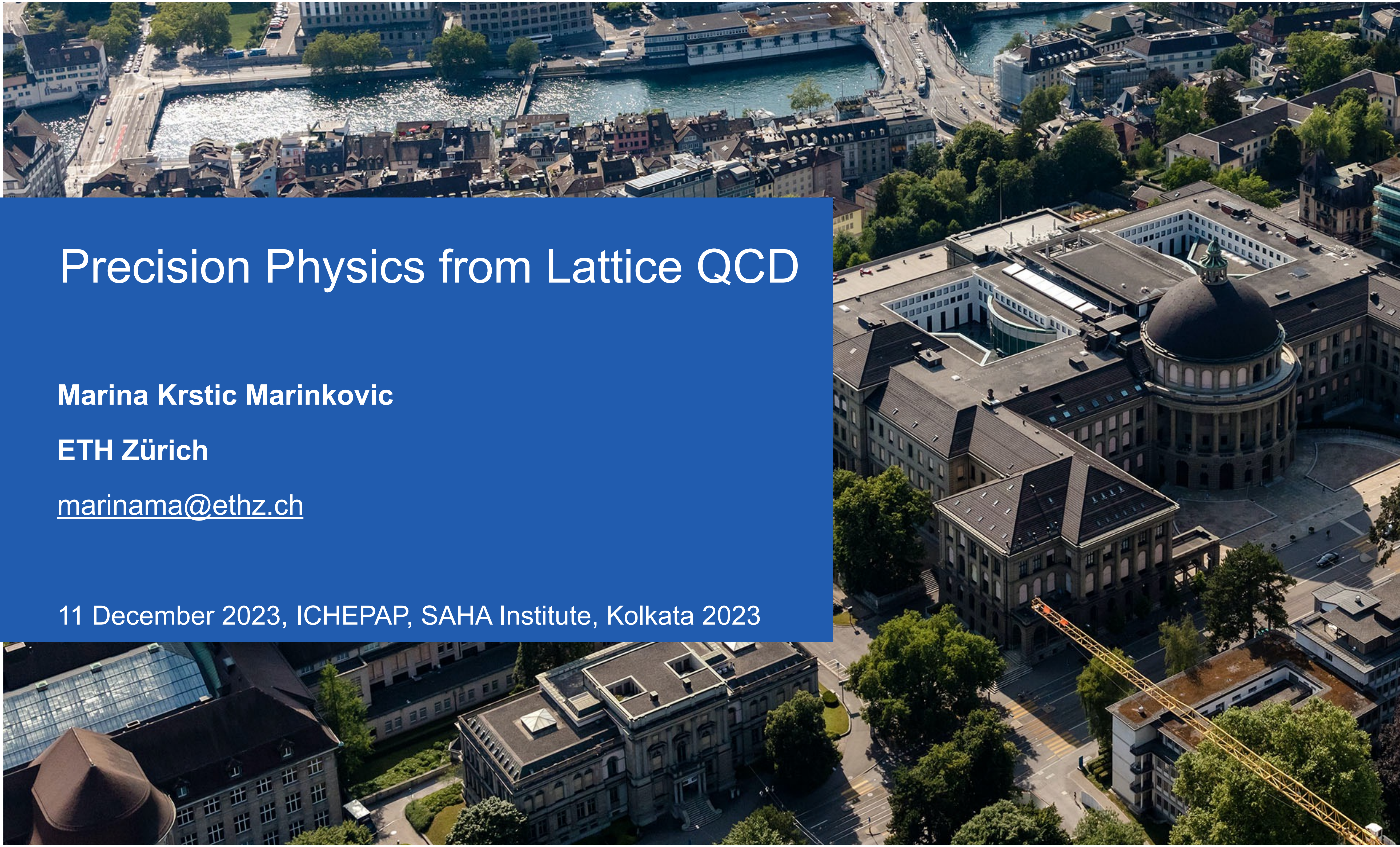


Precision Physics from Lattice QCD

Marina Krstic Marinkovic

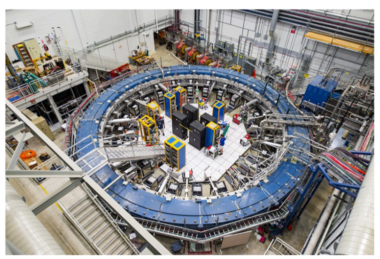
ETH Zürich

marinama@ethz.ch

11 December 2023, ICHEPAP, SAHA Institute, Kolkata 2023

Outline

1. Setting the Stage: **Anomalous Magnetic Moment of the Muon** on the lattice



https://muon-gm2-theory.illinois.edu/white-paper/

arXiv:2006.04822

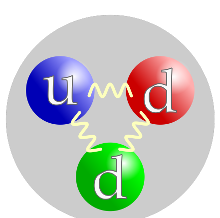
STRONG 2020

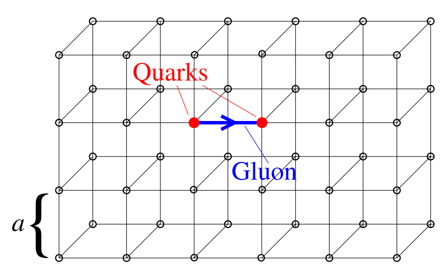
arXiv:2201.12102

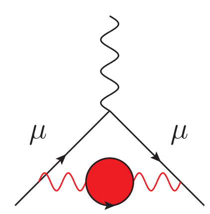
Snowmass 2021

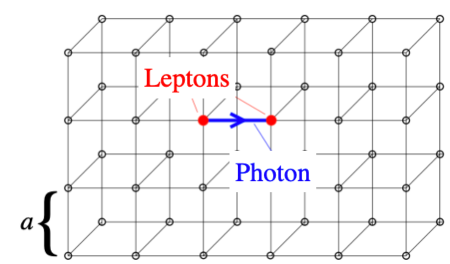
arXiv:2203.15810

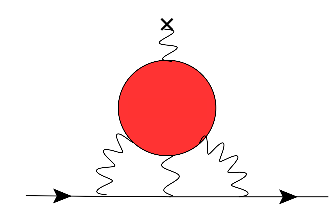
2. **HLbL** from Lattice QCD and QCD+QED



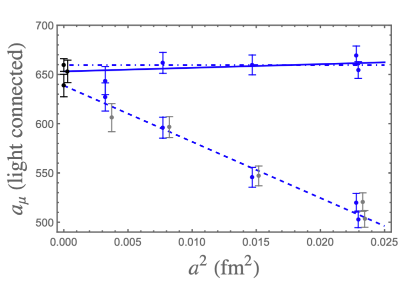




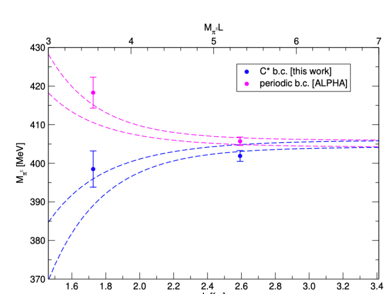




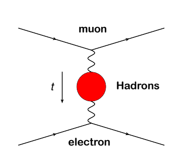
3. **HVP:** Status and New Approaches

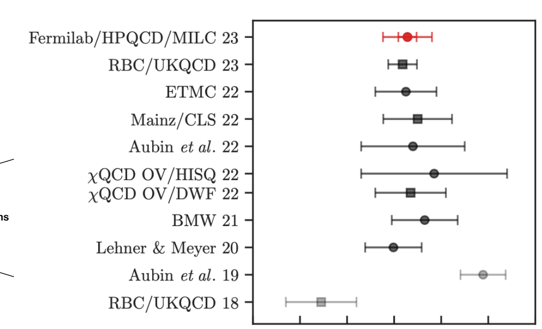


arXiv:2204.12256

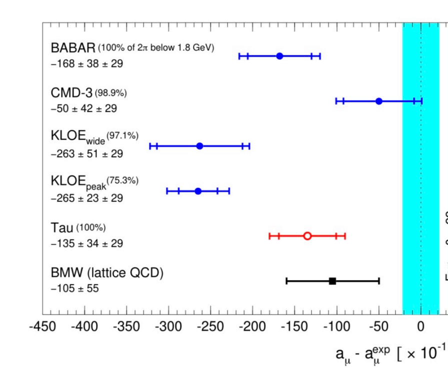


arXiv:2209.13183





arXiv:2301.08274



arXiv: 2312.02053

The magnetic moment of the muon: $a_\mu = \frac{g_\mu - 2}{2}$

EXPERIMENT:

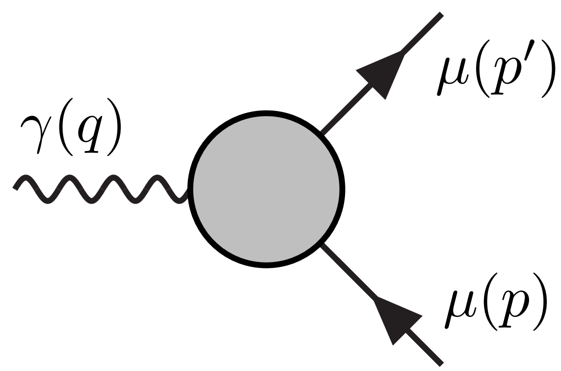
a_μ^{exp} = 11659208.0(6.3) × 10 $^{-10}$ (0.54ppm) [BNL E821, 2006-2008]

a_μ^{exp} = 11659205.9(2.2) × 10 $^{-10}$ (0.19ppm) [Fermilab Muon g-2, 2018-2023]

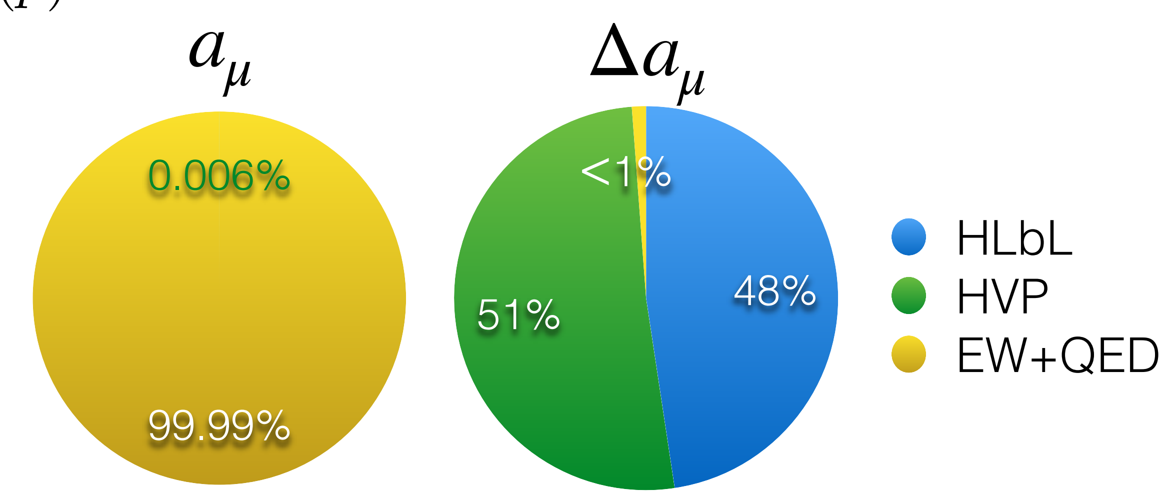
THEORY:

a_μ^{th} = 11659181.0(4.3) × 10 $^{-10}$ (0.37ppm) [Muon g-2 Theory Initiative, Phys.Rept. 887 (2020), 1-166]

TENSION: **3.7 σ / 4.2 σ**



$$a_\mu = \frac{g_\mu - 2}{2}$$

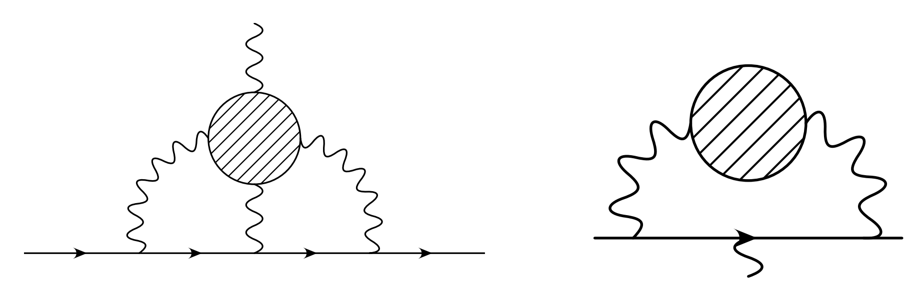


Standard Model:

(1) Q.E.D.

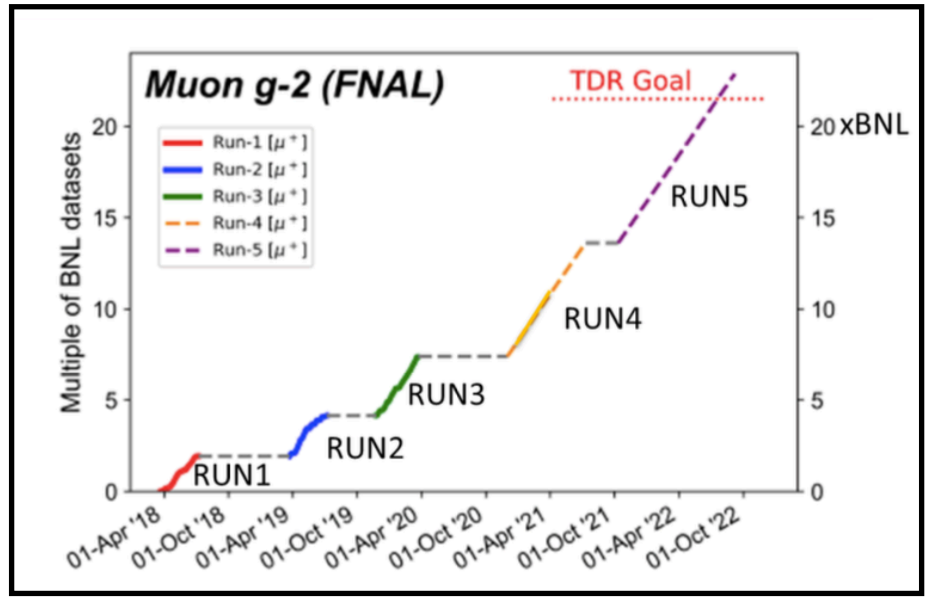
(2) Electroweak

(3) Hadronic



[WP20] T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166
White Paper Muon g-2 Theory Initiative: https://muon-gm2-theory.illinois.edu

a_μ from experiment: FNAL E989, J-PARC, ...



PAUL SCHERRER INSTITUT

arXiv:2201.12102

arXiv:2201.12102

https://cds.cern.ch/record/2677471

D. B. Abi *et al.* (Muon g−2 Collaboration) Phys. Rev. Lett. 126, 141801 – 7 April 2021

D. B. Aguillard *et al.* (Muon g−2 Collaboration) Phys. Rev. Lett. 131, 161802 (2023) – 10 August 2023

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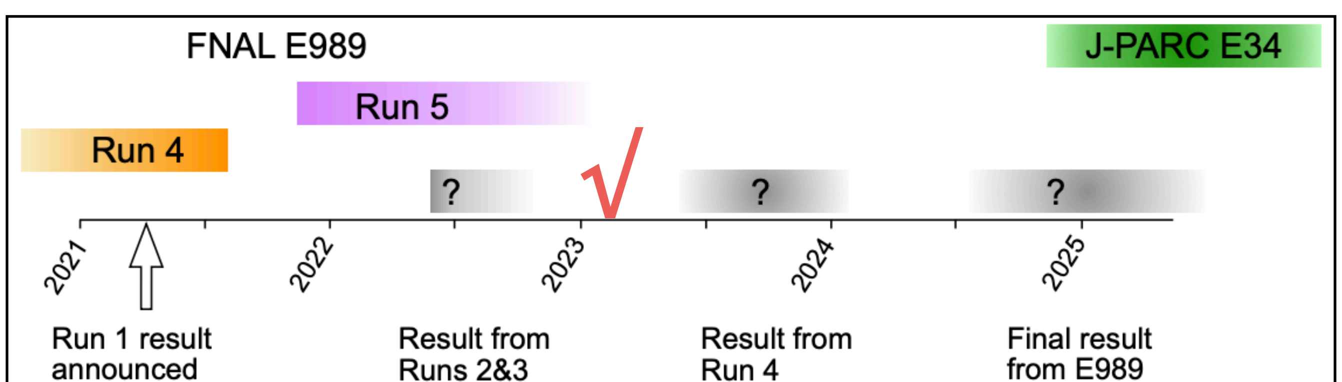
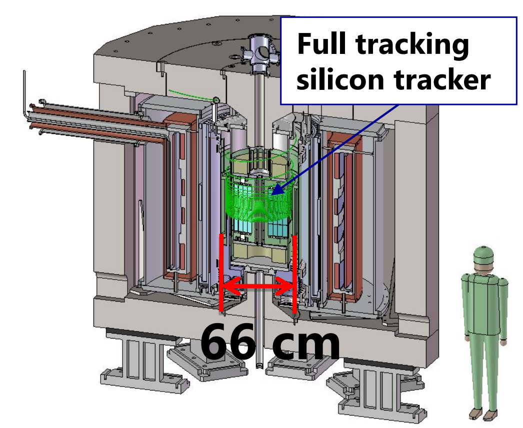




J-PARC

Muon g-2/EDM

Data driven HVP approach: dispersive methods

[see talk by Martin Hoferichter, Tue, 17:45]

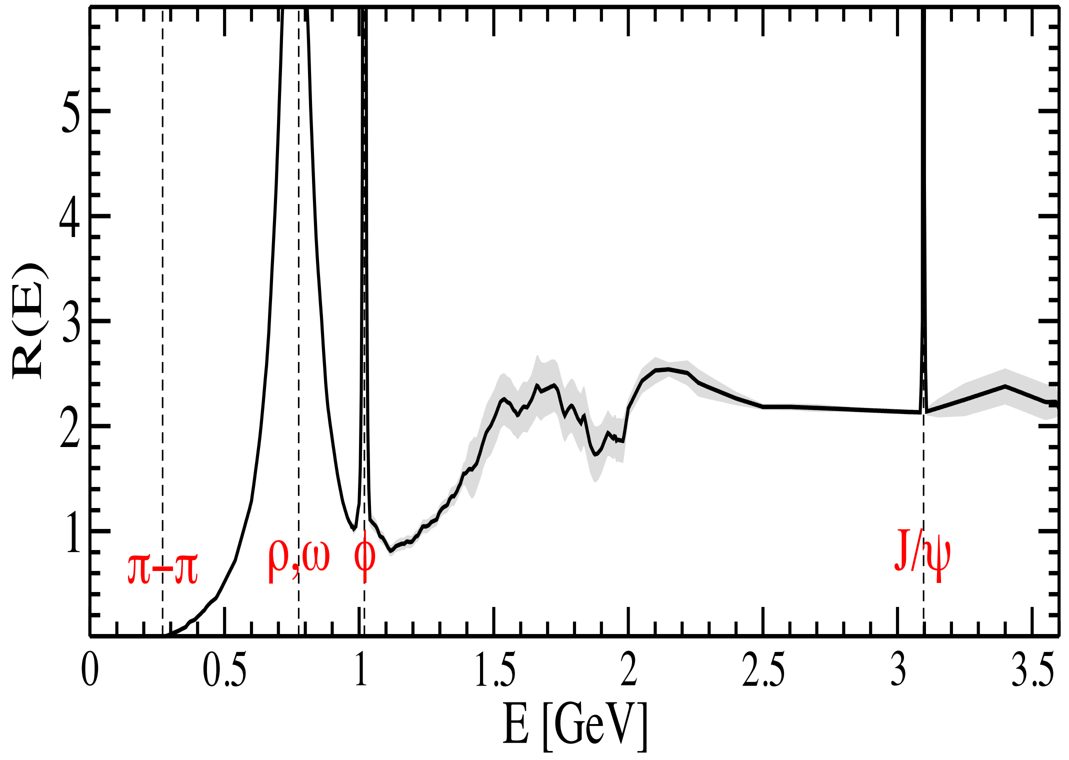
- Relation between the $\mathcal{R}e\,\Pi(Q^2)$ and $\mathcal{I}m\,\Pi(Q^2)$:

$$\Pi(Q^2) - \Pi(0) = \frac{Q^2}{\pi} \int_0^\infty ds \frac{\mathcal{I}m\,\Pi(s)}{s(s-Q^2)}$$

- Imaginary part of $\Pi(s)$ is related to the experimental total cross-section in e+e- annihilation:

$$\mathcal{I}m\,\Pi(s) = \frac{\alpha}{3} R(s)$$

- Important contributions:
- $\rho, \omega, \phi, J/\psi$



Data driven HVP approach: dispersive methods

[see talk by Martin Hoferichter, Tue, 17:45]

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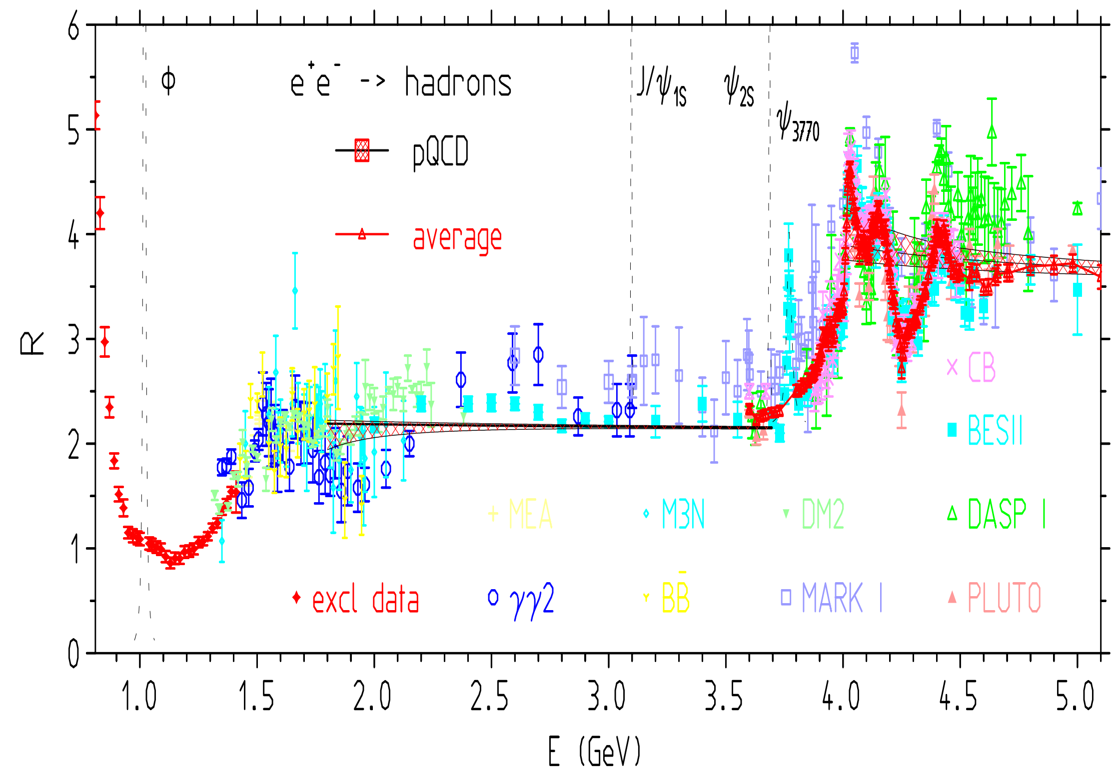
- Imaginary part of $\Pi(s)$ is related to the experimental total cross-section in e+e- annihilation:

$$\mathcal{I}m\,\Pi(s) = \frac{\alpha}{3}R(s)$$

- Other than: $\rho, \omega, \phi, J/\psi$
- **O(1000)** channels
- Model calculations had to be used for some channels
- [Keshavarzi, Nomura, Teubner, Phys.Rev. D97 (2018) no.11]

Lattice QCD (+QED) provide a way to compute these contributions in a model-independent way

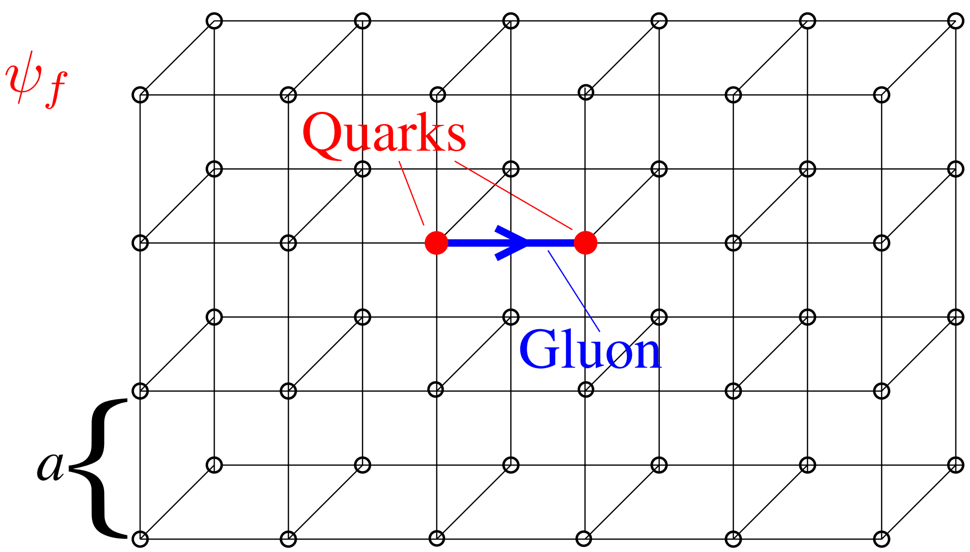


Non-perturbative treatment of QCD

- Systematic method for computing hadronic observables from first principles
- “Sacrifice” $N_f + 1$ observables to fix the parameters (e.g. hadron masses), and everything else is a prediction of the theory

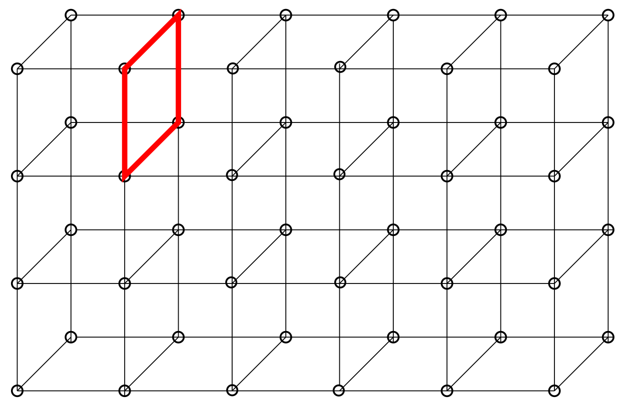
$$\mathcal{L}^E_{QCD} = \frac{1}{2g} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=u,d,s,\ldots} \overline{\psi}_f \left\{ \gamma_\mu \left(\partial_\mu + i A^a_\mu T^a \right) + m_f \right\} \psi_f$$

$$S^E_{QCD} = \int d^4x \, \mathcal{L}^E_{QCD}$$

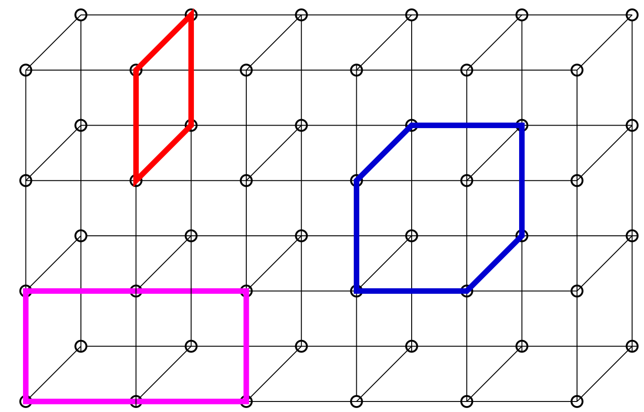


Independent calculations using different choice of the discretization for **S_G** and **S_F**. Results agree once continuum limit is taken!

Lattice QCD: choice of fermion discretization



Wilson's (plaquette) Gauge Action

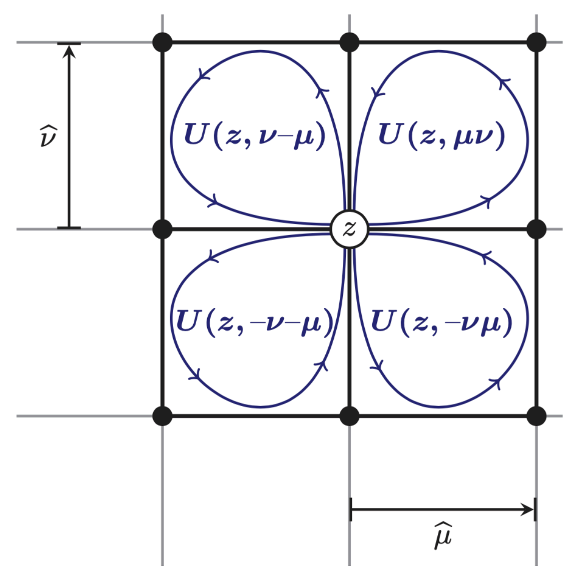


Luscher-Weisz Gauge Action

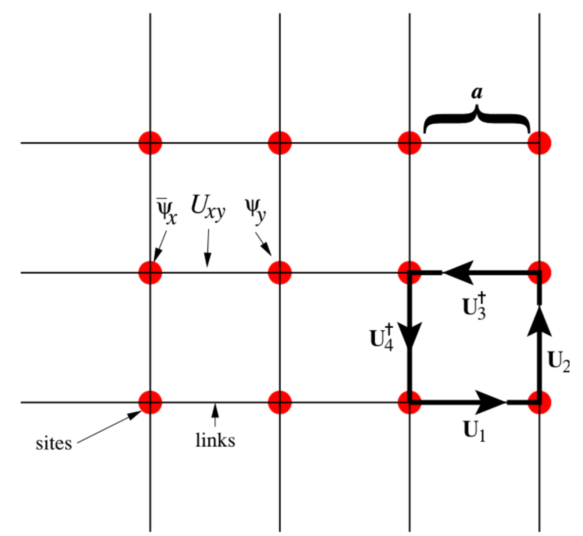
Iwasaki Gauge Action

- Different lattice collaborations

- Different discretization prescription for fermionic action

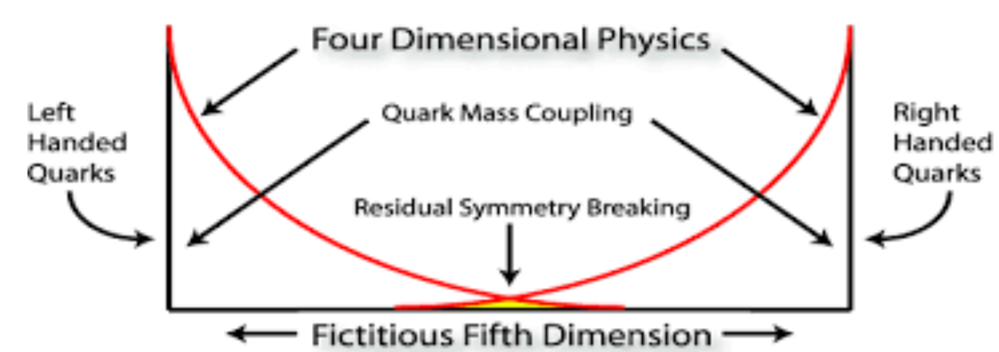


Improved Wilson Fermions



Staggered Fermions

Results agree once continuum limit is taken!



Doman Wall Fermions

Recipe for Lattice QCD Computation

(1) Generate ensembles of field configurations using *Monte Carlo*

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- Compute correlation function of fields, extract Euclidean matrix elements or amplitudes
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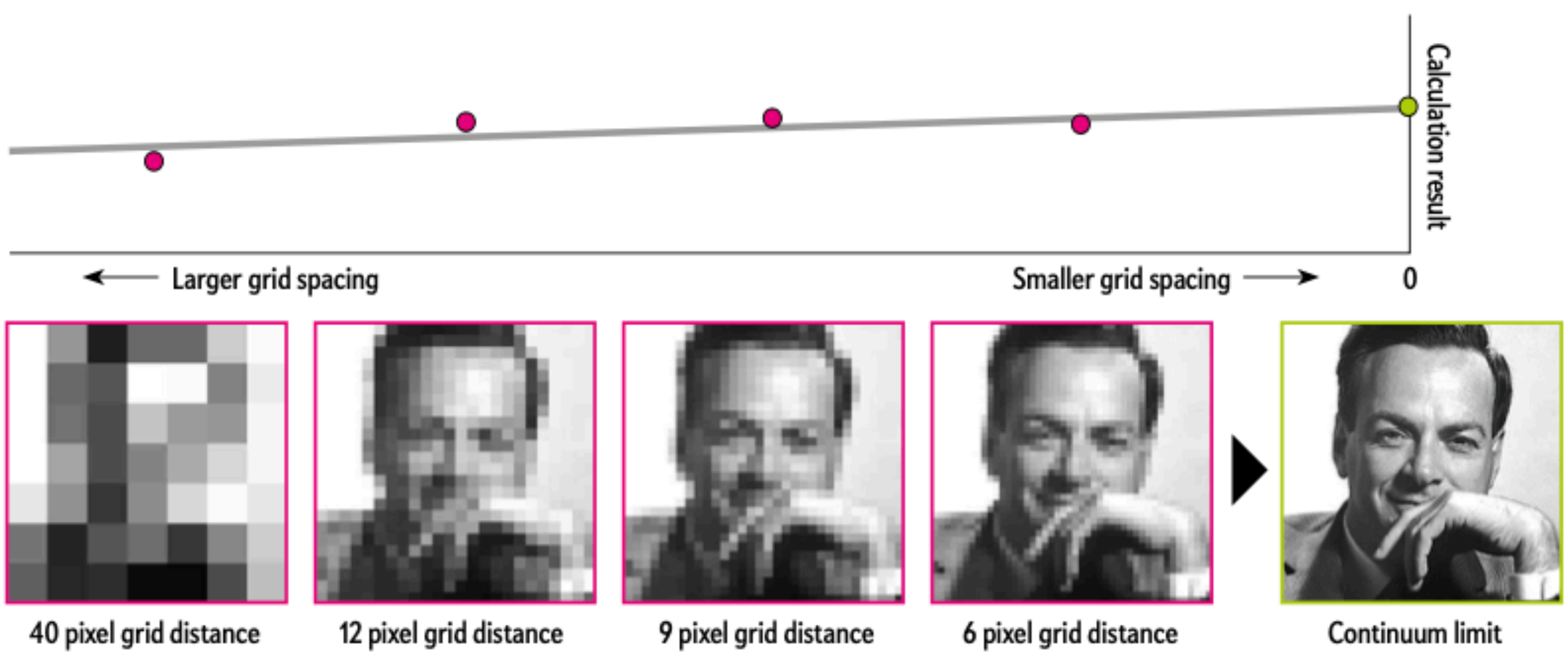
(3) Extrapolate to continuum, infinite volume, physical quark masses (now directly accessible)



Beyond Lattice QCD

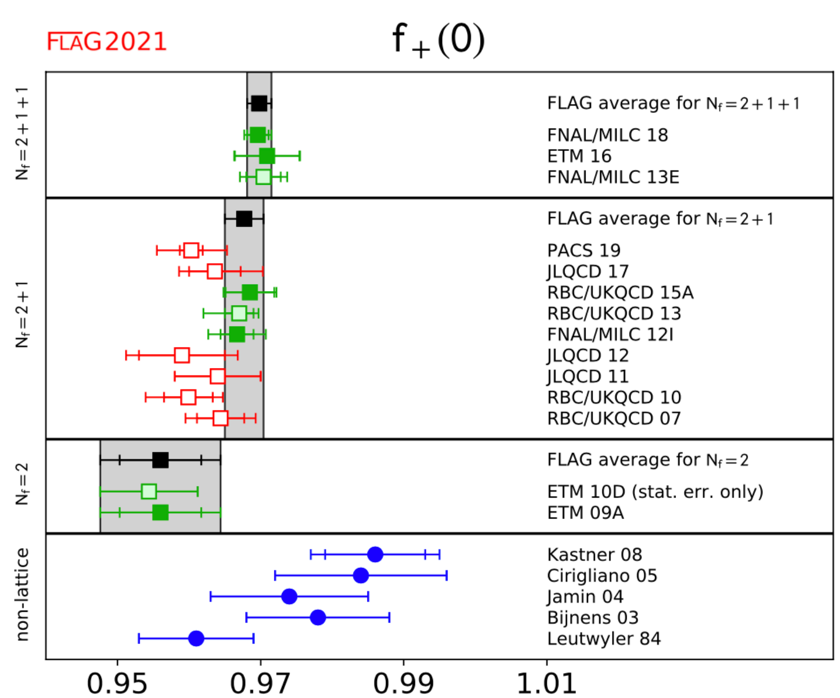
- Important source of systematics:
  - ➡ neglecting/incomplete treatment of **isospin breaking effects**

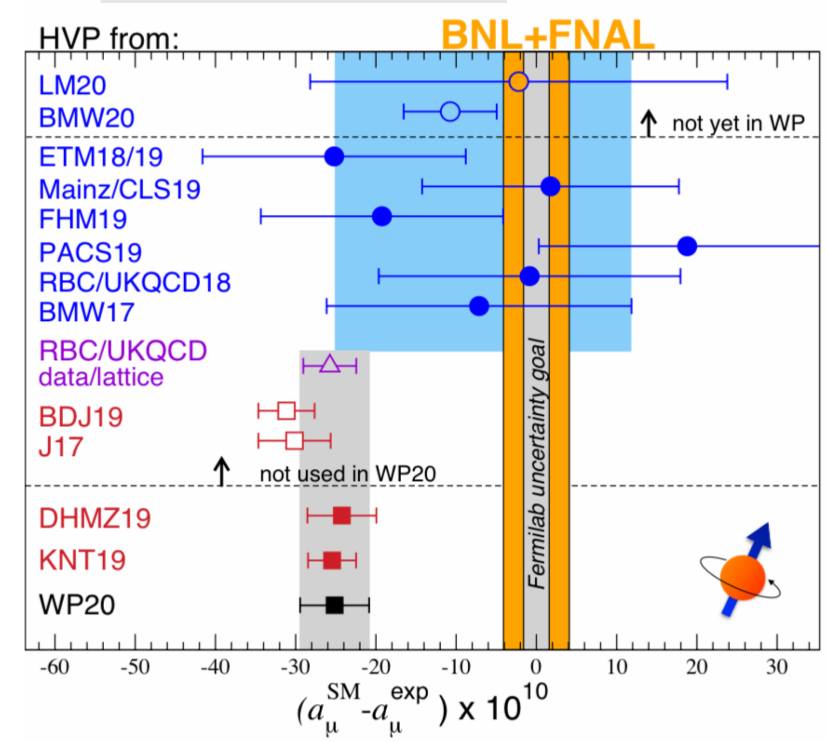
First decades of Lattice QCD, typically: $m_u = m_d$ **and** $\alpha_{em} = 0$

❖ Few percent effects: $\frac{m_u - m_d}{M_p} \simeq 0.3\%$ $\alpha_{\text{EM}} = 0.7\%$ $\frac{M_n - M_p}{M_n} \simeq 0.1\%$

❖ Have to be taken into account for the goal precision: $\frac{\Delta a_\mu^{HVP}}{a_\mu^{HVP}} \approx 0.002$

$\frac{\Delta f_+(0)}{f_+(0)} \approx 0.002 - 0.003$





[FLAG21 2203.15810 [hep-ph]]

[Snowmass22 2203.15810 [hep-ph]]

Beyond Lattice QCD

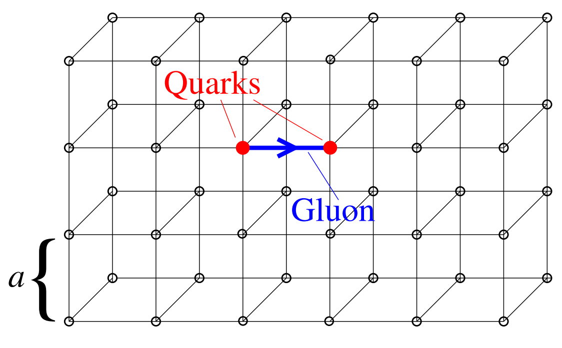
- Important source of systematics:
  - ➡ neglecting/incomplete treatment of **isospin breaking effects**

$$m_u \neq m_d \ \textbf{ and } \ \alpha_{em} \neq 0$$

➡ expand about iso-symmetric theory

QCD TREATED NON-PERTURBATIVELY, α_{em} “SMALL”

[R123: 1303.4896, PRD87(2013)11]

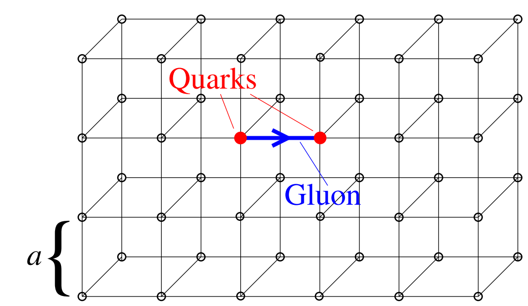


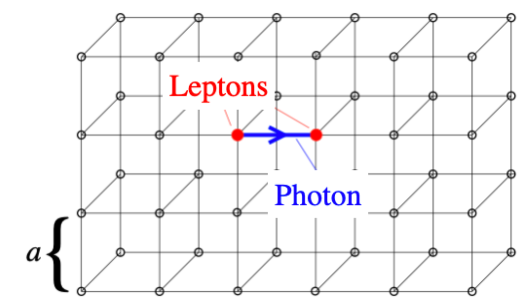
$O(\alpha_{em}), O(m_u - m_d)$

➡ simulate QED+QCD

QCD AND QED TREATED NON-PERTURBATIVELY,

[Duncan,Eichten, hep-lat/9602005, PRL76(1996),
Blum et al. 0708.0484 *PRD* 76 (2007)114508, …]
[BMW: 1406.4088 Science 347 (2015),
QCDSF: 1509.00799, *JHEP* 04(2016) 093,
RCstar: 2209.13183 JHEP03(2023)012, …]





Challenges for IB effects lattice computation

- **FV effects** <—— the way the infrared divergence associated with the zero momentum mode of the photon propagator is canceled on the lattice:

  ➡ continuum: $$\alpha_{em} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \cdots$$ ➡ lattice: $$\frac{\alpha_{em}}{V} \sum_k \frac{1}{k^2} \cdots$$

- Gauss law does not allow a non-zero charge to exist in a finite periodic box: $q(t) = \int d^3x\, \rho(t, \mathbf{x}) \equiv 0$

- Three ways to deal with IR divergence:

  ➡ **Modify gauge filed**: removing the global zero-mode/ spatial zero mode per timeslice ($QED_{\rm TL}/QED_{\rm L}$) [PRL76(1996), Prog. Theor. Phys. 120(2008)413, Science 347 (2015) 1406.4088, …]

  ➡ **Massive photon** [PRL 117 (2016) 7, PoS LATTICE2021 (2022) 281]

  ➡ QED_∞ [Phys. Rev. D 96 (2017), Phys. Rev. D 100 (2019) 094509]

  ➡ **C* boundary conditions** (no zero-mode present) [JHEP 1602, 076, EPJC 80 (2020) 3, JHEP03(2023)012]

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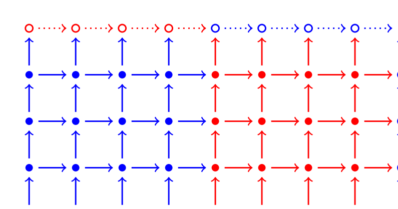
QCD AND QED TREATED NON-PERTURBATIVELY,

[R123: 1303.4896, PRD87(2013)11]

HVP:
- [RBC/UKQCD: JHEP09 (2017) 153, PRL121(2018)022003]
- [BMW: Borsanyi et al. Nature 593 (2021)7857]
- [Mainz: PRD106(2022)11]
- ...

[openQ*D: https://gitlab.com/rcstar/openQxD**]**



[RCstar: Campos et al. Eur.Phys.J.C 80 (2020) 3, Bushnaq et al. JHEP03(2023)012]

HVP: [RCstar: Altherr et al. PoS LATTICE2022 (312), PoS LATTICE2022 (281)]

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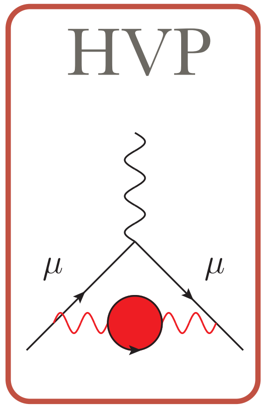
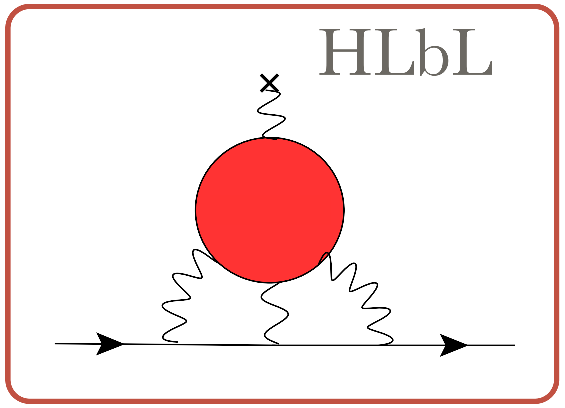
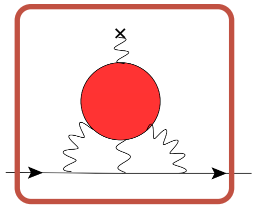




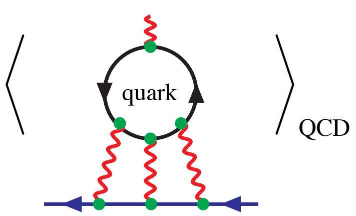


Muon g-2 Light-By-Light Contribution

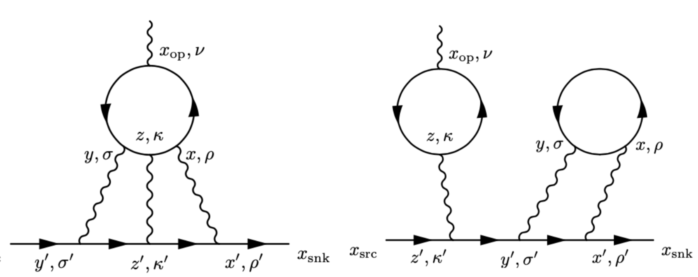
- Target relative precision: $\leq 10\,\%$
  - ➡ Calculations not as mature as the lattice HVP determinations

Two completely independent methods/calculations

❖ RBC/UKQCD ($QCD + QED_{\mathrm{L}}$):



[Blum, Izubuchi, Hayakawa, PoS (LAT2005)353 hep-lat/0509016, +Chowdhury,1407.2923 PRL114(2015)1]



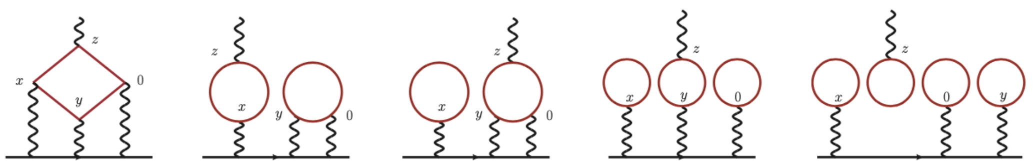
[T. Blum et al, 1610.04603, PRL118 (2017)2, 1911.08123, PRL124(2020)13]

❖ Mainz group ($QCD + QED_{\infty/\mathrm{cont.}}$) :

$$a_\mu^{HLbL} \propto \int f'(q^2) e^{iqx} \langle J_\mu(x_1) J_\nu(x_2) J_\rho(x_3) J_\sigma(x_4) \rangle$$

[P. Rakow (QCDSF), Glasgow 2007]

[Colangelo, Hoferichter, Procura, Stoffer, 1402.7081 JHEP09(2014)091, 1506.01386 JHEP09(2015)074]



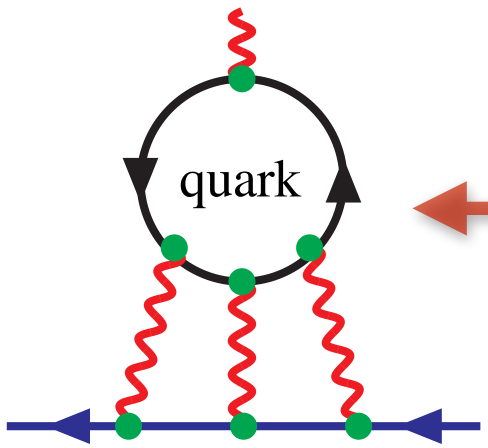
[Chao et al, 2104.02632, EPJC81(2021)651, 2204.08844 EPJC82(2022)8]

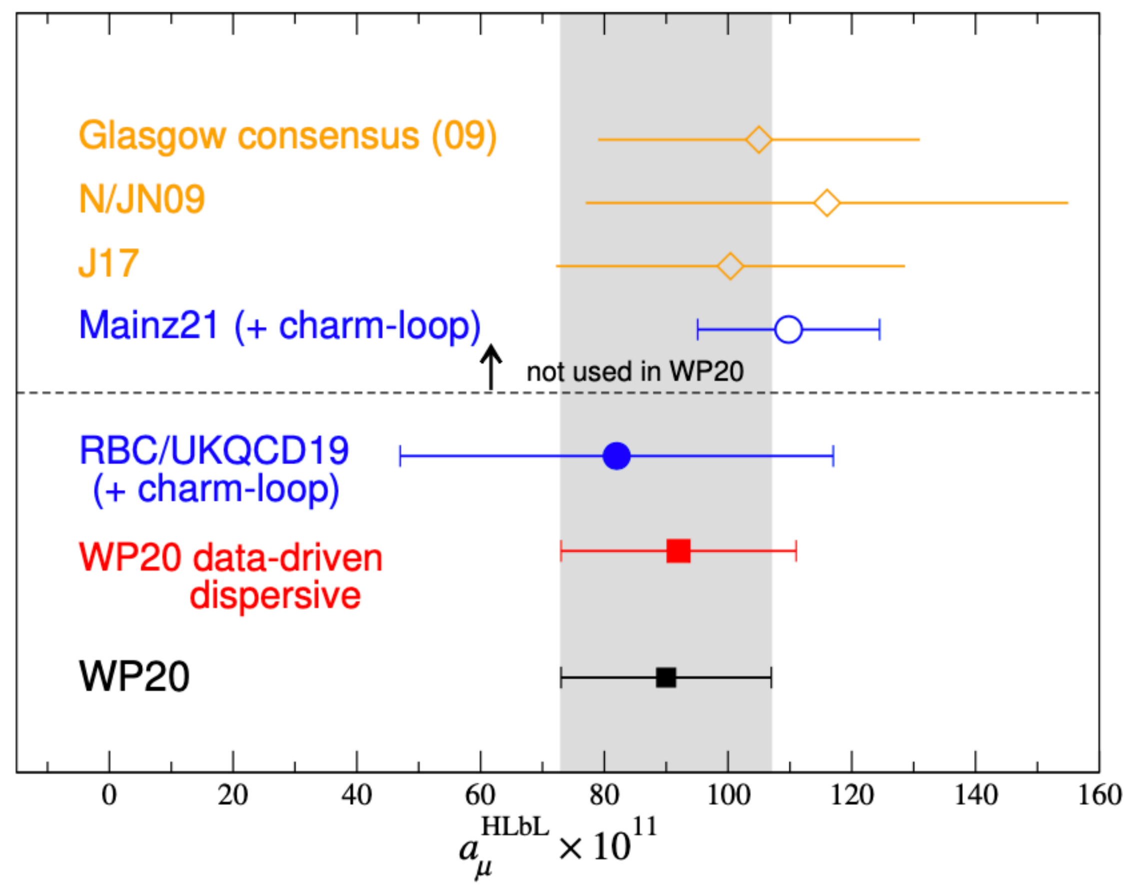
➡ Crosschecks between the two approaches in WP20

https://muon-gm2-theory.illinois.edu/white-paper/

[Phys.Rept.887, arXiv:2006.04822]

Muon g-2 Light-By-Light Contribution





models of QCD +EFT, large N_c

Lattice QCD + QED

data + dispersive approach

[A. El-Khadra, LATTICE 2021]

➡ Pseudoscalar pole contributions to the muon g-2:

[Mainz: Gerardin et. al 1607.08174 PRD94 (2016), Gerardin et. al 1903.09471 PRD100(2019), BMW: Verplanke et. al, PoS LATTICE2022 (332), ETMC: Alexandrou et al., 2212.06704 , 2212.10300 PoS LATTICE2022(306), 2112.03586 PoS LATTICE2021(519)]

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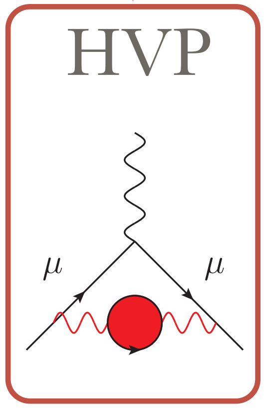
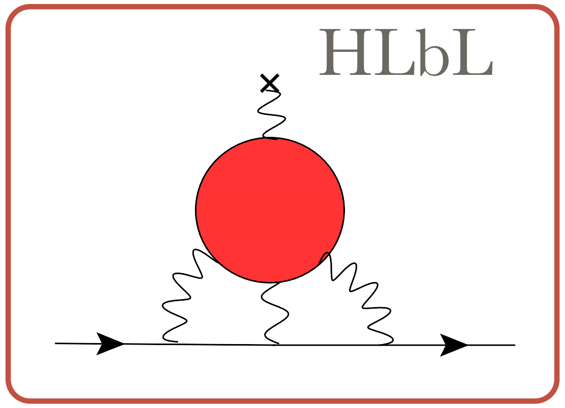
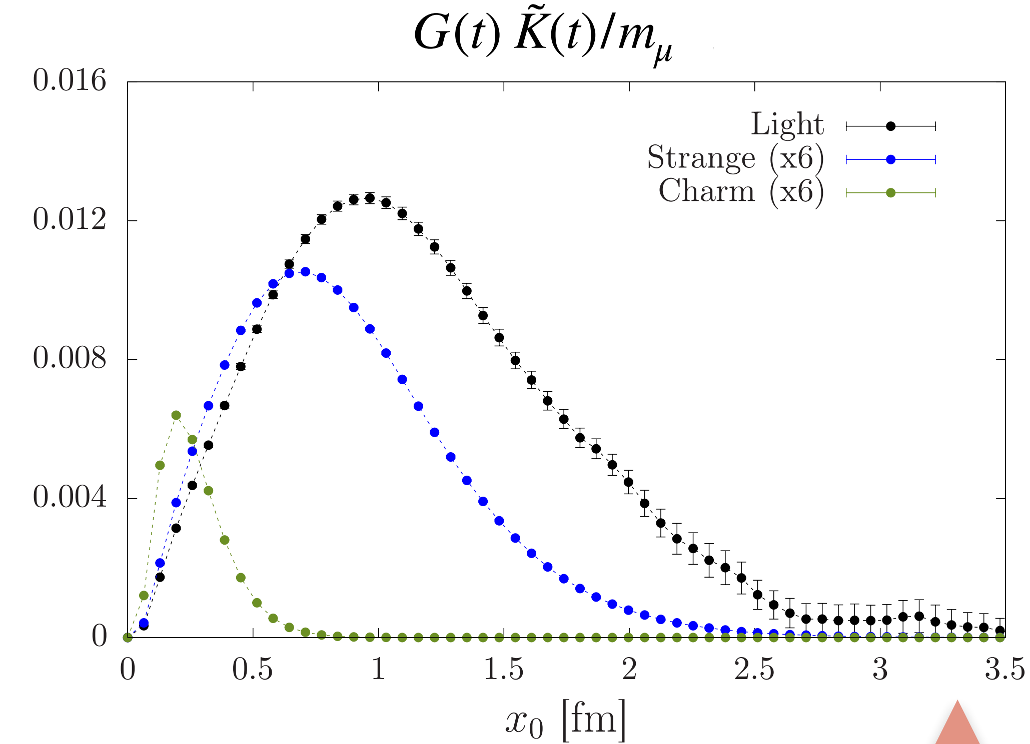




Lattice HVP: dominant sources of errors

- Understanding the systematics is extremely important:
  - ➡ deterioration of signal at large distances
  - ➡ disconnected diagrams
  - ➡ discretization effects
  - ➡ continuum extrapolation …

[Bernecker & Meyer, 1107.4388 EPJA47 (2011) 148]



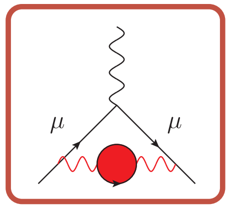
Mainz/CLS [H. Wittig@Lat18]

Time-momentum representation: current-current correlator, summed over all distances:

$$G^f(t) = -\frac{1}{3}\sum_k \sum_{\mathbf{x}} \langle j_k^f(\mathbf{x},t) j_k^f(\mathbf{0},0)\rangle; \qquad f = u,d,s,c$$

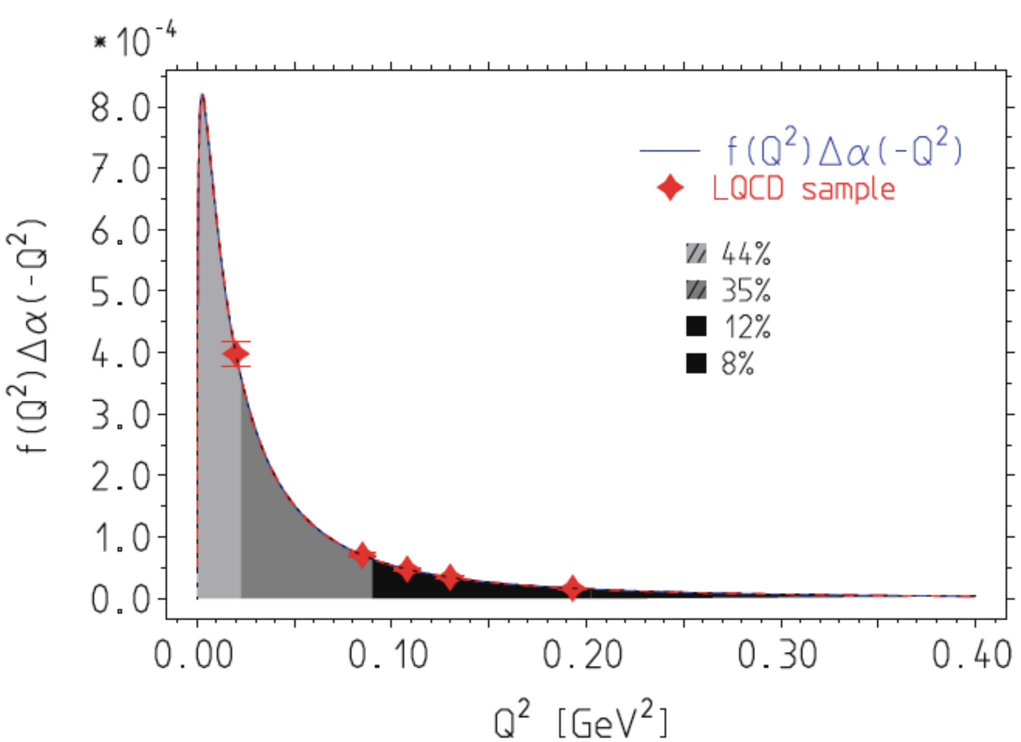
And only then integrated in time:

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G(t)\,\tilde{K}(t,m_\mu)$$

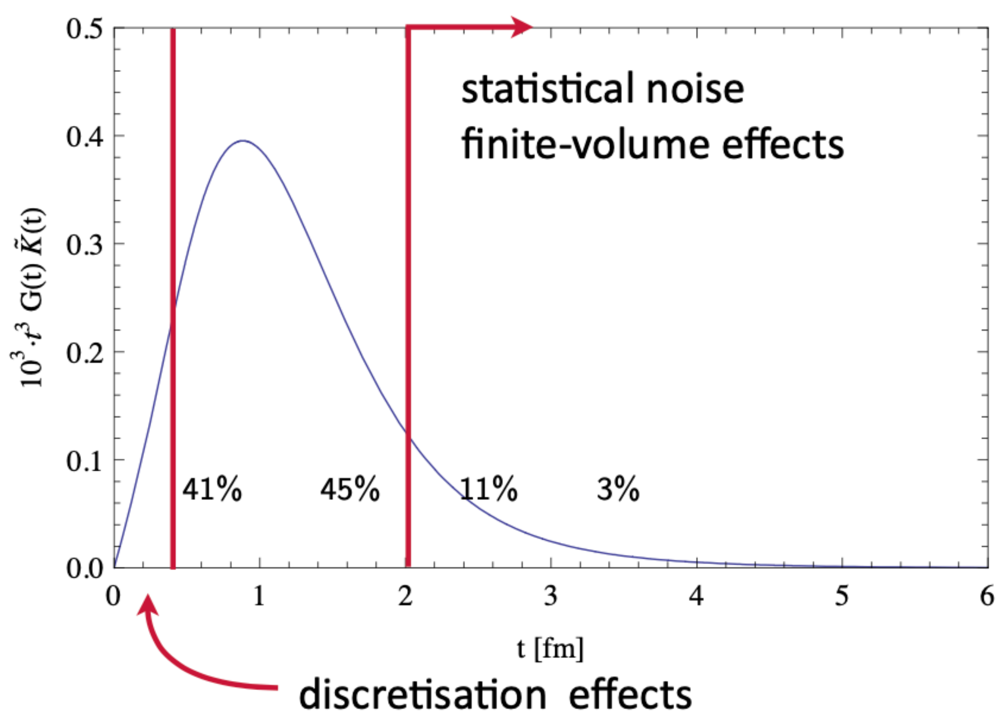


Lattice HVP: dominant sources of errors

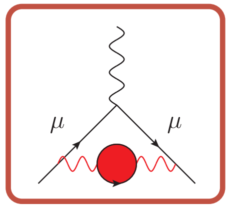
- Understanding the systematics is extremely important:
  - ➡ deterioration of signal at large distances (Q^2 —> 0)
  - ➡ disconnected diagrams
  - ➡ discretization effects
  - ➡ continuum extrapolation …
  - ➡ isospin breaking effects
  - ➡ scale setting error
  - ➡ finite volume effects
  - ➡ charm quark



[F. Jegerlehner, 2017]



[H.Wittig, SchwingerFest22]



HVP Euclidean Window Quantities

$$a_\mu^{\text{HVP, LO}} = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}\,,$$

$$a_\mu^{\text{SD}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)[1 - \Theta(x_0, t_0, \Delta)]\,,$$

$$a_\mu^{\text{W}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)[\Theta(x_0, t_0, \Delta) - \Theta(x_0, t_1, \Delta)]\,,$$

$$a_\mu^{\text{LD}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)\Theta(x_0, t_1, \Delta)\,,$$

$[0.4\text{fm} - 1.0\text{fm}] \approx 30\,\%$ of a_μ^{HVP}

[RBC/UKQCD: Blum et al.1801.07224 PRL121(2018)022003]

[BMW: Borsanyi et al. 2002.12347 Nature 593 (2021)7857]

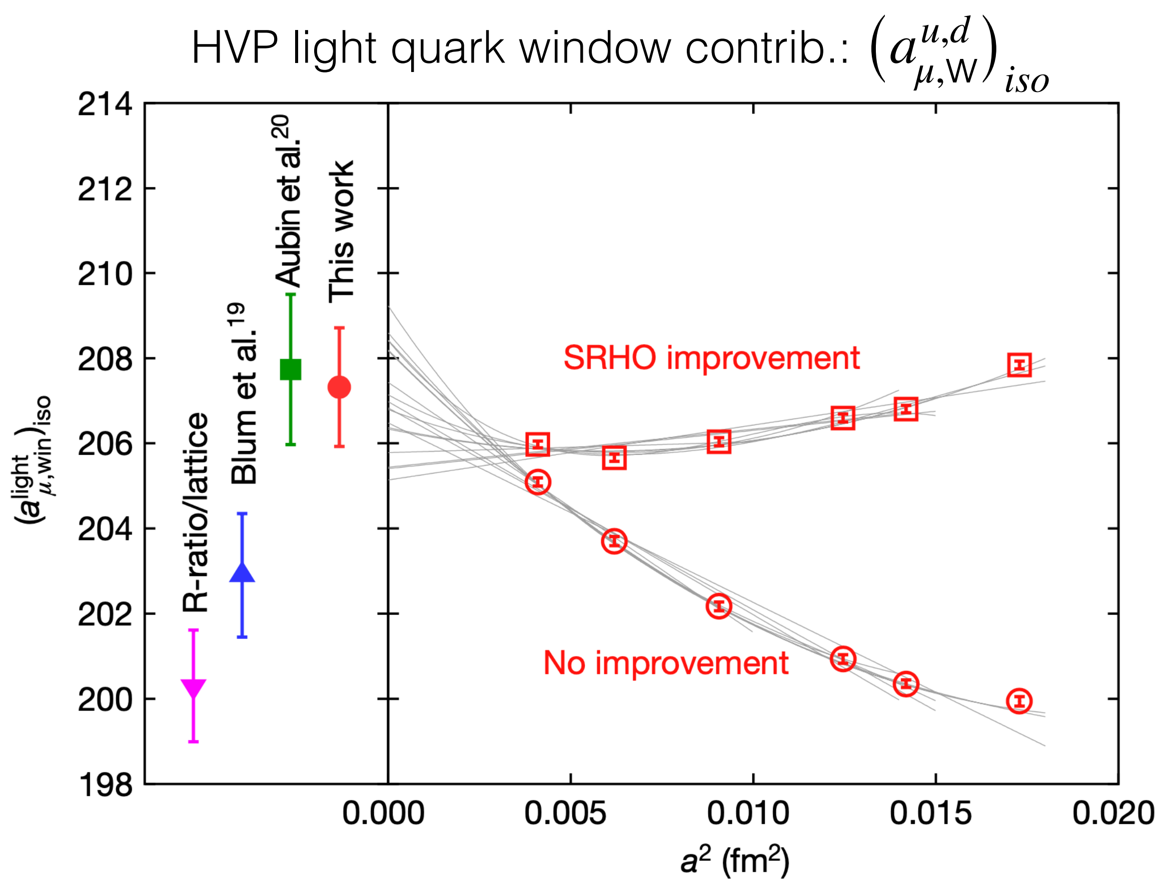
[Phys.Rept.887, arXiv:2006.04822]

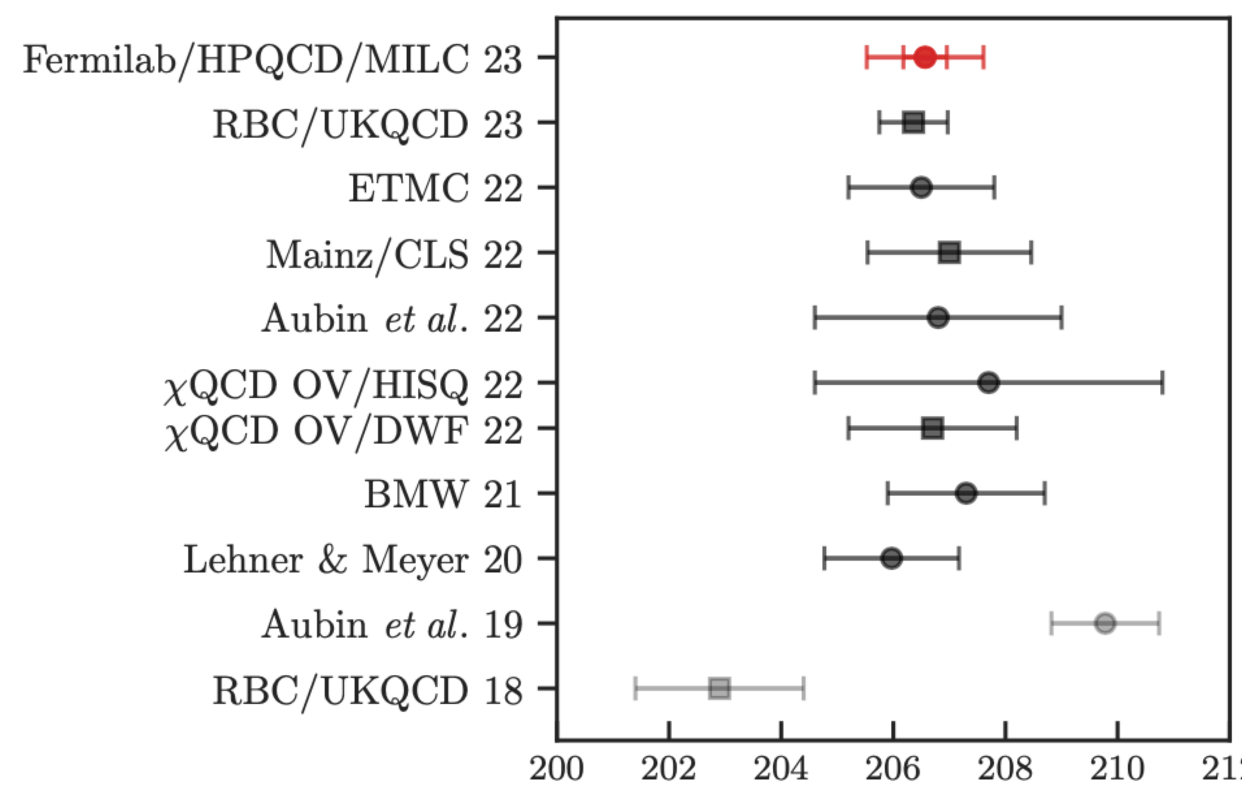
[Mainz 2002.12347 PRD106(2022)11]

[Fermilab/HPQCD/MILC 2301.08274]

- Easy to compare with dispersive methods:

[Colangelo et al. 2205.12963 Phys.Lett.B 833 (2022)]

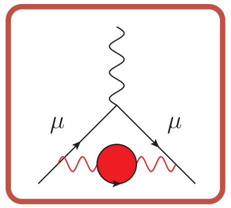




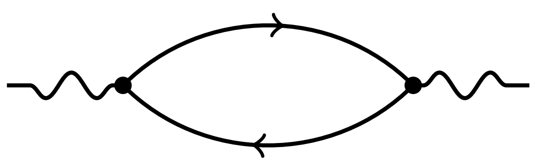
[Borsanyi et al. 2002.12347 Nature593(2021)7857]

[Fermilab/HPQCD/MILC 2301.08274 [hep-lat]]

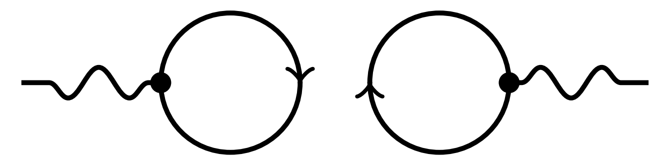
Lattice computation of $a_\mu^{\mathrm{HVP,LO}}$



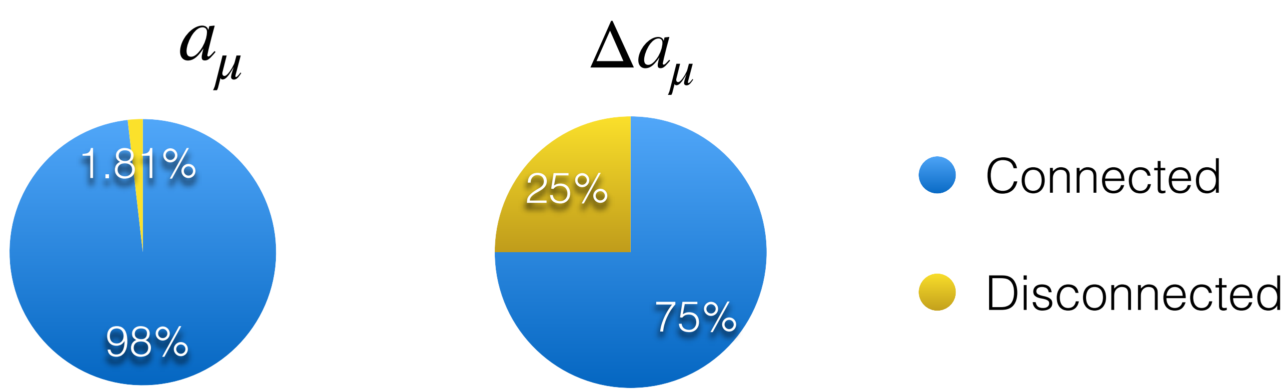
- Quark-connected contribution:



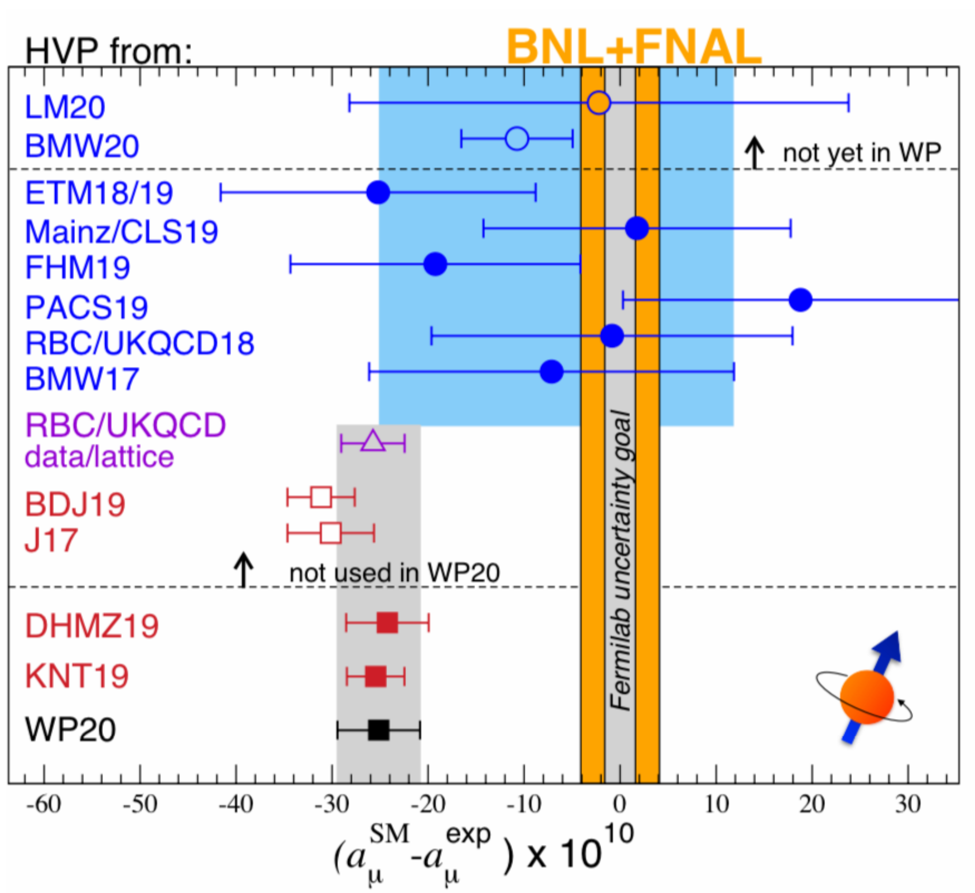
- Quark-disconnected contribution:



[RBC/UKQCD: PRL116(2016)232002, … BMW: Borsanyi et al. Nature593 (2021)7857, arxiv:2002.12347]

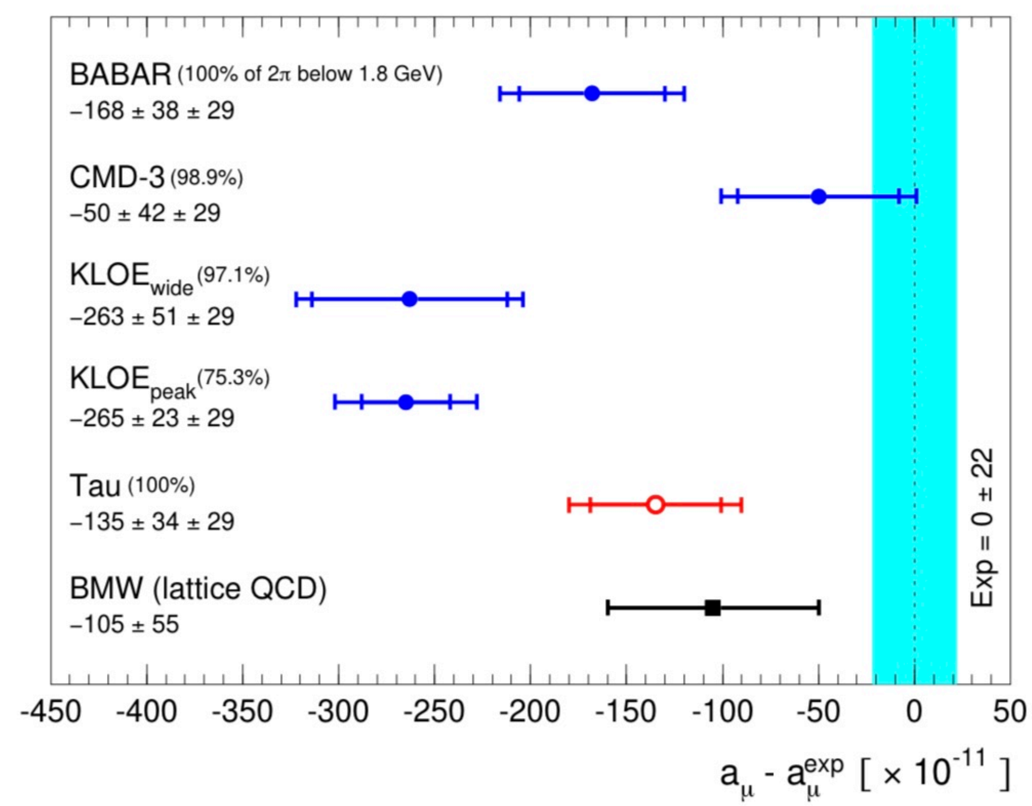


$a_\mu^{\mathrm{W2}} \equiv a_\mu^{\mathrm{win}(1.5,\ 1.9,\ 0.15)}$



[Snowmass22 2203.15810 [hep-ph]]

Difference of dispersive HVP $(\pi^+\pi^-; 1.8\mathrm{GeV})$ and Muon g-2 exp..: $\left(a_\mu^{th} - a_\mu^{exp}\right) \times 10^{-11}$

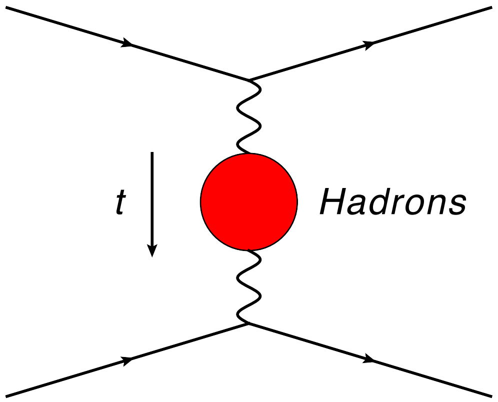


[Davier, Hoecker, Lutz, Malaescu, Zhang 2312.02053 [hep-ph]]

Prospects for the HVP independent measurement

[MUonE Collaboration https://web.infn.it/MUonE/]

[For more details, see LOI]
https://cds.cern.ch/record/2677471



- In space-like (Euclidean) momenta region
- Obtain $a_\mu^{had,LO}$ by utilising the running of α_{QED} in a space-like process

$$a_\mu^{\mathbf{had,LO}} = \frac{\alpha}{\pi}\int_0^1 \mathbf{dx(1-x)\Delta\alpha_{had}[Q^2(x)]}$$

[Lautrup, de Rafael '69, de Rafael, PLB(1994), Blum PRL(2002)]

- Proposal to measure precisely the Q^2 - dependent fine-structure constant:

$$\alpha(Q^2) = \frac{\alpha(O)}{1-\Delta\alpha(Q^2)}$$

[Phys.Lett. B746 (2015) 325-329 by Carloni, Passera, Trentadue, Venanzoni] **@e+e- detector**

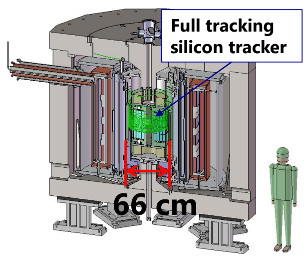
[Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.] **@CERN**

Physics
Long Shutdown (LS)
Beam commissioning
Technical stop

LHC schedule

FNAL E989: Muon g-2

J-PARC E34: Muon g-2/EDM



High Performance Computational Physics @ ETH zürich

FNSNF

PAULI CENTER for Theoretical Studies

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

European Commission

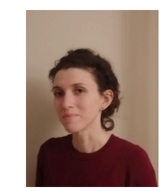
Google Research

- HPCP lab, July 2023

QUANTUM CENTER

ETH AI CENTER

+ A. Altherr, ETH



+ L. Parato, ETH

+ A. Gandon, ETH/IBM

IONQ

IBM Quantum

AQT

Sandia National Laboratories

CSCS
Centro Svizzero di Calcolo Scientifico
Swiss National Supercomputing Centre

IT Services

HLRN

Summary & Outlook

- Exciting time for lattice QCD computations: mature and innovative!
- **Muon anomalous magnetic moment (muon g-2):** good quantity for constraining new physics
  - ➡ **QED** and **EW** contributions known precisely: **hadronic contributions** dominating uncertainties
  - ➡ **Lattice QCD(+QED)** gives an independent theory prediction of **HVP and HLbL**

- **Lattice QCD** not sufficient for sub-percent precision — inclusion of **QED and isospin effects** needed
- **HLbL**: either QCD+QED, or 4-pt function on the lattice (involved but feasible)
- **HVP**: mature, QCD+QED for sub-percent precision, agreement on window observable, full HVP in progress
- **MUonE** experiment: provide an **independent input for HVP** and intermediate milestones
- Experimental input for data driven HVP under scrutiny [BABAR, KLOE, CMD]
- Muon g-2 Theory Initiative as a framework for exchange between **lattice/data-driven/experimental** results
- Update of the White Paper by the Muon g-2 Theory initiative in preparation, https://muon-gm2-theory.illinois.edu

Lattice 2025, TFIR, Mumbai, India,
3-8 November 2025
42nd International Symposium on Lattice Field Theory

LOC:

Debasish Banerjee (SINP, Kolkata)	Subhasish Basak (NISER, Bhubaneswar)
Dipankar Chakraborty (IIT, Kanpur)	Saumen Datta (TIFR, Mumbai)
Sanatan Digal (IMSc, Chennai)	Rajiv Gavai (IISER Bhopal)
Prasad Hegde (IISc, Bengaluru)	**Nilmani Mathur (TIFR, Mumbai)**
M. Padmanath (IMSc, Chennai)	Indrakshi Raychowdhury (BITS, Goa)
Sayantan Sharma (IMSc,Chennai)	Rishi Sharma (TIFR, Mumbai)
Gunnar Bali (TIFR, Mumbai and Regensburg, Germany