densities can no longer be initial conditions, as inflation will erase all initial densities. The highest temperature of the radiation era is still unknown, but *existing constraints on the Hubble scale during inflation restricts the reheat temperature to be below around 10^{16} GeV*. Hence, DM density may be produced (1) gravitationally during or after reheating, (2) at the reheating epoch from the inflaton, (3) through interactions with the SM bath after reheating

- **The crucial DM distribution function:** In all cases above, the DM distribution function may

- ***(1) remain non-thermal throughout,***

- ***(2) internally thermalize with a temperature $T_{DM} \neq T_{SM}$,***

- ***(3) thermalize with the SM sector with $T_{DM} = T_{SM}$***

- ***In the following, I shall take toy examples from the reheating production after inflation of the initial distribution and illustrate these three possibilities for the DM distribution function***

Scenario-1: Dark Matter phase-space distribution does not Thermalize

Gravitational production is always present for massive particles due to the time-dependent metric — its efficiency depends upon the Hubble rate, the particle mass and spin, and whether the coupling is minimal or with additional couplings to the Ricci scalar

Apart from this, small additional interactions can also lead to non-thermalized production. There are a large number of possibilities: mixing with, or small couplings to SM particles, reheating etc.

Example-1: singlet fermion DM (sterile neutrino) produced from small mixing with SM neutrinos $\mathscr{L} \supset \overline{L}HN$

Example-2: DM produced in reheating — distribution depends on details of the reheating era: (i) reheat temperature, (ii) duration of reheating, (iii) inflaton mass

Consider a simple scenario in which at the end of slow-roll, the inflaton field undergoes a *damped oscillation around the minimum of the potential*, approximated by a quadratic form near the minimum: *equivalent to a field theory of massive spin-0 particles with negligible velocity.*

Production in perturbative reheating: *Initial Distribution Function*

Example: Light Singlet fermion DM (ψ) produced in reheating from the inflaton (ϕ) — ψ does not have any renormalizable self or SM interaction terms

$$\mathscr{L}_{\rm int} \supseteq -\mu_H H^\dagger H \phi - \lambda \overline{\psi}\psi\phi\,.$$

Large literature

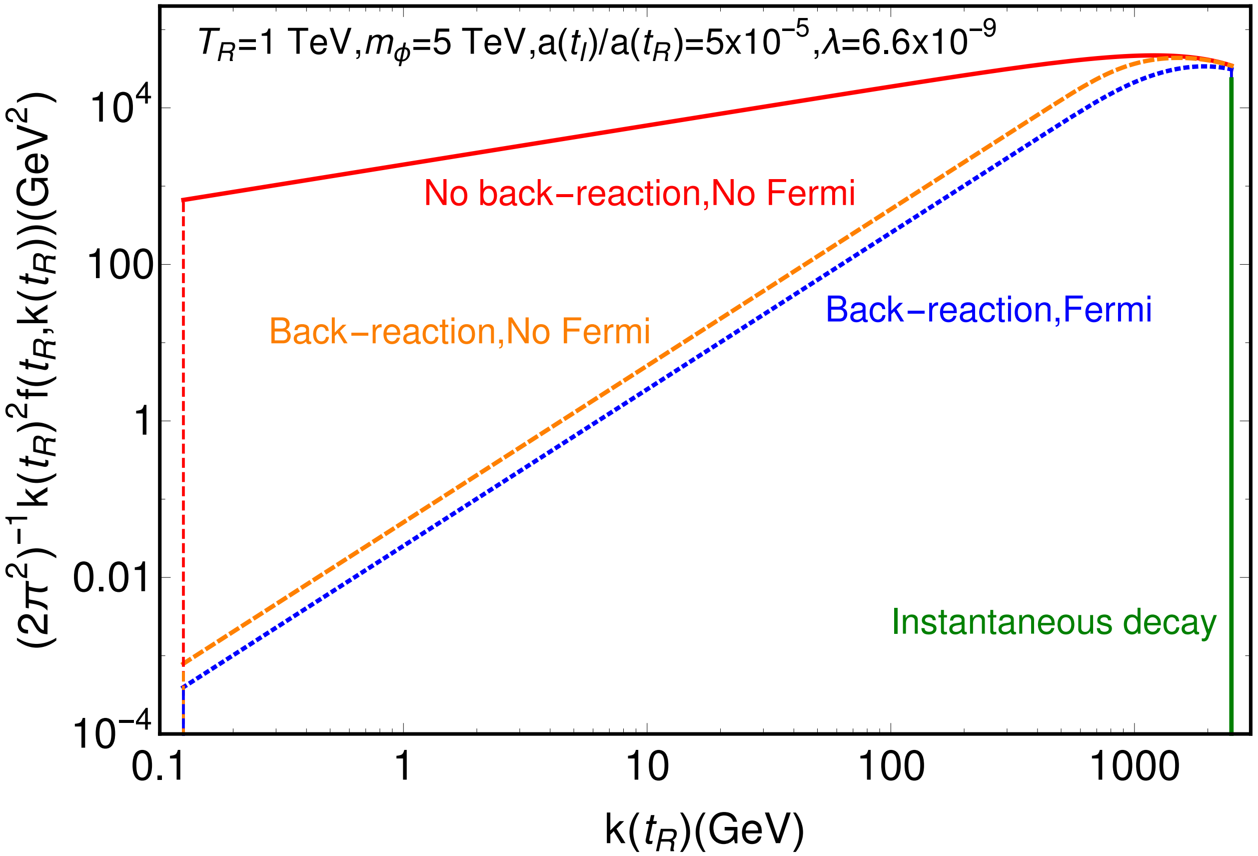
Moroi, Yin, 2020

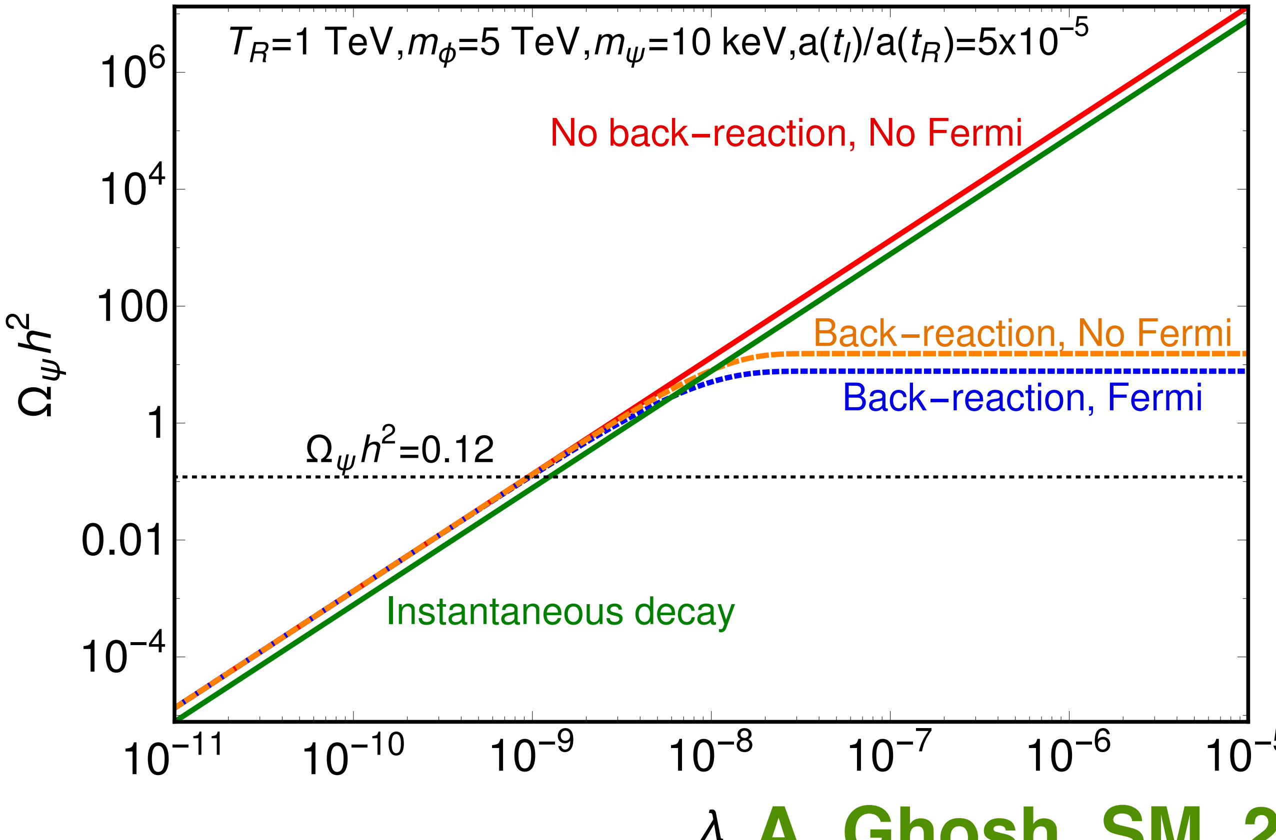
A. Ghosh, SM, 2022

In this case, the initial DM distribution function can be computed from

$$\frac{\partial f_\psi(\vec{k},t)}{\partial t} - H\,\vec{k}\,.\,\vec{\nabla}_{\vec{k}}\, f_\psi(\vec{k},t) = -\frac{1}{2E_{\vec{k}}}\int \frac{d^3\vec{k}\,'}{(2\pi)^3\,2E'_{\vec{k}'}}\,\frac{d^3\vec{p}}{(2\pi)^3\,2E_{\vec{p}}}(2\pi)^4\delta^4(p-k'-k)$$

$$\times 2\left[\,|M|^2_{\psi\psi\to\phi}\, f_\psi(\vec{k},t) f_\psi(\vec{k}\,',t)(1+f_\phi(\vec{p},t)) - |M|^2_{\phi\to\psi\psi}\, f_\phi(\vec{p},t)(1-f_\psi(\vec{k},t))(1-f_\psi(\vec{k}',t))\right].$$





A. Ghosh, SM, 2022

Perturbative reheating: *Evolution of distribution function beyond simple redshift*

Question : Since in this scenario the inflaton ***necessarily couples*** to both SM and DM fields, can inflaton-mediated scatterings become important in DM cosmology?

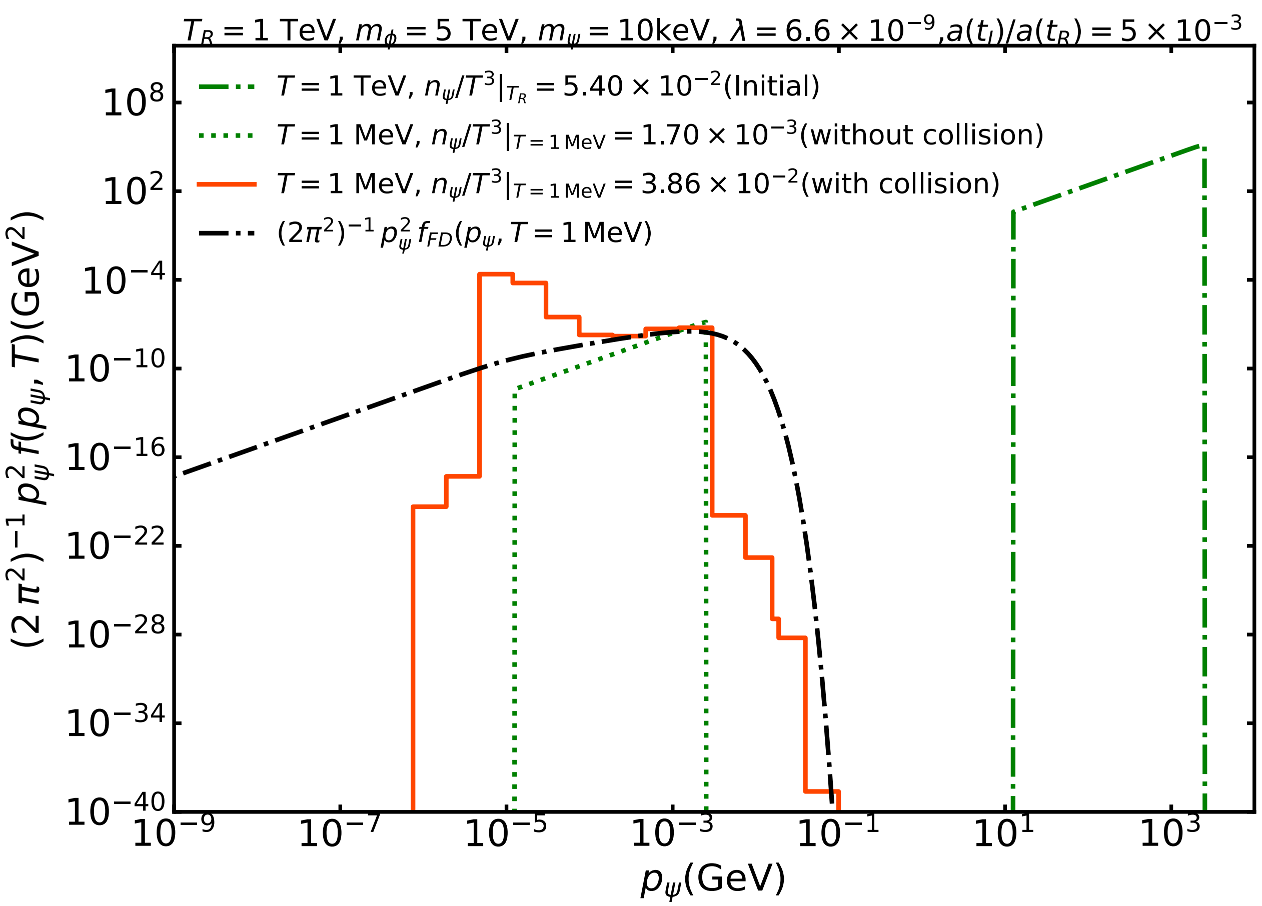
Solve the integro-differential kinetic equation with collision terms from 2 to 2 scatterings

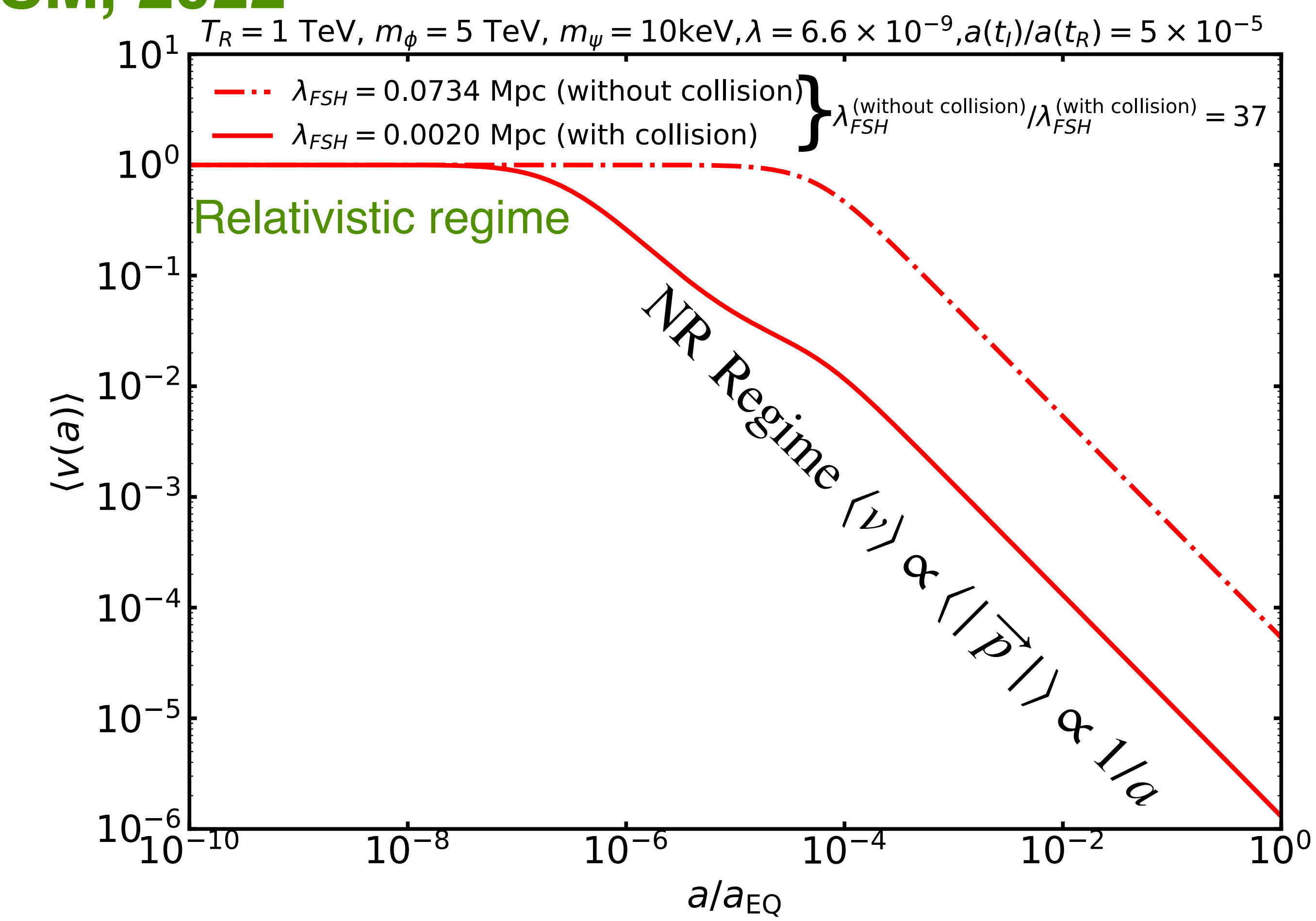
$$\psi(p_1)\overline{\psi}(p_2) \leftrightarrow \psi(p_3)\overline{\psi}(p_4)$$
$$\psi(p_1)h(p_2) \leftrightarrow \psi(p_3)h(p_4)$$
$$\psi(p_1)\overline{\psi}(p_2) \leftrightarrow h(p_3)h(p_4)$$

Co-moving free-streaming length

$$\lambda_{\rm FSH} = \int_{t_{\rm dec}}^{t_{\rm EQ}} \frac{\langle v(t)\rangle}{a(t)} dt,$$

A. Ghosh, SM, 2022





Thermalization is not achieved, but the average velocity at MR equality can be modified by a factor of 40 —> important implications for structure formation. Scatterings populate lower momentum modes, reducing the average velocity.

Scenario-2: DM self-thermalizes

No significant scatterings with the SM bath $T_{\rm DM}(\neq T_{\rm SM})$

- If DM is a scalar, it can naturally have *self-scatterings and number-changing annihilations*, which are necessary for internal thermalization.

- With no significant scatterings with the SM bath, its initial density can come from reheating, or gravitational production.

- Post-reheating, in the limit of a heavy inflaton compared to the reheat temperature, DM undergoes only self-interactions, which may lead to internal thermalization

- The DM phase-space density evolves following the same Eqn. but now with a richer collision term

$$\frac{\partial f}{\partial t} + \frac{d\vec{x}}{dt}.\nabla_x f + \frac{d\vec{p}}{dt}.\nabla_p f = C[f]_{\text{2DM} \rightarrow \text{2DM} + \text{3DM} \rightarrow \text{2DM}}$$

Internal Thermalization: how background cosmology changes

- The zeroth moment of the kinetic equation, at zeroth order in an FRW background gives the evolution of the number density

$$\frac{dn_\chi(t)}{dt} + 3Hn_\chi(t) = g_\chi \int \frac{d^3\mathbf{p}}{(2\pi)^3} C[f_\chi]$$

And the *second moment* at zeroth order gives the evolution of the DM temperature, *if the self-interactions lead to internal thermalization*

$$\frac{dT_\chi}{dt} + 2HT_\chi + \frac{T_\chi}{n_\chi}\left(\frac{dn_\chi}{dt} + 3Hn_\chi\right) - \frac{H}{3}\left\langle \frac{|\mathbf{p}|^4}{E^3} \right\rangle = \frac{1}{n_\chi}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{|\mathbf{p}|^2}{3E} C[f_\chi]$$

Where $$T_\chi \equiv \frac{g_\chi}{n_\chi}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{|\mathbf{p}|^2}{3E} f_\chi(\mathbf{p}, t)\,.$$

We can solve this coupled system of Eqns. to determine the DM temperature and number density as a function of time.

The initial condition for the DM temperature is unknown, and can be set by the reheating dynamics.

D.Ghosh, S. Gope, SM, *to appear*

Self-interacting singlet scalar DM

Low-energy toy model: $\mathscr{L}_{\rm int} = -\frac{\mu}{3!}\chi^3 - \frac{\lambda}{4!}\chi^4 .$

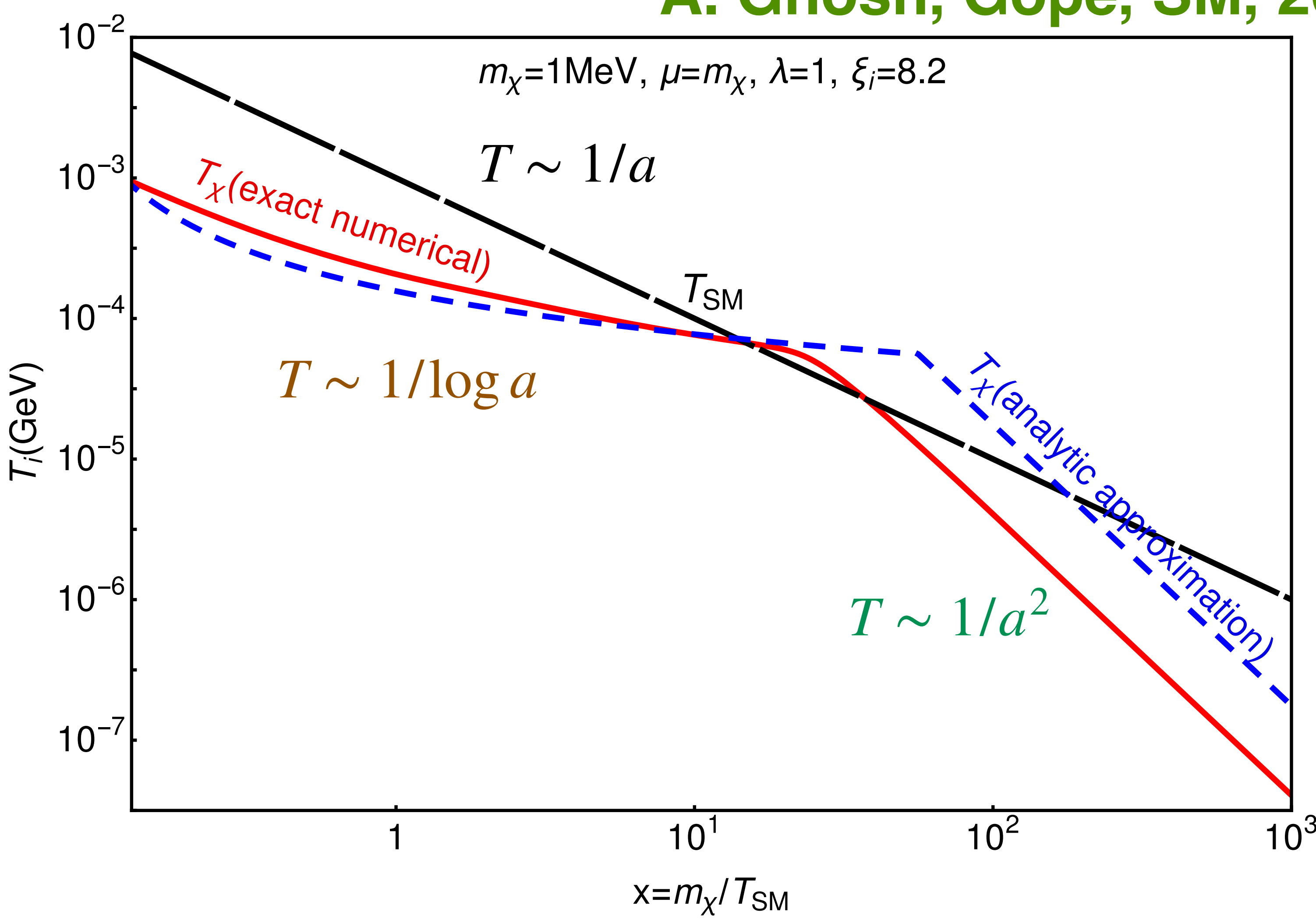
Leads to elastic and inelastic scatterings (the latter sets $\mu_\chi = 0$)

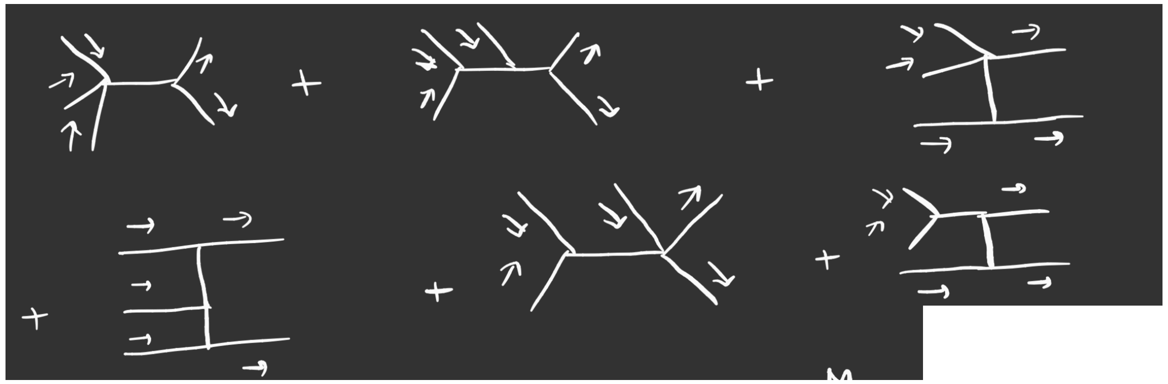
Non-standard temperature evolution with 3 to 2 processes: $T_\chi(t) \sim 1/\log a(t)$

Dolgov, 1980; Carlson, Machacek, Hall, 1992; … Ghosh, Gope, SM, 2022

$\xi(a) = T_{\rm SM}(a)/T_\chi(a)$

A. Ghosh, Gope, SM, 2022





3 to 2: minimal number changing process -> if absent density determined by initial conditions (such as at reheating)

Self-interacting singlet scalar DM: cosmological probes

CMB: $$\frac{T_\chi(a_{\rm LS})}{m_\chi} < 10^{-5}, \qquad \text{at } T_{\rm SM}(a_{\rm LS}) \sim 0.26 \text{ eV},$$

Implies strong upper bound on DM Temperature

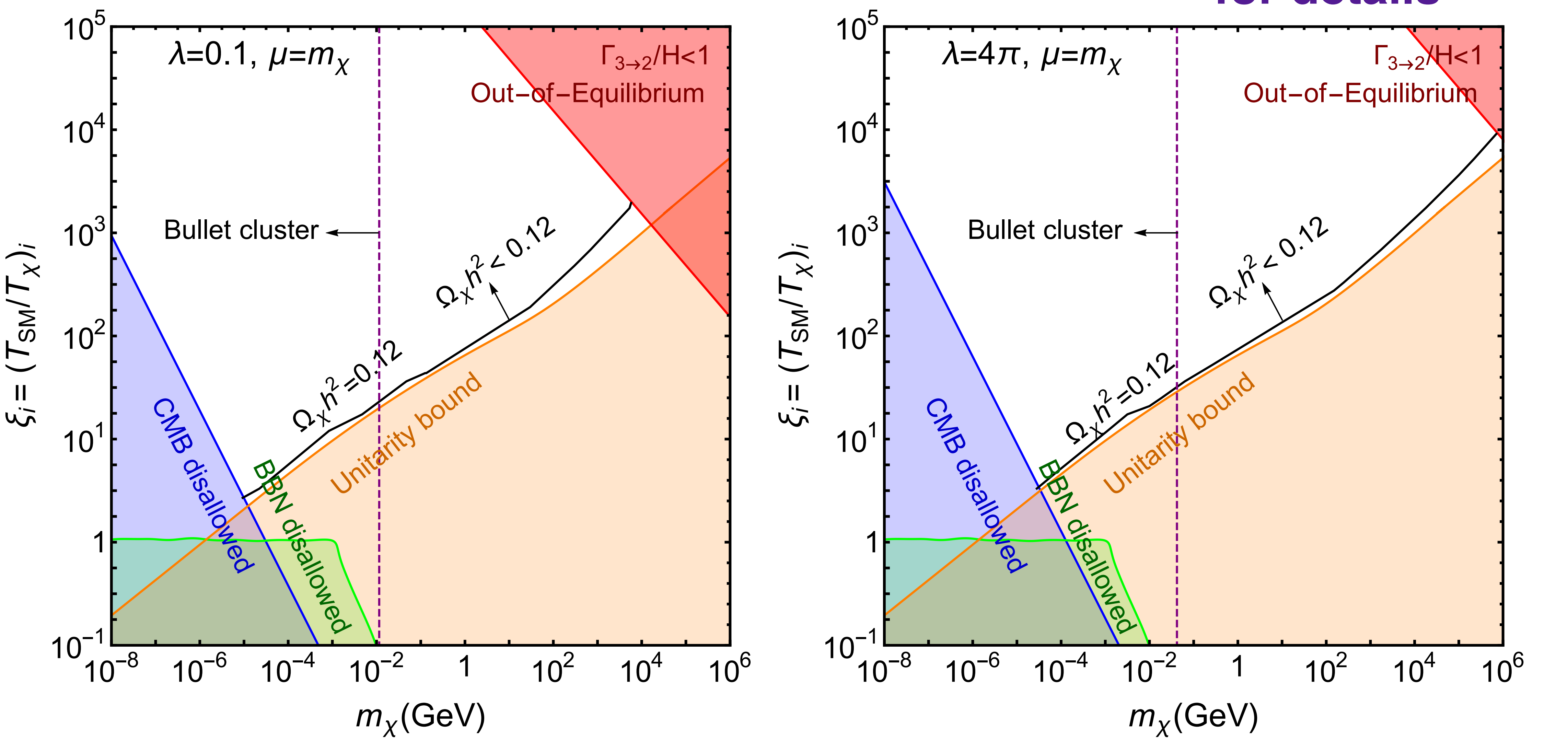
Heimersheim, Schoeneberg, Hooper, Lesgourgues 2020; Buen-Abad, Emami, Schmaltz, 2018

BBN: $\xi_i > 1.07,\qquad 95\,\%\ \text{C.L. Constraint from BBN, for}\ \ m_\chi \xi_i < T_{\rm BBN}$

Very cold DM: No equilibrium for 3 to 2 at T ~ m, density from initial conditions → leads to a lower bound on DM temperature

A. Ghosh, Gope, SM, 2022

See Poster by S. Gope for details



How different can the dark matter temperature be in a reheating production scenario?

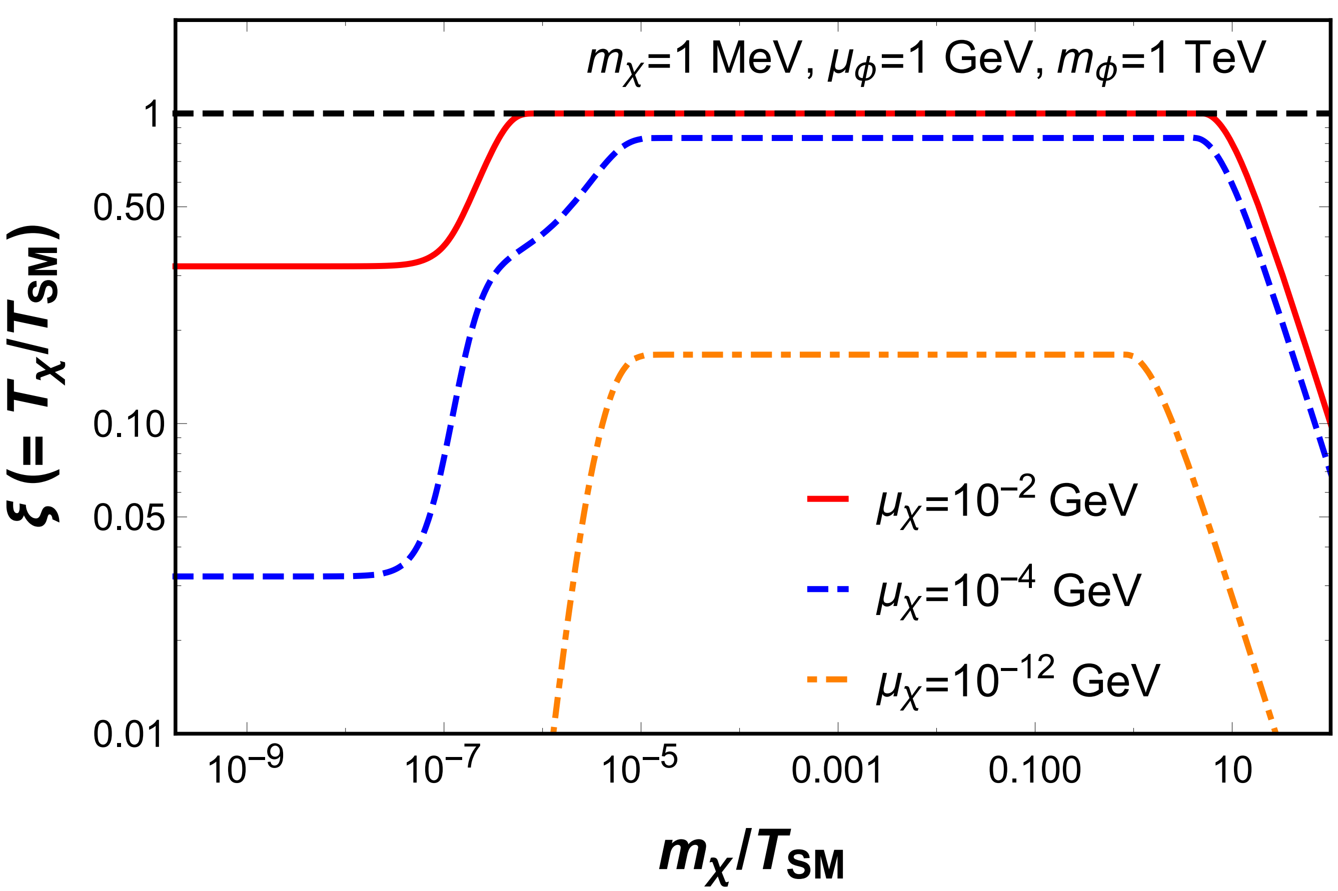
D.Ghosh, S. Gope, SM, *to appear*

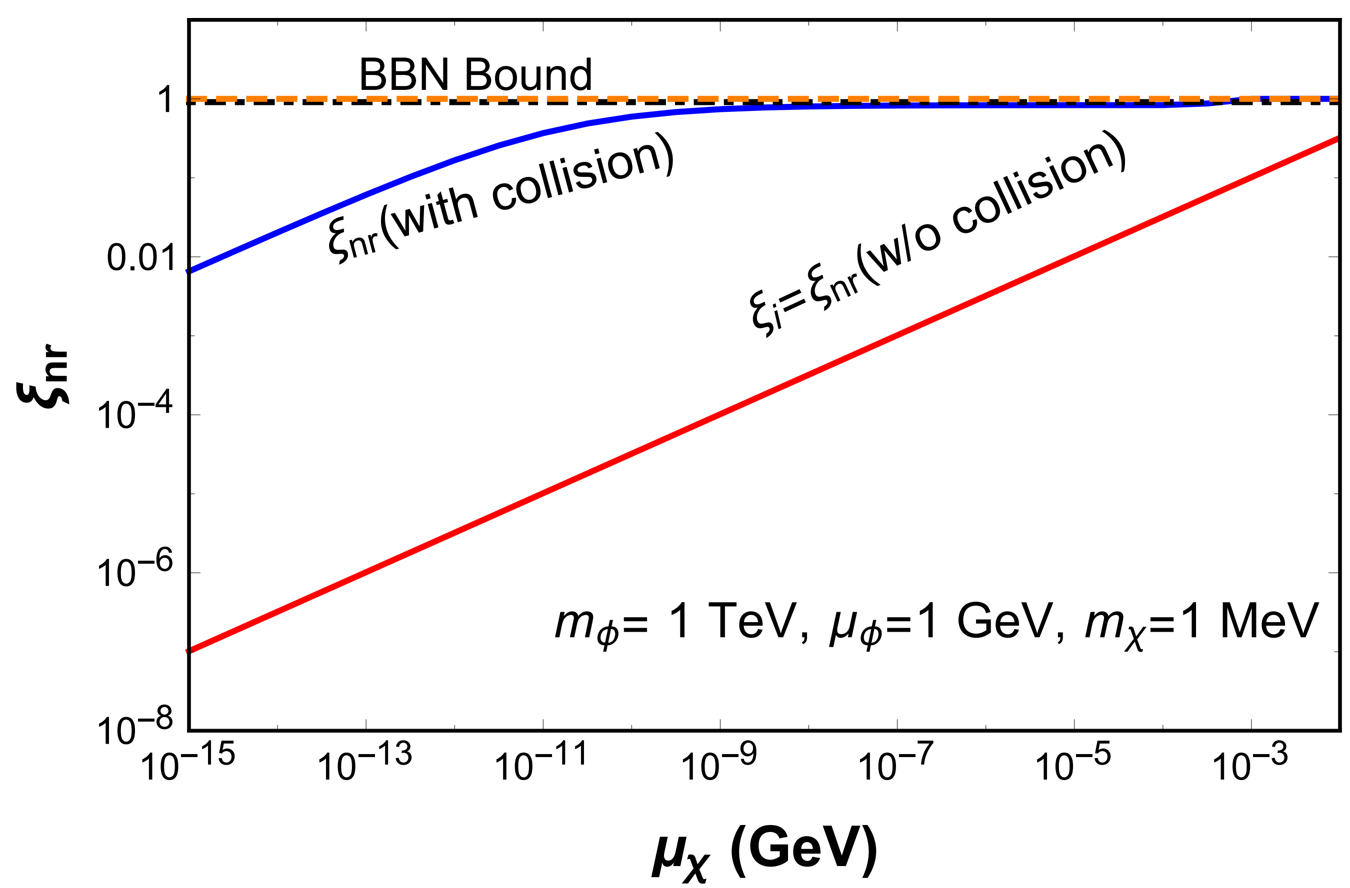
In general, many SM-inflaton operators can contribute to reheating

$$\mathcal{L} \supset \mu_\phi \phi H^\dagger H + \frac{\lambda_\phi}{2}\phi^2 H^\dagger H + \frac{\mu_\chi}{2}\phi\chi^2 + \frac{\lambda}{4}\phi^2\chi^2 + \frac{1}{\Lambda}\phi\bar{L}He_R + \frac{1}{\Lambda}\phi\bar{Q}\tilde{H}u_R + \frac{1}{\Lambda}\phi\bar{Q}Hd_R$$
$$+\frac{1}{\Lambda}(\partial_\mu\phi)(g_L\bar{f}_L\gamma^\mu f_L + g_R\bar{f}_R\gamma^\mu f_R) + \frac{1}{\Lambda}\phi B_{\mu\nu}B^{\mu\nu} + \frac{1}{\Lambda}\phi W^{a\mu\nu}W^a_{\mu\nu} + \frac{1}{\Lambda}\phi G^{a\mu\nu}G^a_{\mu\nu},$$

Key message: the DM temperature at reheating can be significantly modified by subsequent *necessarily present* inflaton mediated DM-SM scattering processes, fixed by the same couplings

Example: If the Higgs-inflaton operator dominates the SM coupling:





Scenario-3: DM thermalizes with SM bath

$$T_{\mathrm{DM}} = T_{\mathrm{SM}}$$

This naturally is the most predictive and widely tested scenario. There are many simple models which are very much viable at present.

The simplest model with one free parameter (the DM mass): DM is the neutral component of a multiplet of the electroweak gauge group. Different spin and representations possible.

Several talks in this conference

The next to simplest model with two free parameters (mass, Higgs coupling): DM is a singlet fermion coupled to the Higgs boson.

We may even set a theoretical upper bound on the thermal dark matter mass in this scenario with standard cosmology: from S-matrix Unitarity

Briefly discuss next

The possible existence of a DM chemical potential may change the standard picture considerably.

How high can the mass of thermal dark matter be?

Griest, Kamionkowski, 1990; Hui, 2001

Unitarity of the S-matrix ($SS^\dagger = S^\dagger S = I$) implies the optical theorem for a 2-particle initial state α

$$\mathrm{Im}(\mathscr{M}_{\alpha\alpha}) = 2|\vec{p}|E_{\mathrm{CM}}\sum_\beta \sigma_{\alpha\to\beta}$$

which implies an upper bound on the total inelastic scattering cross-section for the given initial state

$$\sigma_{\mathrm{inelastic}} \leq \sum_\ell \frac{\pi}{|\vec{p}|^2}(2\ell+1)$$

The maximum reaction rate for a 2 ⟶ k process is thus:

$$\langle \sigma_{2\to k} v_{\mathrm{rel}}\rangle_{\max} = \sum_\ell (2\ell+1)\frac{4\sqrt{\pi}}{m_\chi^2}\sqrt{x}e^{-(k-2)x}, \quad x = m_\chi/T$$

Using detailed balance in chemical equilibrium for the process k ⟷ 2, we can then show

$$\langle \sigma_{k\to 2} v_{\mathrm{rel}}^{k-1}\rangle_{\max} = \sum_\ell (2\ell+1)\frac{2^{\frac{3k-2}{2}}}{g_\chi^{k-2} m_\chi^{3k-4}}(\pi x)^{\frac{3k-5}{2}}, \quad x = m_\chi/T$$

Bhatia, SM, 2021

How high can the mass of thermal dark matter be?

Specific cases:

$$\langle \sigma_{2\to 2} v_{\rm rel} \rangle_{\rm max,\ s-wave} = \frac{4\sqrt{\pi}}{m_\chi^2}\sqrt{x}$$

Griest, Kamionkowski, 1990

$$\langle \sigma_{3\to 2} v_{\rm rel}^2 \rangle_{\rm max,\ s-wave} = \frac{8\sqrt{2}(\pi x)^2}{g_\chi m_\chi^5}$$

Bhatia, SM, 2021

The maximum inelastic cross-section implies the minimum surviving number density of DM.

This in turn implies the maximum DM mass, if it saturates the observed total density.

Bhatia, SM, 2021

Symmetry	Annihilation channels	$\ell = 0$	$\ell = 1$	$\ell = 0 + 1$
Z_2	$\chi + \chi^* \to \mathrm{SM} + \mathrm{SM}$	127.7 TeV	220 TeV	253.5 TeV
Z_3	$3\chi^{(*)} \to 2\chi^{(*)}$	1.15 GeV	1.72 GeV	1.91 GeV
Z_2	$4\chi \to 2\chi$	6.9 MeV	9.4 MeV	10.1 MeV

Brief remarks on the DM Chemical Potential

- Baryons and electrons have a small chemical potential, $\eta_B \sim 10^{-9}$ $(\mu_B/m_B \sim 10^{-4})$

- The neutrino chemical potential is only weakly constrained by BBN
$$|\mu_\nu/T| \lesssim 0.02, \quad T \sim 1 \text{ MeV}$$

- If DM had a thermal equilibrium history, then in general it can have a non-zero chemical potential $\mu_{\rm DM}$

- For $\mu_{\rm DM} \neq 0$ the relationship between the DM mass and annihilation rate is modified **Nussinov, 1985; Griest, Seckel, 1987**
$$n_{\rm DM} + n_{\rm anti-DM} \sim 2e^{-m/T_{\rm FO}} \cosh(\mu_{DM}/T_{\rm FO})$$

- Several possible interesting minimal mechanisms proposed to generate the DM asymmetry from scatterings. **e.g., A. Ghosh, D. Ghosh, SM, 2020**

- A key question: how to observationally discriminate a symmetric DM scenario from an asymmetric one (for bosonic ADM strong constraints exist from the existence of old compact objects)?

- One possibility is to use a neutrino signal, which is viable if DM dominantly annihilates (or decays) to neutrinos. A pair of (anti-)DM may annihilate to a pair of (anti-)neutrinos,. How to differentiate neutrinos from anti-neutrinos with existing/upcoming detectors? **Fukuda, Matsumoto, SM, 2014**
D. Ghosh, R. Gandhi, SM, B. Mukhopadhyaya, *to appear*

Summary: Lessons so far from early Universe cosmology

- The standard cosmological model that fits the CMB anisotropy and LSS data well, *does not require* any non-gravitational interaction of DM — the idealization of collisionless DM works well.

- There is of course some room for optional DM interactions with itself and with photons, baryons and neutrinos — constraints on them have been derived using the CMB and LSS data in many studies. *The power spectrum at small scales usually get suppressed due to elastic scatterings.* The velocity-independent DM-photon scattering should be around six orders of magnitude below the Thompson scattering, and DM-baryon scattering around three orders of magnitude below the same for a 1 GeV DM.

- The total matter density enters through the Poisson equation for the gravitational potential, and is constrained well. The baryonic mass density is constrained both by CMB and BBN, and hence the total DM mass density.

- Low **velocity dispersion** is important, the free-streaming length should not be larger than around 100 kpc. *Hence the phase-space distribution function is the key.*

- The DM particle mass does not enter the collisionless perturbation equations directly — hence CDM cosmology does not constrain it in a model independent way. The mass and couplings enter through the DM production mechanism in the early Universe.

- Generically, the DM phase space distribution function may (1) remain non-thermal, (2) may internally thermalize or (3) may thermalize with the SM bath

- Cosmological probes constrain the distribution function properties and the DM temperature

A light-hearted outlook :)

- We are trying to search for a particle for which we neither know the mass nor the interaction strength, both of which can span several orders of magnitude.

- Contrast this with previously predicted particles which were successfully found later: positron, pion, neutrino, the weak gauge bosons, the Higgs boson.

- The Higgs was perhaps the hardest of these, and I am reminded of the closing paragraph in the famous “A phenomenological profile of the Higgs boson” by Ellis, Gaillard and Nanopoulos (1976) which summarized the challenge:

- *“We should perhaps finish with an apology and a caution. We apologize to experimentalists for having no idea what is the mass of the Higgs boson, unlike the case with charm, and for not being sure of its couplings to other particles, except that they are probably all very small. For these reasons we do not want to encourage big experimental searches for the Higgs boson, but we do feel that people performing experiments vulnerable to the Higgs boson should know how it may turn up.”*