Seesaw determination of dark matter relic density

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Setting the stage

- Motivations:
  1. Two of the biggest unsolved mysteries: Origin of neutrino masses and Dark matter relic density $\Rightarrow$ Can they be interrelated?
  2. Can dark matter be detected (at least indirectly) in recent future, even if it is very *feebly* coupled to SM?
- Neutrino mass is very elegantly explained by Type-I seesaw mechanism:

$$\mathscr{L}_{\text{seesaw}} = i\overline{N_R}\partial\!\!\!/ N_R - \frac{1}{2}m_N(\overline{N_R}N_R^c + \overline{N_R^c}N_R) - (Y_\nu\overline{N_R}\tilde{H}^\dagger L + h.c.)\,,$$

- The light neutrino masses are given by:

$$m_\nu = -\frac{v^2}{2}Y_\nu^T\, m_N^{-1}Y_\nu$$

- Note, we need at least three heavy neutrinos to explain the three light neutrino masses.
- Only one of the Yukawa couplings can be very small given $\Delta m^2_{\rm sol} \sim 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} \sim 10^{-3}$ eV^2.
- To explain the dark matter we next add a neutrino portal to the hidden sector:

$$\delta\mathscr{L} = -Y_\chi \overline{N} \phi \chi + h.c..$$

- Here both χ and ϕ are SM singlets.
- One or both of them can be dark matter candidates. χ is a Majorana fermion.
- Given the smallness of the Yukawa couplings dark matter is produced by freeze-in mechanism.

Dark matter production

- We assume that $m_N < m_{Z,W,h}$ and $m_{N_{2,3}} > m_h$.
- N_2 and N_3 do not take part in DM production and is assumed to have very small neutrino portal interactions.
- DM is produced via freeze-in primarily from $N \to \phi\,\chi$ decay (controlled by y_χ).
- Because of this, the comoving number density $Y_N\big|_{T\sim m_Z} = Y_\phi\,(T_0) = Y_\chi\,(T_0)$.
- Hence it is sufficient to calculate Y_N (controlled by the seesaw couplings, Y_ν) and thereby establishing an one-to-one correspondence between the DM and seesaw parameters!
- Important: The relic density becomes independent of y_χ (hence the correspondence!) only if the two body decay is the dominant mode of production (more on this later).

- N is produced dominantly from decays: $h \to N\nu, W^{\pm} \to N l^{\pm}, Z \to N\nu$.
- The decay width of $V \to N f$ is given by:

$$\Gamma_{V\to N f} = \frac{1}{48\pi} m_V |Y_{\nu i}|^2 f(m_N^2/m_V^2).$$

where $f(x) = (1-x)^2(1+2/x)$ and V is $W^{\pm}$ or Z.

- For $m_N < m_V$ the gauge boson decay width is enhanced by a factor of m_V^2/m_N^2 wrt that of h.
- Freeze-in condition entails: $\Gamma_V/H|_{T\simeq m_Z} \lesssim 1 \Rightarrow$ $\sum_i |Y_{\nu i}|^2 \lesssim 1\cdot 10^{-16}\cdot\left(\frac{m_N}{10\,\text{GeV}}\right)^2$
- After solving a simple Boltzmann Eq. we get $Y_{\rm DM}^{\rm today} = 3\times 10^{-4} \sum_{i=h,Z,W} \frac{g_i \Gamma_i}{M_i^2}$

- Hence, one finally obtains

$$\Omega_{DM}h^2 \simeq 10^{23} \sum_i |Y_{\nu i}|^2 \left(\frac{m_\chi + m_\phi}{1\,\text{GeV}}\right) \left(\frac{10\,\text{GeV}}{m_N}\right)^2 .$$

- Equating this to 0.12 we get:

$$\sum_i |Y_{\nu i}|^2 \simeq 10^{-24} \cdot \left(\frac{m_N}{10\,\text{GeV}}\right)^2 \left(\frac{1\,\text{GeV}}{m_\chi + m_\phi}\right). \qquad (1)$$

- Using $m_{\nu_1} < \sum_i |Y_{\nu i}|^2 v^2/(2m_N)$ we get

$$m_{\nu_1} < 4 \cdot 10^{-12}\,\text{eV} \cdot \frac{m_N}{10\ \text{GeV}} \cdot \left(\frac{1\,\text{GeV}}{m_\chi + m_\phi}\right). \qquad (2)$$

- $f\bar{f} \to NL$: only 20% of the total N number density.
- The one-to-one correspondence holds iff: $\Gamma_{N\to\phi\,\chi} > \sum_f \Gamma_{N\to\nu f\bar{f}} + \Gamma_{N\to l f \bar{f}'}$

Three-body decays, neutrino line ...

- The two body decay width is given by:

$$\Gamma_{N\to\chi\phi} \simeq \frac{1}{16\pi} m_N |Y_\chi|^2 \left(1 + \frac{2m_\chi}{m_N}\right)$$

- The three body width is given by:

$$\Gamma_{N\to\nu f\bar{f}} = \frac{N_c}{1536\pi^3} |Y_{\nu i}|^2 \frac{g_2^2}{\cos\theta_W^2} (g_L^2 + g_R^2) \frac{m_N^3}{m_Z^2},$$

  and similarly for $N \to \ell f \bar{f}'$.

- Therefore $\Gamma_{N\to\phi\chi} > \sum_f \Gamma_{N\to\nu f\bar{f}} + \Gamma_{N\to l f\bar{f}'}$ implies a lower limit on y_χ:

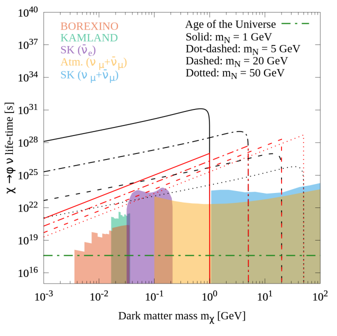
$$|Y_\chi|^2\Big|_{\min} \simeq 10^{-4} \sum_i |Y_{\nu i}|^2 (m_N/10\,\text{GeV})^2 \quad (3)$$

- Further, if $m_\chi > m_\phi$ then it can dominantly decay (with life-time > age of the Universe) to produced a neutrino line.

- The decay width is given by:

$$\Gamma_{\chi\to\phi\nu} = \frac{1}{32\pi}|Y_\chi|^2 \frac{\sum_i |Y_{\nu i}|^2 v^2}{m_N^2} m_\chi \left(1 - \frac{m_\phi^2}{m_\chi^2}\right)^2 \tag{4}$$

- This life-time has a lower limit as dictated by several neutrino experiments[1] $\Rightarrow y_\chi^2|_{\max}$. Thus, Using (1) and (3) in (4) we get the black lines as upper-limit on τ_χ:



[1] JHEP05 (2021) 101 (Coy, Hambye)

Constraints

- BBN: Constraints from BBN is not a matter of concern because the number of N particles decaying is very limited, and they negligibly contribute to the total energy density at this time (hence to the Hubble expansion rate) even if N decays into two particles which are relativistic.
- Moreover, the decay is into χ and ϕ, which do not cause any photo-disintegration of nuclei since they do not produce any electromagnetic or hadronic material.
- Structure Formation: Imposing that DM, which has kinetic energy $\sim m_N/2$ when produced from N decay, redshifts enough so that it is non-relativistic when $T \sim$ keV gives an upper bound on the χ lifetime (the red lines in the plot)

$$\tau_\chi \lesssim 10^{28}\,\text{sec}\left(\frac{m_{DM}}{m_N}\right)^2\left(\frac{m_N}{10\,\text{GeV}}\right). \tag{5}$$

A closer look at structure formation

- More formally, we should calculate $\lambda_{\rm fs}$ and compare it with the limit obtained from Ly-α.
- Free streaming length is given by:

$$\lambda_{FS} = \int_{t_i}^{t_0} \frac{\langle v(t)\rangle}{a(t)} dt = \frac{1}{m_\chi} \int_{t_i}^{t_0} \frac{\langle p_\chi(t)\rangle}{a(t)} dt$$

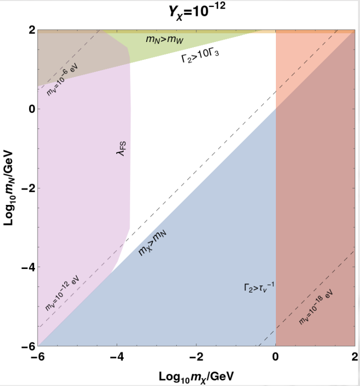
- Here, $\langle p_\chi \rangle = \int d^3 p_\chi f_\chi p_\chi / (\int d^3 p_\chi f_\chi)$.
- The distribution function f_χ is solved via:

$$\hat{L} f_N = Hx \frac{\partial f_N}{\partial x} = \mathscr{C}_N(Z \to N\nu) + \mathscr{C}_N(W^\pm \to N\ell^\pm) - C_N(N \to \chi\phi)$$

where, $\hat{L} = \left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)$, $x \equiv m_N/T$, and

$$Hx\frac{\partial f_\chi}{\partial x} = \mathscr{C}_\chi(N \to \chi\phi) = \frac{Y_\chi^2\left((m_N+m_\chi)^2 - m_\phi^2\right)}{16\pi p_\chi E_\chi} \int_{E_{N(\chi),-}}^{E_{N(\chi),+}} dE_N f_N$$

- Using the distribution function we can find $\lambda_{\rm fs}$ and enforce $\lambda_{\rm fs} < 66$ kpc.
- The allowed parameter space for a typical y_χ [2]:



- Theoretically, $y_\chi \lesssim 10^{-10}$ since $\Gamma_\chi > \tau_{\rm universe}^{-1}$.

[2] JCAP 02 (2023) 028, Rupert Coy, AG

A second scenario: Relativistic Freeze-out

- Consider that the heaviest particle among χ and ϕ has a lifetime $<$ the age of the universe $\Rightarrow$ much larger values of y_χ.
- In this case, DM is made of only the lightest species and no neutrino line can be observed.
- A large y_χ coupling $\Rightarrow$ thermalisation of N, χ and ϕ.
- The thermalised hidden sector is characterized by a temperature, $T' < T$.
- The one-to-one connection is lost ?

– Yes, if DM undergoes a non-relativistic, secluded freeze-out in the hidden sector.

– But here, since $m_\phi < m_N, m_\chi$, the ν-portal annihilation processes ($\phi\phi \leftrightarrow \chi\chi$ etc) will not decouple when DM is non-relativistic but when DM is relativistic.

$\Rightarrow$ DM relic doesn't depend on the annihilation cross section but only on T'/T.

– T'/T is set by SM→N freeze-in and $\sim 10^4\, y_\nu^{1/2}\, \sqrt{10\,\mathrm{GeV}/\mathrm{m_N}}$.

- T'/T can be estimated by considering that at the peak of N freeze-in production, when $T \simeq m_Z$, each N has an energy $\simeq m_Z$, so that the dark sector energy density is

$$\rho_{DS}|_{T\simeq m_Z} \simeq n_N|_{T\simeq m_Z}\, m_Z = (\pi^2/30) g^{\star}_{HS} T'^4\,, \qquad (6)$$

with n_N given by $Y_N = n_N/s$ found earlier.

- Knowing T'/T we can find the relic density by[3]:

$$\Omega_{DM} = 1.74 \times 10^{11} \left(\frac{m_\phi}{1\,TeV}\right) \left(\frac{T'}{T}\right)^3 \left(\frac{g_{\mathrm{DM}}}{g^s_\star}\right) \qquad (7)$$

where $n_{\mathrm{DM}} \sim T'^3$ and entropy conservation at decoupling time is used.

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[3] Phys.Lett.B 807 (2020) 135553, Hambye, Lucca, Vanderheyden.

- Using (6) in (7) we get:

$$\Omega_{DM}h^2 \simeq 2.5\times 10^{18}\left(\sum_i |Y_{\nu i}|^2\right)^{3/4} \cdot g_{DM}\left(\frac{1\,\mathrm{GeV}}{m_N}\right)^{3/2}\left(\frac{m_{DM}}{100\ \mathrm{MeV}}\right), \qquad (8)$$

- Note that this requires slightly smaller values of Y_ν couplings than the first scenario, because the dark sector thermalisation process increases the number of DM particles.
- T'/T can be more accurately calculated using [4]:

$$\frac{d\rho_{\mathrm{DS}}}{dt}+4H\rho_{\mathrm{DS}}=\frac{1}{a^4}\frac{d(\rho_{\mathrm{DS}}\,a^4)}{dt}=-\sum_{i=Z,h,W}\frac{g_i}{2\pi^2}m_i^3\,T\,\Gamma_i K_2(m_i/T)$$

- The results are in good agreement with Eq.(7).

[4] JCAP05(2012)034, Chu, Hambye, Tytgat

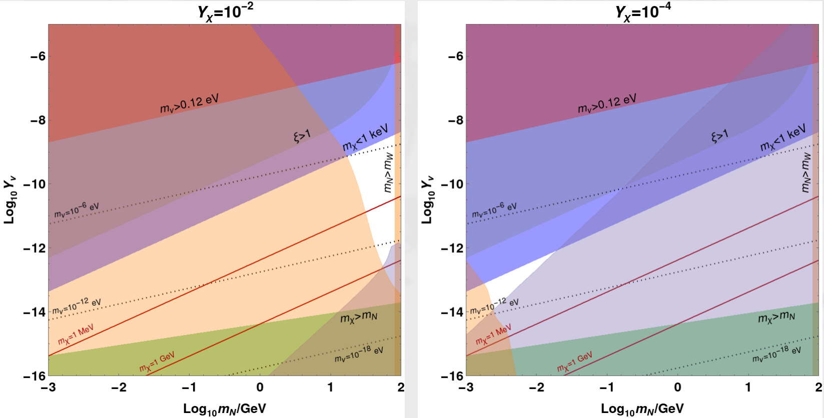
y_χ dependence in Scenario-II

- This scenario is analogous to the freeze-out of neutrinos.
- N acts as the heavy mediator instead of W/Z.
- T' is hence determined by annihilations like $\chi\chi \to \phi\phi$ with N as a mediator ($m_N > T' > m_{\rm DM}$).
- Using $n\langle\sigma v\rangle_{\rm FD} \sim H(T)$ and the expression for T'/T we get,

$$T'_{\rm dec} \sim 10 \left(\frac{10^{-12}}{Y_\nu}\right)\left(\frac{0.01}{Y_\chi}\right)^4 \left(\frac{m_N}{\rm GeV}\right)^3 \ \text{keV}\,. \qquad (9)$$

- For relativistic freeze-out we should have $T' > m_{\rm DM} \Rightarrow$ upper limit on y_χ.
- But, before all these one should explicitly check that whether the dark sector particles have indeed themalised among themselves.
- This is controlled by annihilations of the type $\chi\chi \to NN$.
- Condition for thermalisation gives a lower bound on y_χ.

- A collection of y_χ dependent and independent constraints are shown below for typical values of y_χ [5]:



- We find: $10^{-4} \lesssim y_\chi \lesssim 10^{-2}$.

[5] JCAP 02 (2023) 028, Rupert Coy, AG

Summary

- Seesaw-induced W, Z and h decays could be at the origin of the DM relic density, even though DM is not a seesaw sterile neutrino.
- the usual type-I seesaw model turns out to have sufficient flexibility to allow freeze-in production of DM from these decays in a way which is determined only by the seesaw parameters and the mass of the DM particle.
- As always for freeze-in, these scenarios are not easily testable because they are based upon the existence of tiny interactions.
- The first scenario predicts a neutrino-line within reach of existing or near-future neutrino telescopes.
- Moreover, both scenarios are falsifiable as they predict a small mass for the lightest neutrino.
- Scenario-I is less restrictive than Scenario-II as far as the 1-to-1 correspondence is concerned.

THANK YOU

Backup: Distribution functions and number densities

- The distribution of f_N controls when the χ production stops:

$$f_N(x, y_N) \propto \exp\left[\frac{-\Gamma_N}{2x^2 H(x)}\left(x\sqrt{x^2+y_N^2} - y_N^2 \tanh^{-1}\frac{x}{\sqrt{x^2+y_N^2}}\right)\right]$$

- Comoving number density: