# CPT violation and Sakharov Conditions

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# PLAN OF TALK

MATTER ANTIMATTER ASYMMETRY

THREE CONDITIONS OF SAKHAROV

MEANING OF CONDITIONS

IN THERMAL EQUILIBRIUM

REVISED CONDITIONS IN THE PRESENCE OF CPT VIOLATION

# Matter Antimatter Asymmetry

In very early universe due to pair production and annihilation, particle-antiparticle pairs did maintain a certain form of equilibrium in terms of the number densities of particles and antiparticles (not thermal equilibrium).

$$n_B \approx n_{\bar{B}} \approx n_\gamma .$$

During the early moments in the history of universe, when pair production stopped (temp $\sim 1\ \mathrm{MeV} \sim 10^{10}$ K ), matter and antimatter combined to form photons. During recombination era (temp $\sim 0.25$ eV $\sim 3000$ K) primordial radiation free streamed; now known as Cosmic Microwave Background Radiation (CMBR)

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-10} .$$

Here $n_B, n_{\bar{B}}, n_\gamma$ denote **number densities** of baryons anti-baryons and photons.

N.B: Astronomers often use the term ‘baryonic matter’, to refer to ordinary matter; it’s a bit of a misnomer, because it includes electrons (which are leptons) … and it generally excludes neutrinos (and anti-neutrinos)

# Matter Antimatter Asymmetry

## EXPERIMENTAL SIDE

Baryon to Photon ratio can be determined in two different ways which are independent of each other.

- Abundances of light elements in the intergalactic medium.

Reference: G. Steigman, Ann. Rev. AA 14, 339 (2007)

- From power spectrum of temperature fluctuations in CMBR.

Reference: Planck Collab, Astron & Astrophys. A13, 594 (2016) and A6, 641 (2018).

## THEORETICAL SIDE

1. Theoretical conditions necessary for the generation of asymmetry
2. Showing that the numerical value matches with observed values (Boltzmann Equation Analysis)

# Matter Antimatter Asymmetry

- Abundances of light elements in intergalactic medium:
- $\boldsymbol{\eta_{BBN} = 6.07 \pm 0.33 \times 10^{-10}}$

A precise value of $\eta$ is a crucial input for determining abundances of light elements produced during primordial nucleosynthesis.

- Power spectrum of temperature fluctuations of CMBR:
- $\boldsymbol{\eta_{CMB} = 6.16 \pm 0.15 \times 10^{-10}}$

Observations of Cosmic Microwave Background radiation (CMBR) which is the afterglow of the big bang provide such important constraints on baryon to photon ratio.

**There are only six baryons (protons & neutrons) for one billion photons**

# Matter Antimatter asymmetry

As we generally expect that laws of Physics are same for matter and anti-matter sectors, it is an interesting question as to how such a tiny difference between amount of matter and anti-matter existed in the **early universe**, when matter and antimatter annihilated each other and photons started free streaming.

- This could be due to interplay between two rates. Namely, the rate of B violation and the expansion rate of the universe. If expansion rate is less, then forward and backward reactions are in thermal equilibrium, no asymmetry is generated. But if expansion rate is more then back reaction is blocked. Then an asymmetry can be generated. **(In SM with $m_H$=125 GeV out-of-equilibrium condition is un-likely to be satisfied)**
- This could also be due to CPT violation, which can naturally introduce a small mass difference between particles and antiparticles.

$$m \neq \bar{m}.$$

# Sakharov conditions

The Sakharov conditions refer to the **three necessary conditions** proposed by the Russian physicist **Andrei Sakharov** to explain the asymmetry between matter and antimatter and the predominance of matter in the universe.

1. **Baryon Number Violation**
2. **C and CP violation**
3. **Out of equilibrium condition**

These conditions, when combined, offer a plausible explanation for the observed predominance of matter in the universe. We study various theoretical models and experimental observations to understand how these conditions might have been satisfied in the early universe during baryogenesis, leading to the matter-antimatter asymmetry we observe today. **(comment on SM as well as it's extensions)**

A. D. Sakharov, JETP Letters **5**, 24 (1967).

# Baryon number violation

The first condition is easiest to prove. Consider Hamiltonian **H** describing interactions and it's eigenstates are $|u_i\rangle$. These are quantum mechanical microstates. Suppose Baryon number commutes with Hamiltonian. In other words let's see what happens if it is an unbroken symmetry. We can see that such an assertion leads to an unacceptable conclusion.

Time evolution of Baryon number is:

$B(t) = e^{iHt}\, B(0)\;\, e^{-iHt}$ = $B(0)\, e^{iHt}\; e^{-iHt}$ = $B(0)$ : because $[B(0), H] = 0.$

We take expectation values of Baryon number operator we get,

**Eq(1)** $\langle u_i|B(t)|u_i\rangle = \langle u_i|B(0)|u_i\rangle$

Then if we need *B(t)* to be different from *B(0)* then the commutator $[B(0), H]$ must be non-zero, or we must break the Baryon number.

**We can think in terms of electric charge. Because it is strictly conserved the earth is neutral, solar system is neutral and the universe is as a whole charge neutral.**

# Thermal average in quantum mechanics

Let's consider thermal average of an operator *O*, which is given in terms of the inverse temperature $\beta = \frac{1}{k_B T}$ and the equilibrium probability distribution $p_i$ as

$$\langle O \rangle_T = \frac{\mathrm{Tr}[\rho\, O]}{\mathrm{Tr}[\rho]}.$$

where $\rho$ contains the information on temperature and the probability distribution $p_i$ of quantum states at that temperature.

$$\rho = \sum_i p_i \, |u_i\rangle \langle u_i| = \frac{\mathrm{e}^{-\beta H}}{\sum_k \mathrm{e}^{-\beta E_k}}.$$

We have used **classical statistical probabilities** for quantum states $|u_i\rangle$ which are eigenstates of *H*. This is a usual method for quantum systems at finite temperature. Because we are using probabilities at equilibrium, it may also be called the **equilibrium average**.

This statistical approach using Boltzmann statistics allows us to calculate the expectation value of an operator based on the probabilities of different energy states in a system at thermal equilibrium.

# Thermal average in quantum mechanics

- Maxwell-Boltzmann statistics become appropriate in the non-relativistic regime, where particles are well-separated and do not exhibit quantum mechanical effects like Fermi-Dirac or Bose-Einstein statistics. Therefore, in certain stages of the universe's cooling phase, when the temperature dropped and particles became non-relativistic, the Maxwell-Boltzmann statistics provided a suitable framework for describing their behavior.

- Hadronization phase transition occurs at (*150-200 MeV*) temperatures roughly on the order of $T \approx 10^{12-13}$K.

# An useful relation

We can describe the universe as a thermal ensemble, characterized by Hamiltonian $H$, and density matrix $\rho(0)$. The density matrix is also time dependent,

$$\rho(t) = e^{-iHt}\rho(0)\, e^{+iHt}.$$

Thermal average of Baryon Number is given by,

$$\langle B(0)\rangle_T = \frac{\mathrm{tr}[\rho(0)B(0)]}{\mathrm{tr}[\rho(0)]}.$$

$$\langle B(t)\rangle_T = \frac{\mathrm{tr}[\rho(t)B(t)]}{\mathrm{tr}[\rho(t)]} = \frac{\mathrm{tr}\left[\rho(t)\mathrm{e}^{-\mathrm{i}Ht}B(0)\mathrm{e}^{\mathrm{i}Ht}\right]}{\mathrm{tr}[\rho(t)]} = \frac{\mathrm{tr}\left[\mathrm{e}^{\mathrm{i}Ht}\rho(t)\mathrm{e}^{-\mathrm{i}Ht}B(0)\right]}{\mathrm{tr}[\rho(0)]} = \frac{\mathrm{tr}[\rho(0)B(0)]}{\mathrm{tr}[\rho(0)]} = \langle B(0)\rangle_T.$$

As a consequence if by some mechanism $\langle B(0)\rangle_T$=0, it cannot be generated at later time.

# C and CP violation

Let us say that there is an operator:

$$\boldsymbol{O \equiv C \text{ or } CP}.$$

which commutes with the Hamiltonian $[H, O] = 0$, and let $O$ satisfies,

$$\boldsymbol{O\, B(0)\, O^{-1} = -B(0)},$$

Ref: A. D. Dolgov hep-ph/0511213

so that the baryon number is odd under $O$. Then the equilibrium average is,

$$\langle B(0)\rangle_T = \frac{\mathrm{tr}[\rho(0)B(0)]}{\mathrm{tr}[\rho(0)]} = \frac{\mathrm{tr}[O^{-1}O\,\rho(0)B(0)]}{\mathrm{tr}[\rho(0)]} = \frac{\mathrm{tr}[O\rho(0)B(0)O^{-1}]}{\mathrm{tr}[\rho(0)]} = \frac{\mathrm{tr}[\rho(0)\,OB(0)O^{-1}]}{\mathrm{tr}[\rho(0)]} = -\langle B(0)\rangle_T.$$

We obtain $\boldsymbol{\langle B(0)\rangle_T = 0}$. Therefore at a later time, $\boldsymbol{\langle B(t)\rangle_T = 0}$. **(See page 11)** There is no asymmetry. Therefore both C and CP must be violated.

# Out of Equilibrium condition

In a relativistic quantum field theory, Hamiltonian $H$ is $CPT \equiv \theta$ invariant and then $[H, \theta] = 0$, then $[\theta, \rho(0)] = 0$. That is CPT commutes with the density matrix. Now we can calculate the thermal equilibrium average of Baryon number operator at $t = 0$,

$$\langle B(0)\rangle_T = \frac{tr[\rho(0)B(0)]}{\mathrm{tr}[\rho(0)]} = \frac{\mathrm{tr}[\theta^{-1}\theta\rho(0)B(0)]}{\mathrm{tr}[\rho(0)]} = \frac{\mathrm{tr}[\theta\rho(0)B(0)\theta^{-1}]}{\mathrm{tr}[\rho(0)]} = \frac{tr[\rho(0)\theta B(0)\theta^{-1}]}{\mathrm{tr}[\rho(0)]} = -\langle B(0)\rangle_T$$

This shows explicitly that Baryon asymmetry vanishes.

- **Either**: Equilibrium assumption is removed Ref: Huge Literature, See Kolb and Turner.
- **Or**: $[\rho(0), \theta] = 0$ meaning $[H, \theta] = 0$, condition should be removed. CPT VIOLATION

First choice leads to out of equilibrium baryogenesis and second choice leads to CPT violating baryogenesis. Furthermore it is clear that in the second case there is no need to go out of equilibrium. Our revised conditions become: **1) B violation 2) C and CP violation 3) CPT violation.**

# Assumptions behind proof of CPT theorem

**THEOREM: It states that any fundamental physical theory that respects the laws of quantum mechanics and special relativity must also respect the combined symmetries of Charge conjugation (C), Parity transformation (P), and Time reversal (T).**

**J. Schwinger, G. Luders, W. Pauli, J. S. Bell…**

-- it imposes constraints on the possible interactions and properties of elementary particles.

The proof of the CPT theorem generally relies on a few fundamental assumptions:

Lorentz Invariance

Local Quantum Field Theory

Unitarity and Hermiticity of the S-Matrix

Charge Conservation

These assumptions are foundational to the proof of the CPT theorem. From these assumptions, one can mathematically demonstrate that any theory that satisfies these conditions will also exhibit the combined symmetries of Charge (C), Parity (P), and Time Reversal (T) when all three operations are applied together (CPT symmetry).

# Equilibrium Baryogenesis

R. Bertolami, D. Colladay, V. Kostelecky, R. Potting, Phys. Lett. **B395**, 178 (1997)

CPT conservation leads to equality of masses of particles and antiparticles. CPT violating theories do not guarantee this. Probability distribution functions(say as a function of mass, momentum and chemical potential) become different for particles and antiparticles.

$$f(\mu, p, m) = \frac{1}{\mathrm{e}^{\left[\frac{E-\mu}{T}\right]} \pm 1} \text{ where } E^2 = p^2 c^2 + m^2 c^4$$

If particle and antiparticle have different mass, then,

$$n - \bar{n} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} [f(\mu, p, m) - f(\mu, p, \bar{m})]$$

# A general proof

Statement:

$J_B^\mu = \bar{\Psi}\gamma_\mu\psi$ and $B = \int J_B^0(x)\, \mathrm{d}^3x$ : Baryonic current and Charge

$H|u_i\rangle = E_i|u_i\rangle$ : Eigenstates of the Hamiltonian

$O|u_i\rangle = \eta|\, u_i\rangle$ where $|\eta| = 1$ : Discrete Symmetry unitary operator defined as $[H, O] = 0$, which has simultaneous eigenstates with Hamiltonian.

$OB(0)O^{-1} = -B(0)$ : Baryon number is odd under *O*. In particular *O* can be CPT

We will show that *O* must be a broken symmetry operator.

# A general proof

Proof:

$\langle B(t)\rangle = \langle u_i|e^{+iHt}B(0)e^{-iHt}|\, u_i\,\rangle$

$\langle B(t)\rangle = \langle u_i|O^{-1}O\; e^{+iHt}O^{-1}O\; B(0)\; O^{-1}O\;\; e^{-iHt}O^{-1}O|\, u_i\,\rangle$

$\langle B(t)\rangle = |\eta|^2\,\langle u_i|O\; e^{+iHt}O^{-1}O\; B(0)\; O^{-1}O\;\; e^{-iHt}O^{-1}|u_i\rangle$ because *[O,H]=0*

$\langle B(t)\rangle = |\eta|^2\,\langle u_i|e^{+iHt}O\; B(0)\; O^{-1}\; e^{-iHt}|u_i\rangle$ because $|\eta|^2$=1

$\langle B(t)\rangle = \langle u_i|e^{+iHt}O\; B(0)\; O^{-1}\; e^{-iHt}|u_i\rangle$ because $O\;B(0)\;O^{-1}$ = *-B(0)*

$\langle B(t)\rangle = -\langle u_i|e^{+iHt}\; B(0)\; e^{-iHt}|u_i\rangle$

$\langle B(t)\rangle$ = -$\langle B(t)\rangle$

$\langle B(t)\rangle$ = 0 Vanishes for all values of i

Thermal average:= $\langle B(t)\rangle_T = \sum_i p_i\,\langle u_i|B(t)|u_i\rangle$ = 0 Independent of distribution

**Therefore $[H,O] \neq 0$** ***QED***

# Microscopic versus Macroscopic Physics

- CPT violation is a microscopic effect happening at the level of elementary particles
- Out – of – equilibrium Physics is a macroscopic effect linked with the Hubble expansion rate of the universe.

In both cases we need to depart from conventional scenario. Either at the macroscopic level or at the microscopic level.

Where do we want to depart from usual Physics? Microscopic or Macroscopic ? Which option will be more natural?

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# Thank You

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