Exotic hadrons from Lattice QCD

Nilmani Mathur

tata institute of fundamental research

Collaborators: Parikshit Junnarkar (GSI, Germany),
M. Padmanath (IMSc.),
Archana Radhakrishnan (TIFR)

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Mesons (quark-antiquark systems)

LIGHT UNFLAVORED ($S = C = B = 0$)

		$I^G(J^{PC})$			$I^G(J^{PC})$
•	$\pi^{\pm}$	$1^-(0^-)$	•	$\rho(1700)$	$1^+(1^{--})$
•	π^0	$1^-(0^{-+})$	•	$a_2(1700)$	$1^-(2^{++})$
•	η	$0^+(0^{-+})$		$a_0(1710)$	$1^-(0^{++})$
•	$f_0(500)$	$0^+(0^{++})$	•	$f_0(1710)$	$0^+(0^{++})$
	aka σ; was $f_0(600)$, $f_0(400-1200)$			$X(1750)$	$?^-(1^{--})$
•	$\rho(770)$	$1^+(1^{--})$		$\eta(1760)$	$0^+(0^{-+})$
•	$\omega(782)$	$0^-(1^{--})$		$f_0(1770)$	$0^+(0^{++})$
•	$\eta'(958)$	$0^+(0^{-+})$	•	$\pi(1800)$	$1^-(0^{-+})$
•	$f_0(980)$	$0^+(0^{++})$		$f_2(1810)$	$0^+(2^{++})$
•	$a_0(980)$	$1^-(0^{++})$		$X(1835)$	$?^?(0^{-+})$
•	$\phi(1020)$	$0^-(1^{--})$	•	$\phi_3(1850)$	$0^-(3^{--})$
•	$h_1(1170)$	$0^-(1^{+-})$		$\eta_1(1855)$	$0^+(1^{-+})$
•	$b_1(1235)$	$1^+(1^{+-})$	•	$\eta_2(1870)$	$0^+(2^{-+})$
•	$a_1(1260)$	$1^-(1^{++})$	•	$\pi_2(1880)$	$1^-(2^{-+})$
•	$f_2(1270)$	$0^+(2^{++})$		$\rho(1900)$	$1^+(1^{--})$
•	$f_1(1285)$	$0^+(1^{++})$		$f_2(1910)$	$0^+(2^{++})$
•	$\eta(1295)$	$0^+(0^{-+})$		$a_0(1950)$	$1^-(0^{++})$
•	$\pi(1300)$	$1^-(0^{-+})$	•	$f_2(1950)$	$0^+(2^{++})$
•	$a_2(1320)$	$1^-(2^{++})$	•	$a_4(1970)$	$1^-(4^{++})$
•	$f_0(1370)$	$0^+(0^{++})$		was $a_4(2040)$	
•	$\pi_1(1400)$	$1^-(1^{-+})$		$\rho_3(1990)$	$1^+(3^{--})$
•	$\eta(1405)$	$0^+(0^{-+})$		$\pi_2(2005)$	$1^-(2^{-+})$
•	$h_1(1415)$	$0^-(1^{+-})$	•	$f_2(2010)$	$0^+(2^{++})$
	was $h_1(1380)$		•	$f_0(2020)$	$0^+(0^{++})$
•	$f_1(1420)$	$0^+(1^{++})$	•	$f_4(2050)$	$0^+(4^{++})$
•	$\omega(1420)$	$0^-(1^{--})$		$\pi_2(2100)$	$1^-(2^{-+})$
	$f_2(1430)$	$0^+(2^{++})$		$f_0(2100)$	$0^+(0^{++})$
•	$a_0(1450)$	$1^-(0^{++})$		$f_2(2150)$	$0^+(2^{++})$
•	$\rho(1450)$	$1^+(1^{--})$		$\rho(2150)$	$1^+(1^{--})$
•	$\eta(1475)$	$0^+(0^{-+})$	•	$\phi(2170)$	$0^-(1^{--})$
•	$f_0(1500)$	$0^+(0^{++})$		$f_0(2200)$	$0^+(0^{++})$
	$f_1(1510)$	$0^+(1^{++})$		$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
•	$f_2'(1525)$	$0^+(2^{++})$		$\eta(2225)$	$0^+(0^{-+})$
	$f_2(1565)$	$0^+(2^{++})$		$\rho_3(2250)$	$1^+(3^{--})$
	$\rho(1570)$	$1^+(1^{--})$	•	$f_2(2300)$	$0^+(2^{++})$
	$h_1(1595)$	$0^-(1^{+-})$		$f_4(2300)$	$0^+(4^{++})$
•	$\pi_1(1600)$	$1^-(1^{-+})$		$f_0(2330)$	$0^+(0^{++})$
•	$a_1(1640)$	$1^-(1^{++})$	•	$f_2(2340)$	$0^+(2^{++})$
	$f_2(1640)$	$0^+(2^{++})$		$\rho_5(2350)$	$1^+(5^{--})$
•	$\eta_2(1645)$	$0^+(2^{-+})$		$X(2370)$	$?^?(?^{??})$
•	$\omega(1650)$	$0^-(1^{--})$		$f_0(2470)$	$0^+(0^{++})$
•	$\omega_3(1670)$	$0^-(3^{--})$		$f_6(2510)$	$0^+(6^{++})$
•	$\pi_2(1670)$	$1^-(2^{-+})$			
•	$\phi(1680)$	$0^-(1^{--})$			
•	$\rho_3(1690)$	$1^+(3^{--})$			

STRANGE ($S = \pm1$, $C = B = 0$)

		$I(J^P)$
•	$K^{\pm}$	$1/2(0^-)$
•	K^0	$1/2(0^-)$
•	K^0_S	$1/2(0^-)$
•	K^0_L	$1/2(0^-)$
•	$K_0^*(700)$	$1/2(0^+)$
	aka κ; was $K_0^*(800)$	
•	$K^*(892)$	$1/2(1^-)$
•	$K_1(1270)$	$1/2(1^+)$
•	$K_1(1400)$	$1/2(1^+)$
•	$K^*(1410)$	$1/2(1^-)$
•	$K_0^*(1430)$	$1/2(0^+)$
•	$K_2^*(1430)$	$1/2(2^+)$
•	$K(1460)$	$1/2(0^-)$
	$K_2(1580)$	$1/2(2^-)$
	$K(1630)$	$1/2(?^?)$
•	$K_1(1650)$	$1/2(1^+)$
•	$K^*(1680)$	$1/2(1^-)$
•	$K_2(1770)$	$1/2(2^-)$
•	$K_3^*(1780)$	$1/2(3^-)$
•	$K_2(1820)$	$1/2(2^-)$
	$K(1830)$	$1/2(0^-)$
	$K_0^*(1950)$	$1/2(0^+)$
•	$K_2^*(1980)$	$1/2(2^+)$
•	$K_4^*(2045)$	$1/2(4^+)$
	$K_2(2250)$	$1/2(2^-)$
	$K_3(2320)$	$1/2(3^+)$
	$K_5^*(2380)$	$1/2(5^-)$
	$K_4(2500)$	$1/2(4^-)$
	$K(3100)$	$?^?(?^{??})$
	aka $K_J^?(3100)$	

CHARMED ($C = \pm1$)

		$I(J^P)$
•	$D^{\pm}$	$1/2(0^-)$
•	D^0	$1/2(0^-)$
•	$D^*(2007)^0$	$1/2(1^-)$
•	$D^*(2010)^{\pm}$	$1/2(1^-)$
•	$D_0^*(2300)$	$1/2(0^+)$
	was $D_0^*(2400)$	
•	$D_1(2420)$	$1/2(1^+)$
•	$D_1(2430)^0$	$1/2(1^+)$
•	$D_2^*(2460)$	$1/2(2^+)$
	$D_0(2550)^0$	$1/2(0^-)$
	$D_1^*(2600)^0$	$1/2(1^-)$
	was $D_J^*(2600)$	
	$D^*(2640)^{\pm}$	$1/2(?^?)$
	$D_2(2740)^0$	$1/2(2^-)$
	was $D(2740)^0$	
•	$D_3^*(2750)$	$1/2(3^-)$
	$D_1^*(2760)^0$	$1/2(1^-)$
	$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = \pm1$, $S = \pm1$) (including possibly non- $q\bar{q}$ states)

		$I(J^P)$
•	$D_s^{\pm}$	$0(0^-)$
•	$D_s^{*\pm}$	$0(?^?)$
•	$D_{s0}^*(2317)^{\pm}$	$0(0^+)$
•	$D_{s1}(2460)^{\pm}$	$0(1^+)$
•	$D_{s1}(2536)^{\pm}$	$0(1^+)$
•	$D_{s2}^*(2573)$	$0(2^+)$
	$D_{s0}(2590)^+$	$0(0^-)$
•	$D_{s1}^*(2700)^{\pm}$	$0(1^-)$
	$D_{s1}^*(2860)^{\pm}$	$0(1^-)$
	was $D_{sJ}^*(2860)$	
•	$D_{s3}^*(2860)^{\pm}$	$0(3^-)$
	$D_{sJ}(3040)^{\pm}$	$0(?^?)$

BOTTOM ($B = \pm1$)

		$I(J^P)$
•	$B^{\pm}$	$1/2(0^-)$
•	B^0	$1/2(0^-)$
•	$B^{\pm}/B^0$ ADMIXTURE	
•	$B^{\pm}/B^0/B^0_s/b$-baryon ADMIXTURE	
	V_{cb} and V_{ub} CKM Matrix Elements	
•	B^*	$1/2(1^-)$
•	$B_1(5721)$	$1/2(1^+)$
	$B_J^*(5732)$	$?(?^?)$
	aka B^{**}	
•	$B_2^*(5747)$	$1/2(2^+)$
	$B_J(5840)$	$1/2(?^?)$
•	$B_J(5970)$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

		$I(J^P)$
•	B_s^0	$0(0^-)$
•	B_s^*	$0(1^-)$
•	$B_{s1}(5830)^0$	$0(1^+)$
•	$B_{s2}^*(5840)^0$	$0(2^+)$
	$B_{sJ}^*(5850)$	$?(?^?)$
	$B_{sJ}(6063)^0$	$0(?^?)$
	$B_{sJ}(6114)^0$	$0(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

		$I(J^P)$
•	$B_c^{\pm}$	$0(0^-)$
•	$B_c(2S)^{\pm}$	$0(0^-)$

$c\bar{c}$ (including possibly non- $q\bar{q}$ states)

		$I^G(J^{PC})$
•	$\eta_c(1S)$	$0^+(0^{-+})$
•	$J/\psi(1S)$	$0^-(1^{--})$
•	$\chi_{c0}(1P)$	$0^+(0^{++})$
•	$\chi_{c1}(1P)$	$0^+(1^{++})$
•	$h_c(1P)$	$0^-(1^{+-})$
•	$\chi_{c2}(1P)$	$0^+(2^{++})$
•	$\eta_c(2S)$	$0^+(0^{-+})$
•	$\psi(2S)$	$0^-(1^{--})$
•	$\psi(3770)$	$0^-(1^{--})$
•	$\psi_2(3823)$	$0^-(2^{--})$
	was $\psi(3823)$, $X(3823)$	
•	$\psi_3(3842)$	$0^-(3^{--})$

$b\bar{b}$ (including possibly non- $q\bar{q}$ states)

		$I^G(J^{PC})$
•	$\eta_b(1S)$	$0^+(0^{-+})$
•	$\Upsilon(1S)$	$0^-(1^{--})$
•	$\chi_{b0}(1P)$	$0^+(0^{++})$
•	$\chi_{b1}(1P)$	$0^+(1^{++})$
•	$h_b(1P)$	$0^-(1^{+-})$
•	$\chi_{b2}(1P)$	$0^+(2^{++})$
	$\eta_b(2S)$	$0^+(0^{-+})$
•	$\Upsilon(2S)$	$0^-(1^{--})$
•	$\Upsilon_2(1D)$	$0^-(2^{--})$
	was $\Upsilon(1D)$	
•	$\chi_{b0}(2P)$	$0^+(0^{++})$
•	$\chi_{b1}(2P)$	$0^+(1^{++})$
•	$h_b(2P)$	$0^-(1^{+-})$
•	$\chi_{b2}(2P)$	$0^+(2^{++})$
•	$\Upsilon(3S)$	$0^-(1^{--})$
•	$\chi_{b1}(3P)$	$0^+(1^{++})$
•	$\chi_{b2}(3P)$	$0^+(2^{++})$
•	$\Upsilon(4S)$	$0^-(1^{--})$
	aka $\Upsilon(10580)$	
	$\Upsilon(10753)$	$?^?(1^{--})$
•	$\Upsilon(10860)$	$0^-(1^{--})$
•	$\Upsilon(11020)$	$0^-(1^{--})$

OTHER

		$I^G(J^{PC})$
	$X_0(2900)$	$?(0^+)$
	$X_1(2900)$	$?(1^-)$
	$T_{cc}(3875)^+$	$?(?^?)$
•	$Z_c(3900)$	$1^+(1^{+-})$
	was $X(3900)$	
	$Z_{cs}(4000)$	$1/2(1^+)$
•	$X(4020)^{\pm}$	$1^+(?^{?-})$
	$X(4050)^{\pm}$	$1^-(?^{?+})$
	$X(4055)^{\pm}$	$1^+(?^{?-})$
	$X(4100)^{\pm}$	$1^-(?^{??})$
	$Z_c(4200)$	$1^+(1^{+-})$
	was $X(4200)^{\pm}$	
	$Z_{cs}(4220)^+$	$1/2(1^+)$
	$R_{c0}(4240)$	$1^+(0^{--})$
	was $X(4240)^{\pm}$	
	$X(4250)^{\pm}$	$1^-(?^{?+})$
•	$Z_c(4430)$	$1^+(1^{+-})$
	was $X(4430)^{\pm}$	
	$X(5568)^{\pm}$	$?(?^?)$
	$X(6900)$	
•	$Z_b(10610)$	$1^+(1^{+-})$
	was $X(10610)$	
•	$Z_b(10650)$	$1^+(1^{+-})$
	was $X(10650)^{\pm}$	
	Further States	

Basket full of Mesons

Baryons (Three quark systems)

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
was $N(2080)$		
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
was $N(2090)$		
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
was $N(1900)$		
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
was $N(2200)$		
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
was $\Lambda(2020)$		
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		*

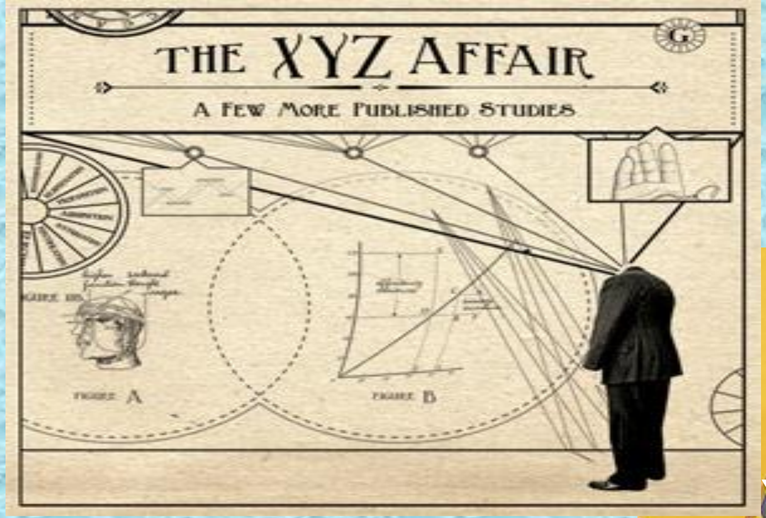
Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
was $\Sigma(1730)$		
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
was $\Sigma(1940)$		
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
was $\Sigma(2000)$		
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2110)$	$1/2^-$	*
was $\Sigma(2160)$		
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		**
$\Sigma(2455)$		*
$\Sigma(2620)$		*
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2012)^-$	$?^-$	***
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$ or $\Sigma_c(2765)$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2923)$		**
$\Xi_c(2930)$		**
$\Xi_c(2970)$	$1/2^+$	***
was $\Xi_c(2980)$		
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3000)^0$		***
$\Omega_c(3050)^0$		***
$\Omega_c(3065)^0$		***
$\Omega_c(3090)^0$		***
$\Omega_c(3120)^0$		***
Ξ_{cc}^+		*
Ξ_{cc}^{++}		***
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6070)^0$	$1/2^+$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^-	$1/2^+$	***
Ξ_b^0	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6100)^-$	$3/2^-$	***
$\Xi_b(6227)^-$		***
$\Xi_b(6227)^0$		***

Particle Data Group

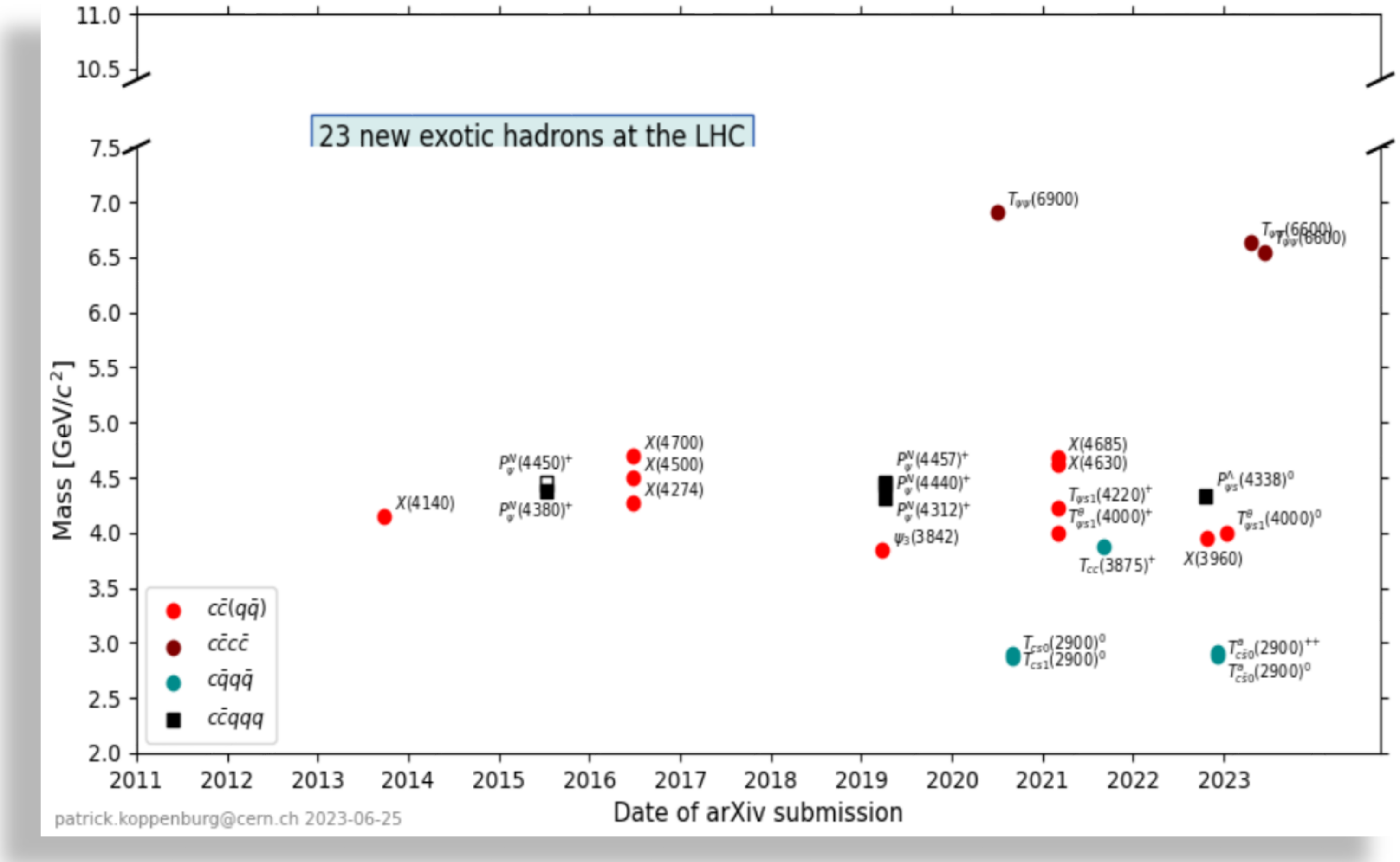
Basket full of Baryons



Newly Found Hadrons

Exciting Discovery

Exotic hadrons at LHC



@LHCb

….And so at Belle, BES III, COMPASS, CMS, ALICE, ATLAS, JLAB

Exotic Hadrons

- Hadrons whose quantum numbers require a valence quark content beyond **qqq** or $q\bar{q}$ are called as **"exotics"**, e.g. $cc\bar{u}\bar{d}$, **glueball**

- Hadrons whose spin, parity and charge conjugation are forbidden in the non-relativistic quark model are also often termed "exotics" (spin exotics)

- Cryptoexotics :
  - mass/width does not fit with meson or baryon spectra
  - overpopulation of the spectra
  - production or decay properties incompatible with standard mesons/baryons

A constituent picture of Hadrons

- QCD : Fundamental degrees of freedoms are quarks (6 flavours) and gluons (8 degrees of freedom)
- Confinement conjecture: quarks and gluons must be combined into colour-neutral combinations of hadrons

Constituents	Combinations	Naming convention (quark model)
$\mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{8}$	Meson
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$	Baryon
$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Glueball
$\bar{\mathbf{3}} \otimes \mathbf{8} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Hybrid
$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Tetraquark/molecule
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35} + \cdots$	Pentaquark
………..	………..	?

A constituent model of hadrons

- However, there can be strong mixings between different hadrons with the same quantum numbers

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

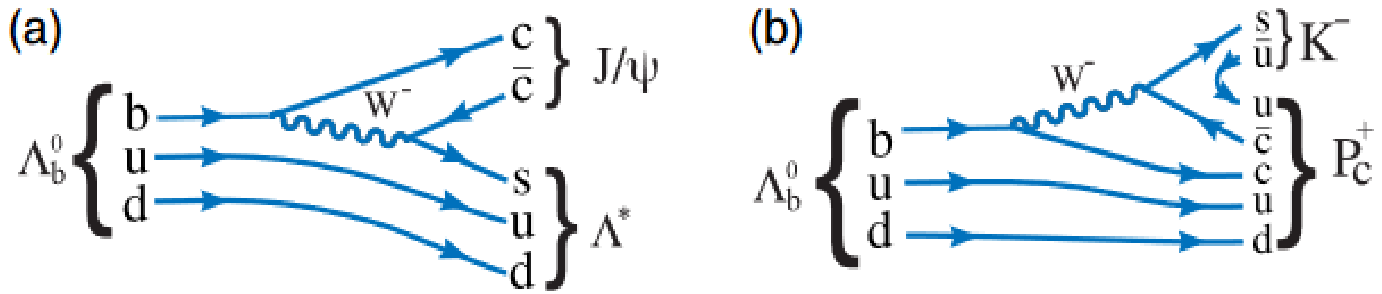
Quest

- Does nature permit what QCD allows?
- Are there subatomic particles beyond mesons and baryons valence structures?

Selected for a Viewpoint in *Physics*
PRL **115,** 072001 (2015) PHYSICAL REVIEW LETTERS week ending 14 AUGUST 2015

Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays

R. Aaij *et al.*[*]
(LHCb Collaboration)



$$\Lambda_b \to J/\psi p K$$

Selected for a Viewpoint in *Physics*
PRL **115,** 072001 (2015) PHYSICAL REVIEW LETTERS week ending 14 AUGUST 2015

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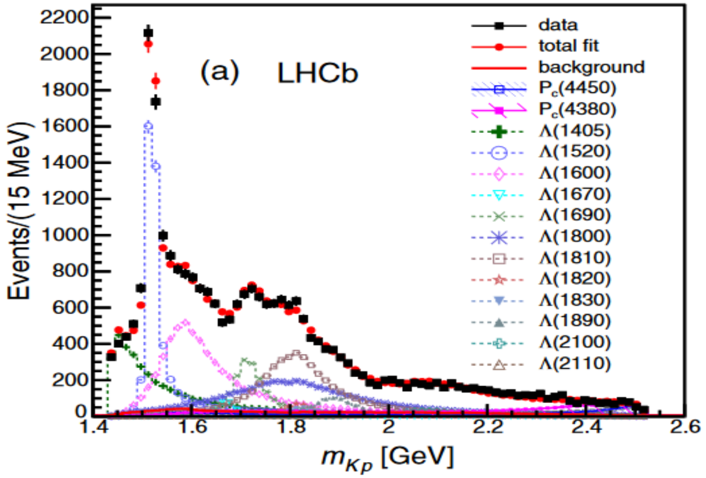
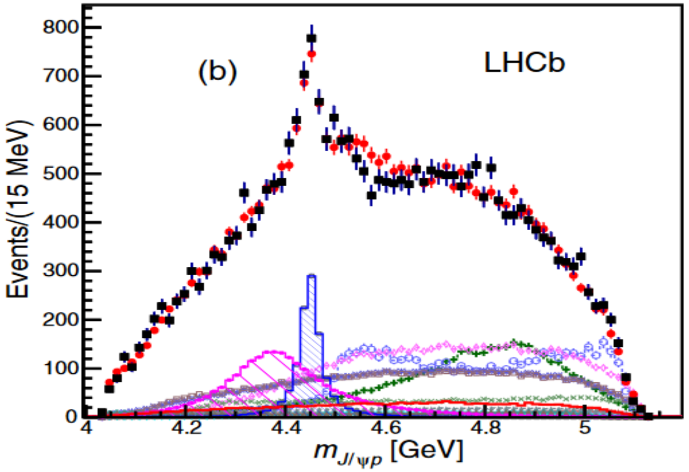
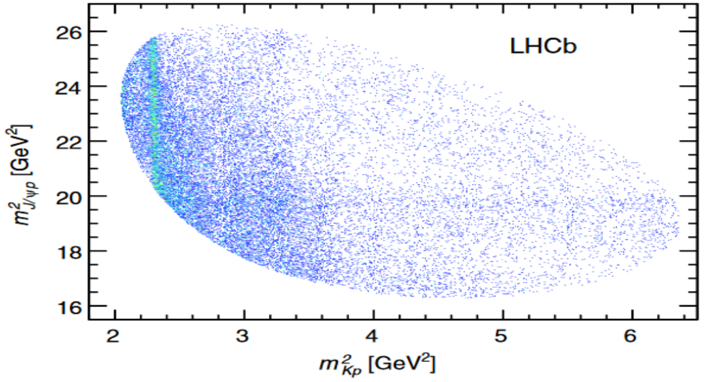
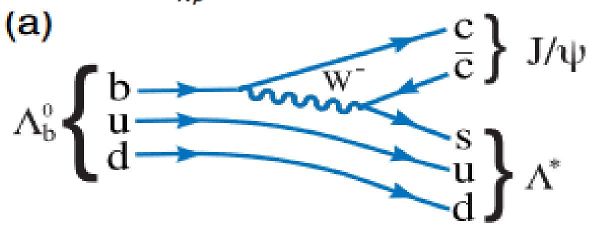
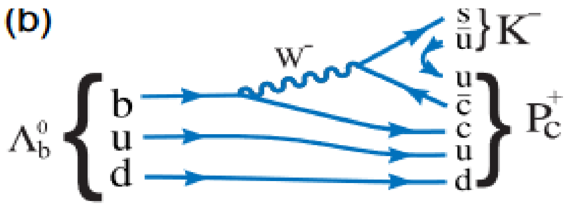
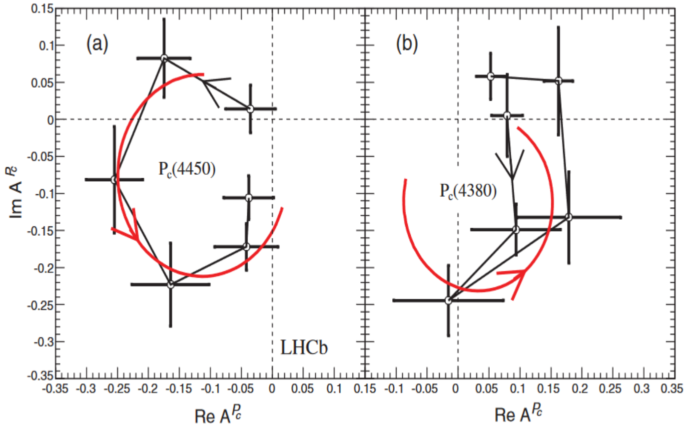












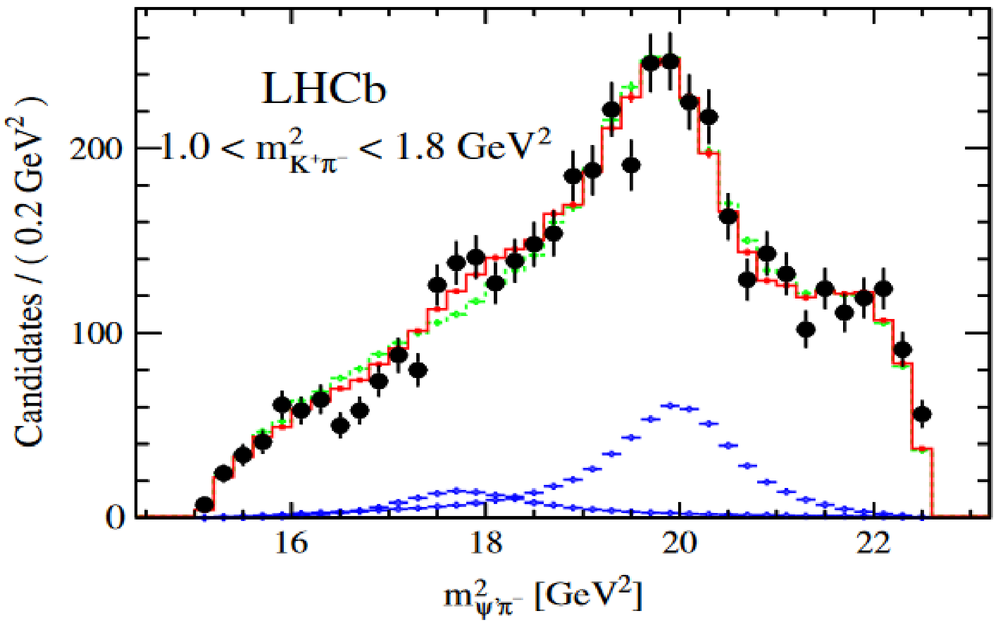
Pc (4380)	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$
Pc (4450)	$4449.8 \pm 1.7 \pm 2.5$	$39 \pm 5 \pm 19$

PRL 112, 222002 (2014) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2014

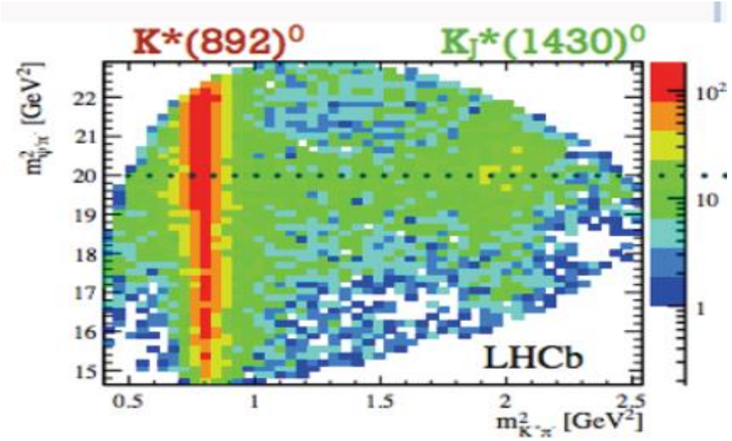
Observation of the Resonant Character of the $Z(4430)^-$ State

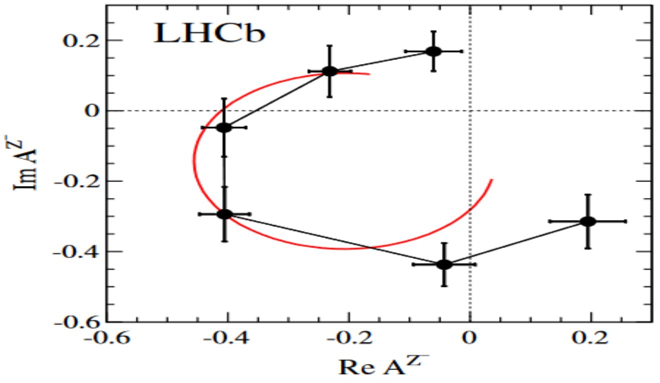
R. Aaij *et al.*[*]
(LHCb Collaboration)

$$B^0 \to \psi' \pi^- K^+$$



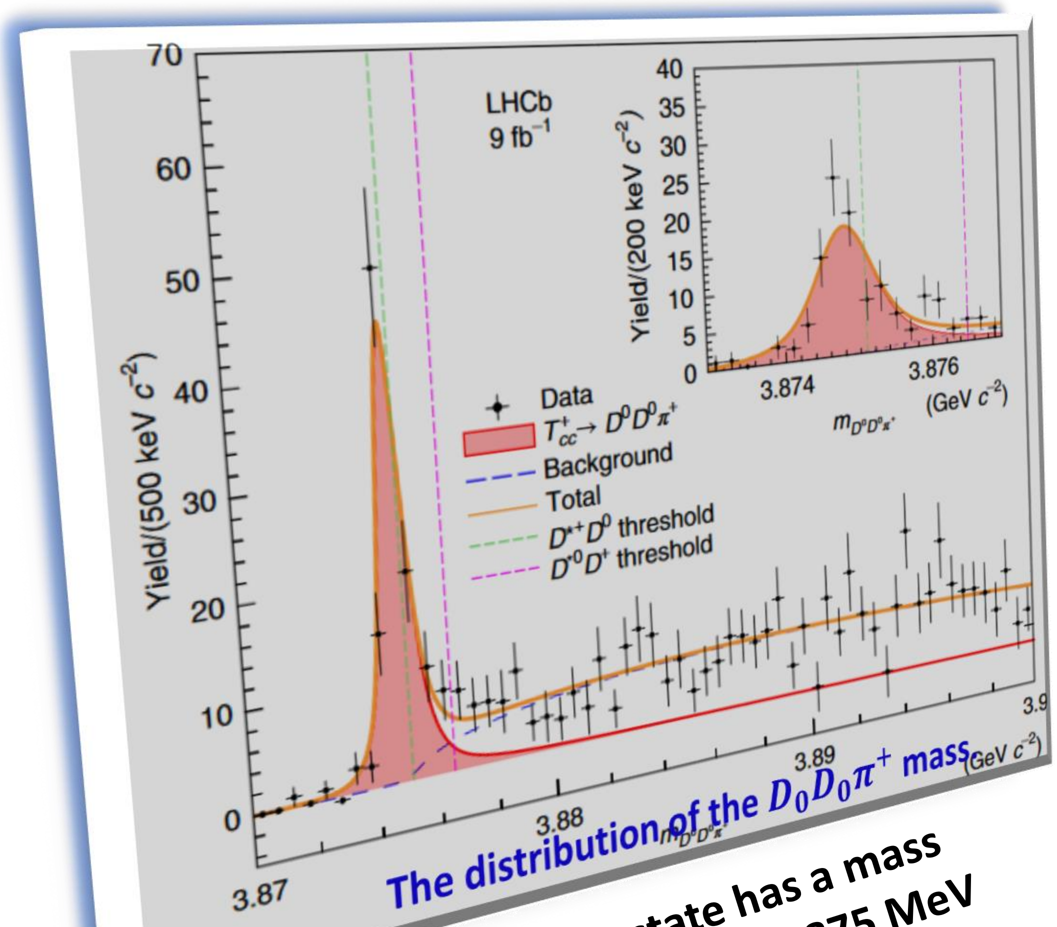
$c\bar{c}d\bar{u}$





$D^*(c\bar{d})D^0(c\bar{u})$ $T_{cc}^+(cc\bar{u}\bar{d})$

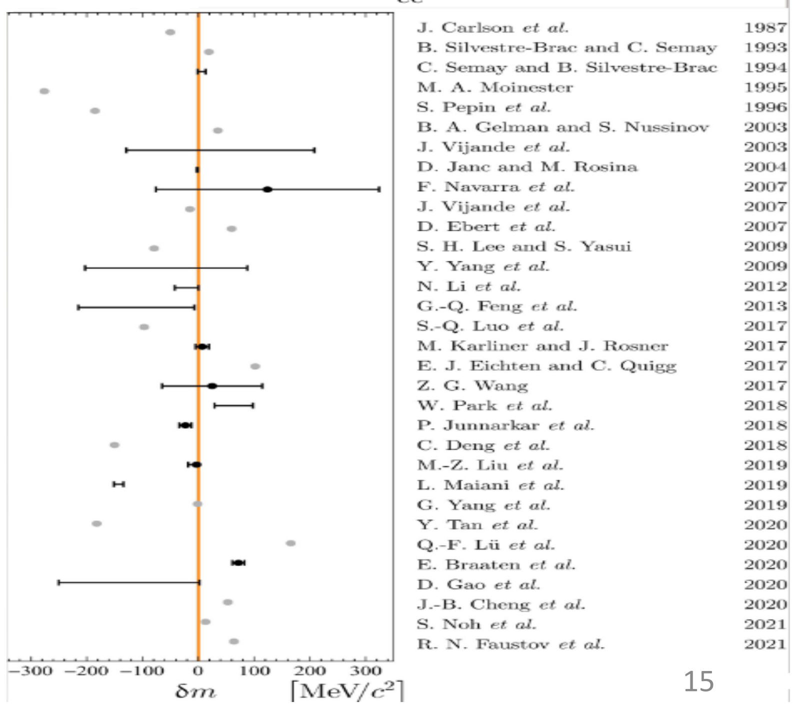
Nature Physics,18, 751(2022) @ LHCb



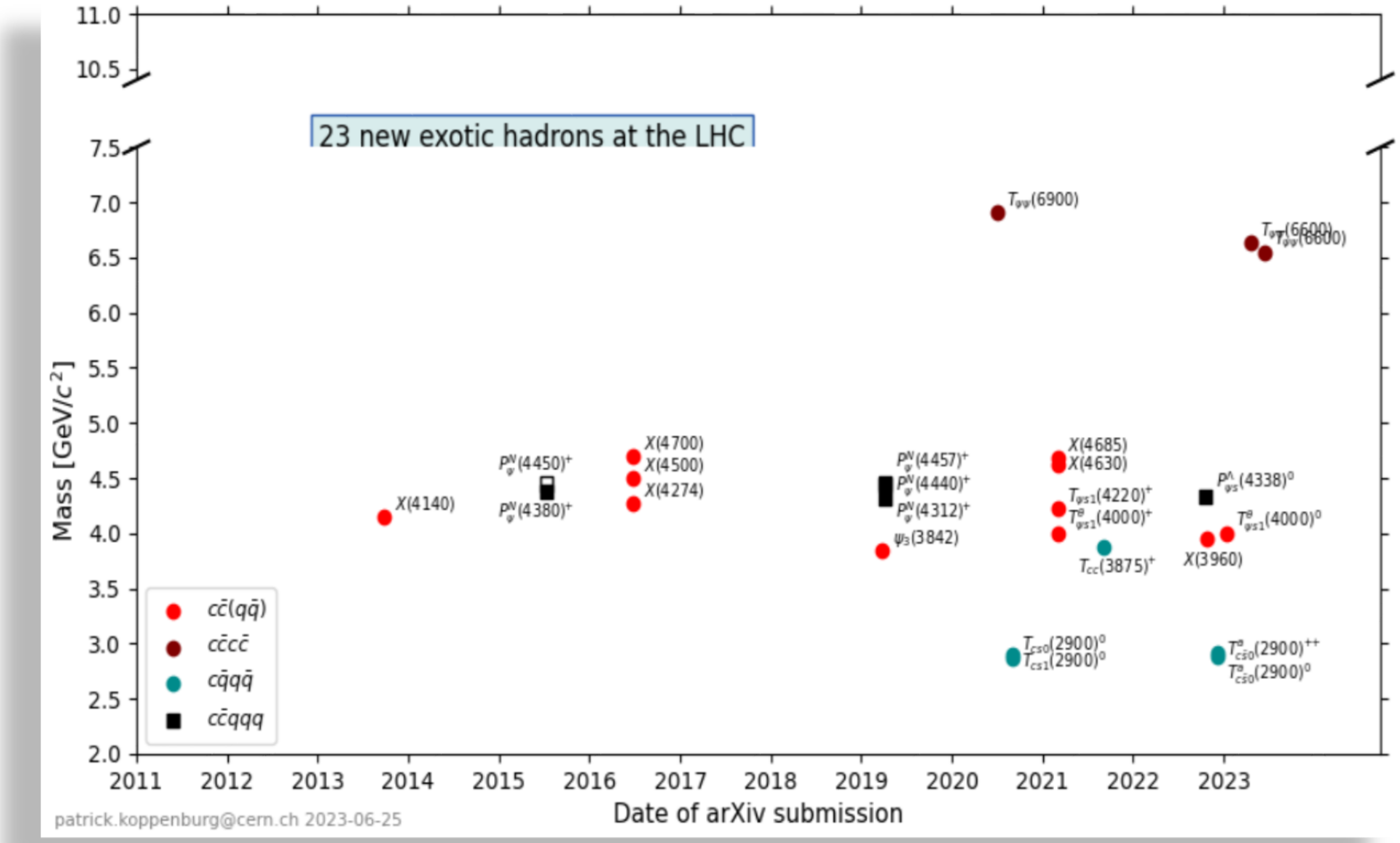
This exotic state has a mass of approximately 3,875 MeV

Parameter	Value
N	117 ± 16
δm_{BW}	$-273 \pm 61\,\mathrm{keV}\,c^{-2}$
Γ_{BW}	$410 \pm 165\,\mathrm{keV}$

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$



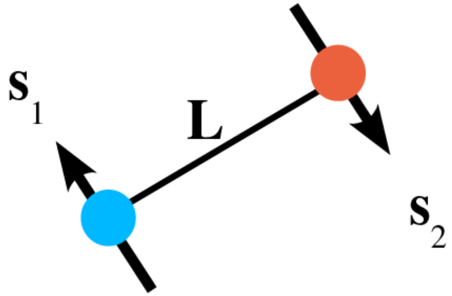
Exotic hadrons at LHC



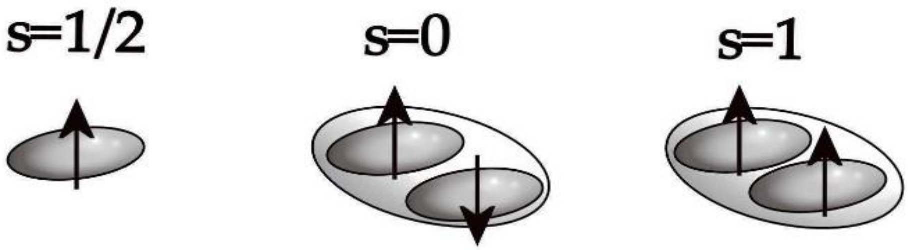
@LHCb

….And so at Belle, BES III, COMPASS, CMS, ALICE, ATLAS, JLAB

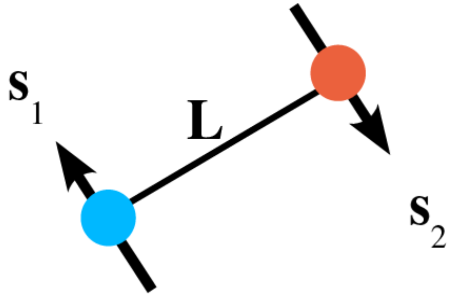
Spin Exotics



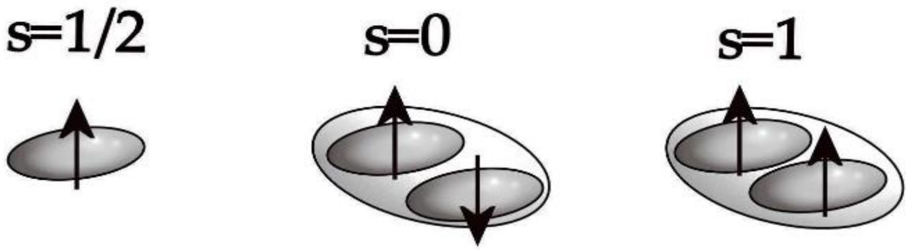
S = 0, 1



Spin Exotics



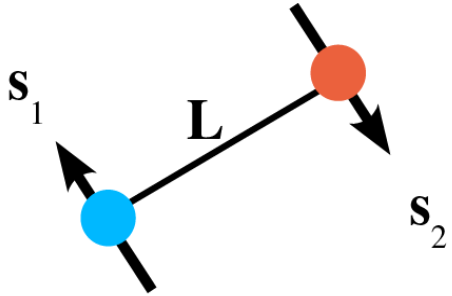
S = 0, 1



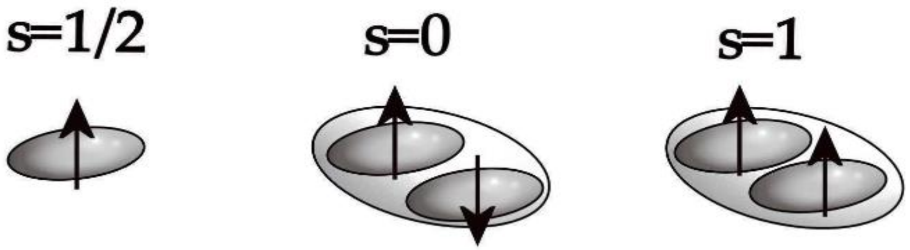
Combine with orbital angular momentum ***L***

$$\vec{J} = \vec{L} \oplus \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

Spin Exotics



S = 0, 1



Combine with orbital angular momentum ***L***

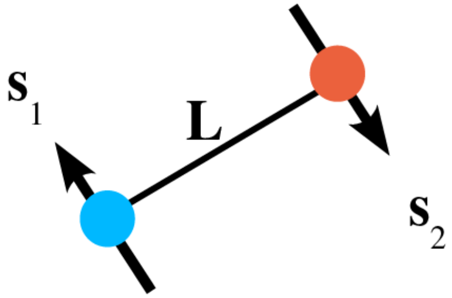
$$\vec{J} = \vec{L} \oplus \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

	L = 0	**L = 1**	**L = 2**	**L = 3**	**....**
Singlet (S = 0)	0^{-+}	1^{+-}	2^{-+}	3^{+-}	....
Triplet (S = 1)	1^{--}	$\{1,2,3\}^{++}$	$\{1,2,3\}^{--}$	$\{2,3,4\}^{++}$	....
	S-wave	P-wave	D-wave	F-wave	

Allowed : $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}, \ldots$

Are these all?

Spin Exotics



$$\vec{J} = \vec{L} \oplus \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

Forbidden (within such a model) quantum numbers :

$$J^{PC}:\ \mathbf{0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, \ldots}$$

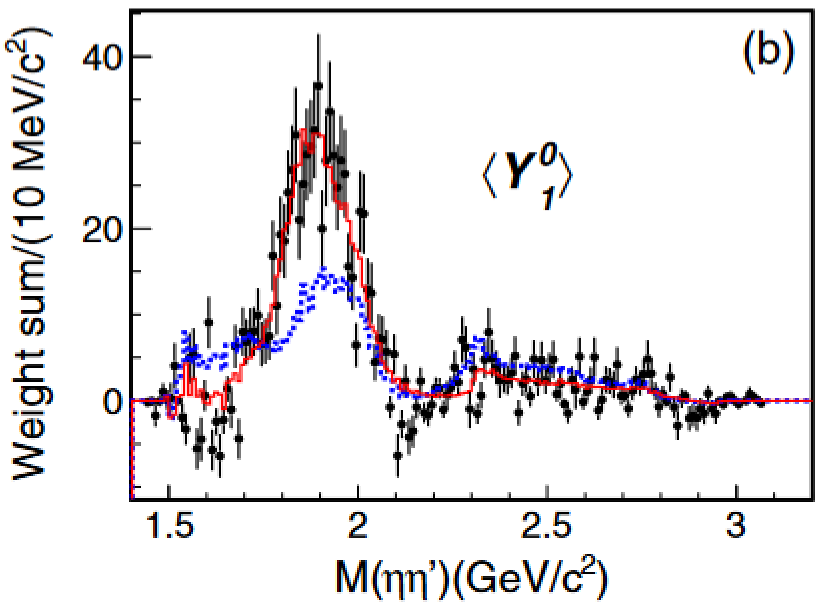
$$\boldsymbol{odd^{-+}, even^{+-}}$$

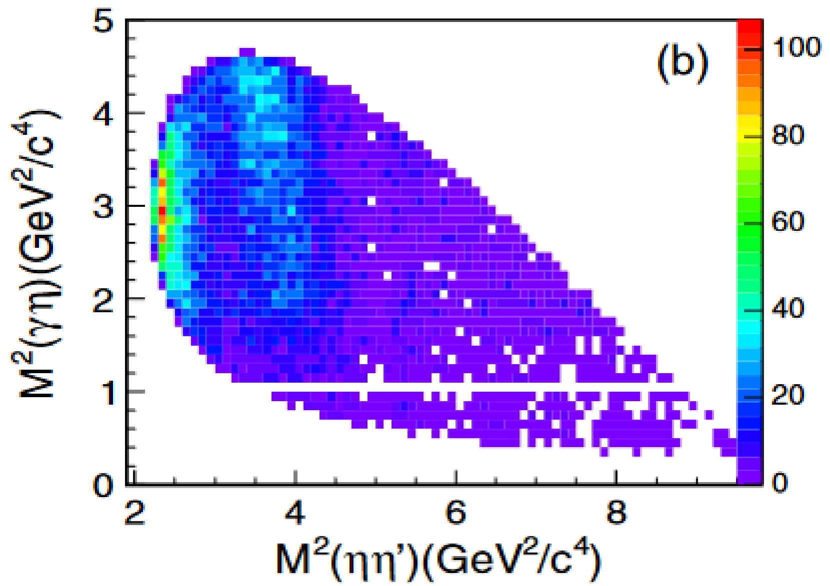
Any meson with these “forbidden” quantum numbers are called

EXOTIC MESON (spin exotic)

Observation of an Isoscalar Resonance with Exotic $J^{PC} = 1^{-+}$ Quantum Numbers in $J/\psi \to \gamma\eta\eta'$

(BESIII Collaboration)

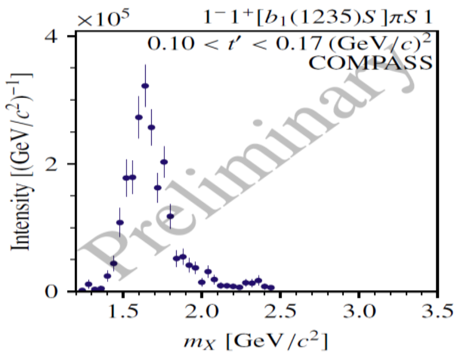




Mass: $1855 \pm 9^{+6}_{-1}$ MeV

Width: $188 \pm 18^{+3}_{-8}$ MeV

Clean signal for a spin-exotic $\pi_1(1600)$



@COMPASS: HQL23

Exotic hadrons and lattice QCD

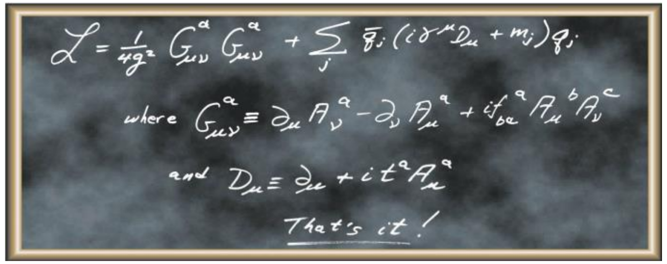
- Tetraquark and pentaquark hadrons have been observed experimentally with heavy quark contents. ….LHC, Belle, BES

- Are their possibilities to find more of those? And other multiquark states?

- What are the structures and properties of these exotic hadrons?

- What can lattice studies do?
    - Can predict more exotic states with possible valence structures and energies
    - Can decipher structures and properties of exotic hadrons

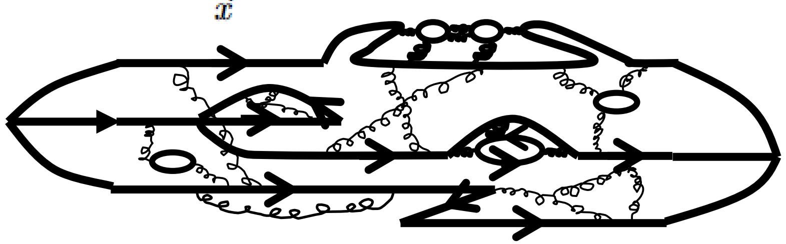
QCD



$$S_{QCD} = \int d^4x \, L_{QCD}(m_{q,} g_s)$$

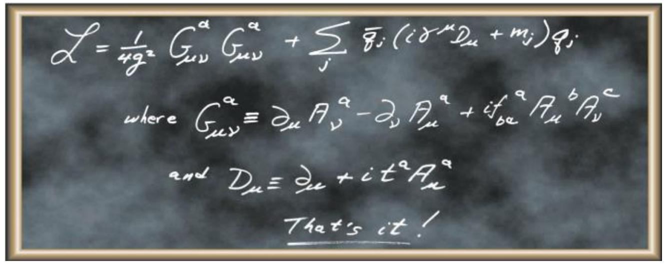
$$\langle C \rangle = \frac{\int DGDqD\bar{q}Ce^{-S_{QCD}}}{\int DGDqD\bar{q}\, e^{-S_{QCD}}}$$

$$C_{\mathcal{O}}(t_i, t_f) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle 0 | \mathcal{O}(\vec{x}_f, t_f) \bar{\mathcal{O}}(\vec{x}_i, t_i) | 0 \rangle$$

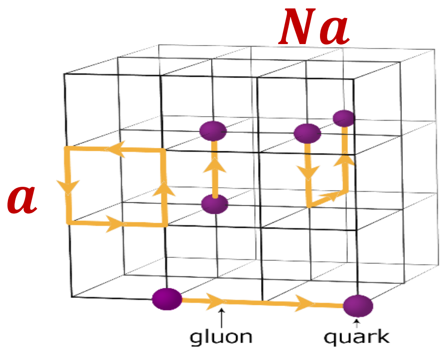


large time $\sim e^{-E_0 t}$

QCD → LQCD



Euclidean time



$$S_{QCD} = \int d^4x\, L_{QCD}(m_q, g_s)$$

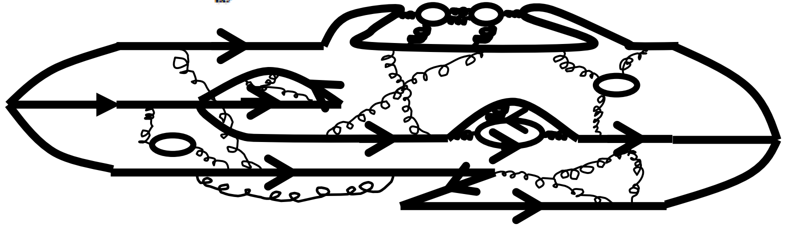
$$S^E_{QCD} = S^E_{QCD}[U, qi, D(U), m_{q_i}, a]$$

$$\langle C \rangle = \frac{\int DGDqD\bar{q}Ce^{-S_{QCD}}}{\int DGDqD\bar{q}\, e^{-S_{QCD}}}$$

$$\langle C \rangle = \frac{\int DUDqD\bar{q}Ce^{-S^E_{QCD}}}{\int DUDqD\bar{q}\, e^{-S^E_{QCD}}} \approx \frac{1}{N}\sum_n C(D^{-1}(U_n))$$

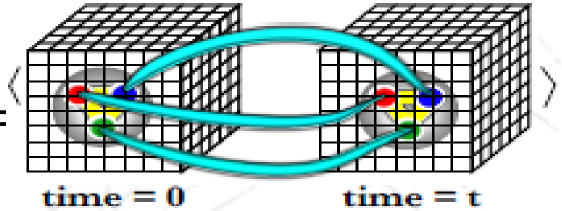
$$\Delta C = \frac{1}{\sqrt{N}} + \text{systematics}$$

$$C_{\mathcal{O}}(t_i, t_f) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle 0|\mathcal{O}(\vec{x}_f, t_f)\bar{\mathcal{O}}(\vec{x}_i, t_i)|0\rangle$$



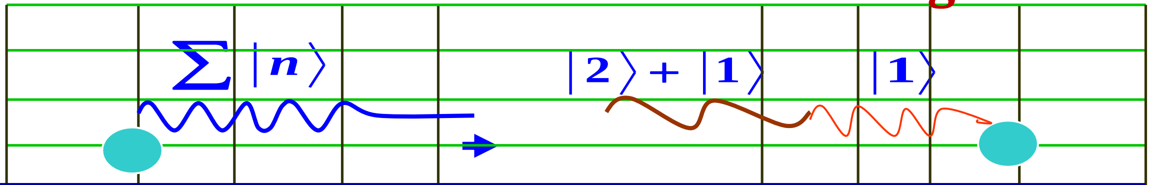
large time $\sim e^{-E_0 t}$

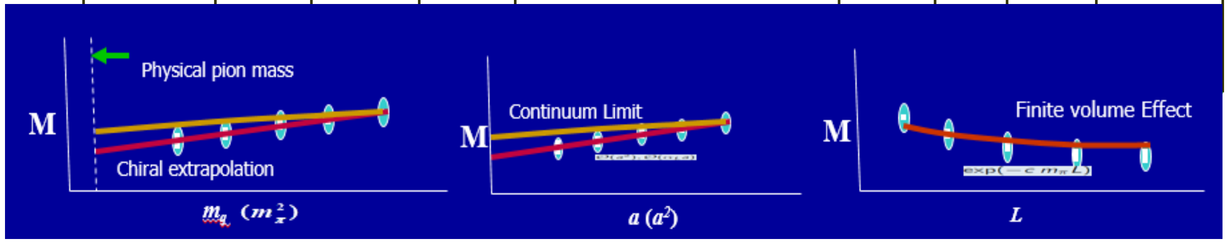
$$\left\langle C^{2pt}_{ab}(t, \vec{P}) \right\rangle =$$



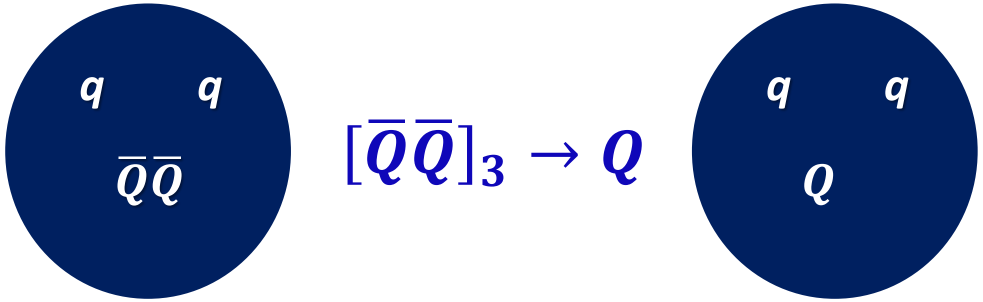
$$= \sum_n Z_{b,n} Z^{\dagger}_{a,n} e^{-E_n t}$$

τ large $\sim e^{-E_0 \tau}$





Heavy four-quark states



A possible structure:

$(qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}')$

How about ?

$\{qq'\}$ $\{\bar{Q}\bar{Q}'\}$

Possible states? : $\bar{b}\bar{b}ud, \bar{b}\bar{b}us, \bar{b}\bar{b}uc, \bar{b}\bar{b}sc,$
$\bar{b}\bar{c}ud, \bar{b}\bar{c}us$ *etc.*

$J = 1, l_1 l_2 \bar{Q}\bar{Q}$ $\qquad$ $J = 0, ll\bar{Q}\bar{Q}$

Possible Interpolating operators for four-quark hadrons

$$(qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}')$$

$$\{\bar{3}, J=0\} \quad \{3, J=1\}$$

Tetraquark type: $[\psi_i(x)\Gamma_1\psi_j(x)][\bar{\psi}_l(x)\Gamma_2\bar{\psi}_k(x)]$ With diquark-antidiquark

Meson-meson type:

$$\left(\sum_x e^{ip_1 \cdot x}\ \bar{\psi}_k(x)\Gamma_1\psi_i(x)\right)\left(\sum_y e^{ip_2 \cdot y}\ \bar{\psi}_l(y)\Gamma_2\psi_j(y)\right)$$

$x = y$: $Local$

$x \neq y$: $Non-local$

with the appropriate color and spin combinations

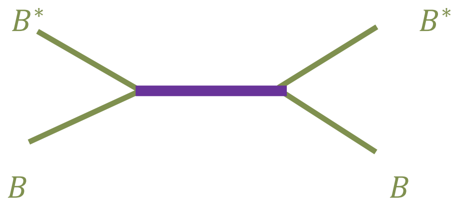
LQCD: bound states of

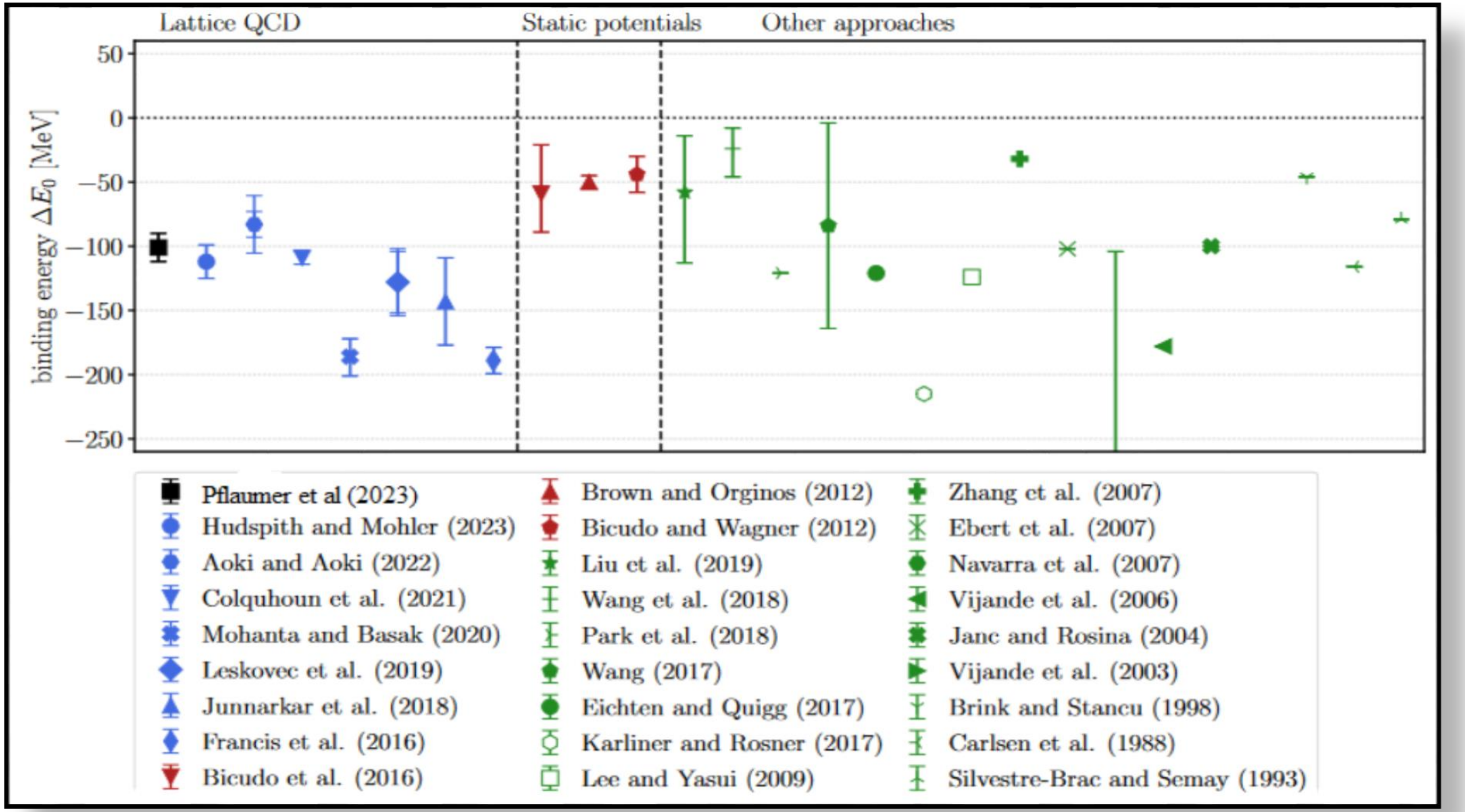
$$T_{bb}(\bar{b}\bar{b}ud), T_{bbs}(\bar{b}\bar{b}us), T_{bc}\,(\bar{b}\bar{c}ud), T_{cc}\,(\bar{c}\bar{c}ud)$$

Expt: $T_{cc}(\bar{c}\bar{c}ud)$

$T_{bb} \equiv \bar{b}\bar{b}ud$

Results so far

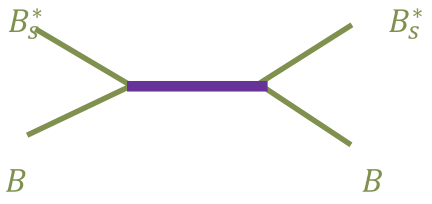


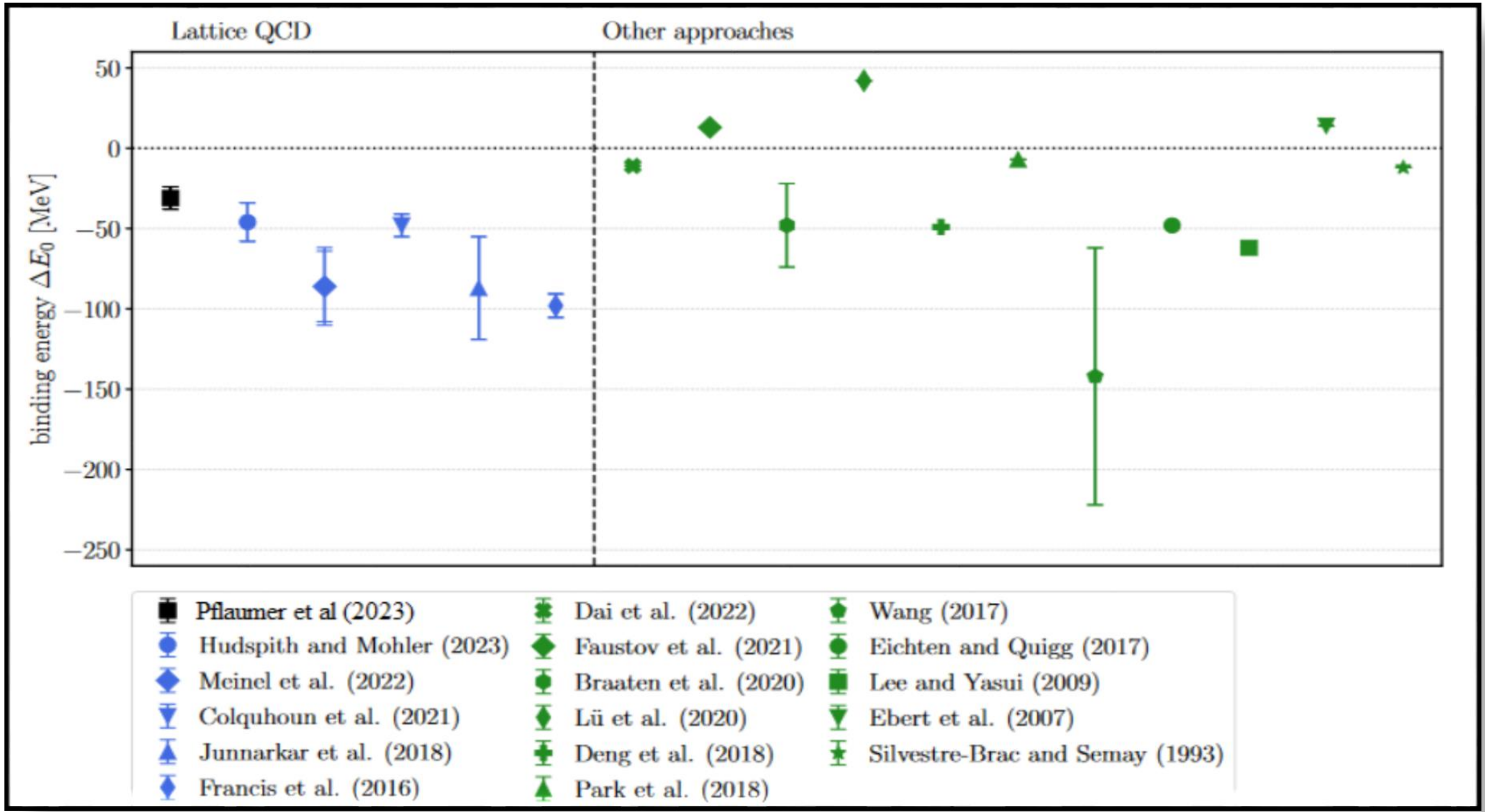


Summary plot: Pflaumer (2023)

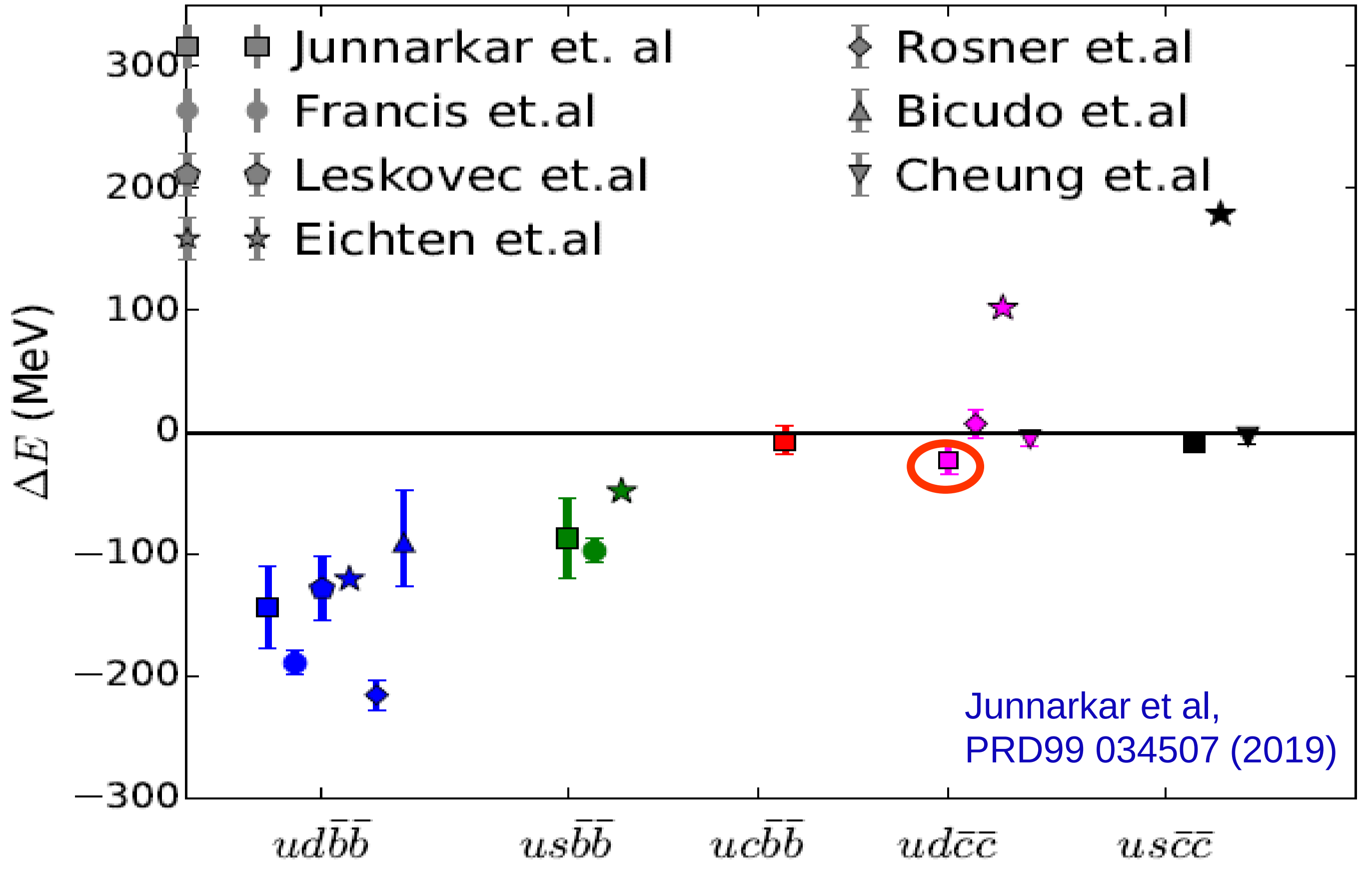
$T_{bb} \equiv \bar{b}\bar{b}us$

Results so far





Summary plot: Pflaumer (2023)



State	ΔE^1 [MeV]	State	ΔE^1 [MeV]
$ud\bar{b}\bar{b}$	-143(34)	$us\bar{b}\bar{b}$	-87(32)
$uc\bar{b}\bar{b}$	-6(11)	$sc\bar{b}\bar{b}$	-8(3)
$ud\bar{c}\bar{c}$	-23(11)	$us\bar{c}\bar{c}$	-8(8)

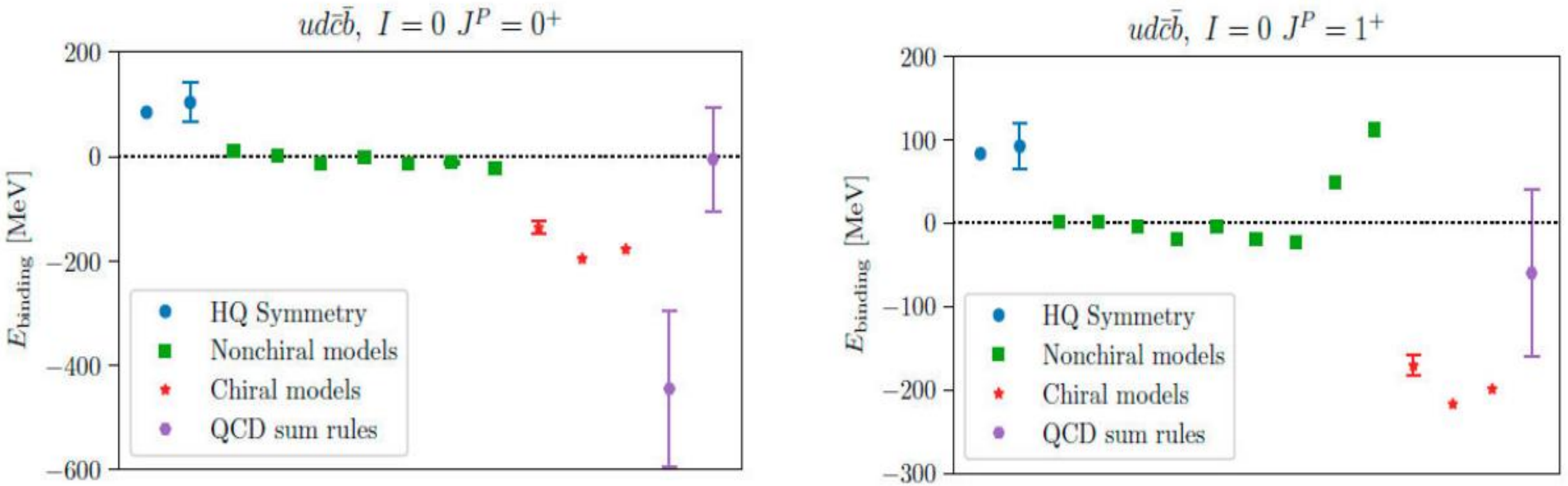
State	ΔE^0 [MeV]	State	ΔE^0 [MeV]
$uu\bar{b}\bar{b}$	-5(18)	$uu\bar{c}\bar{c}$	26(11)
$ss\bar{b}\bar{b}$	3(9)	$ss\bar{c}\bar{c}$	14(4)
$cc\bar{b}\bar{b}$	16(1)		

What about $T_{bc}: \bar{b}\bar{c}q_1q_2$?

Various models predicted mixed results for $ud\bar{b}\bar{c}\ (1^+)$:

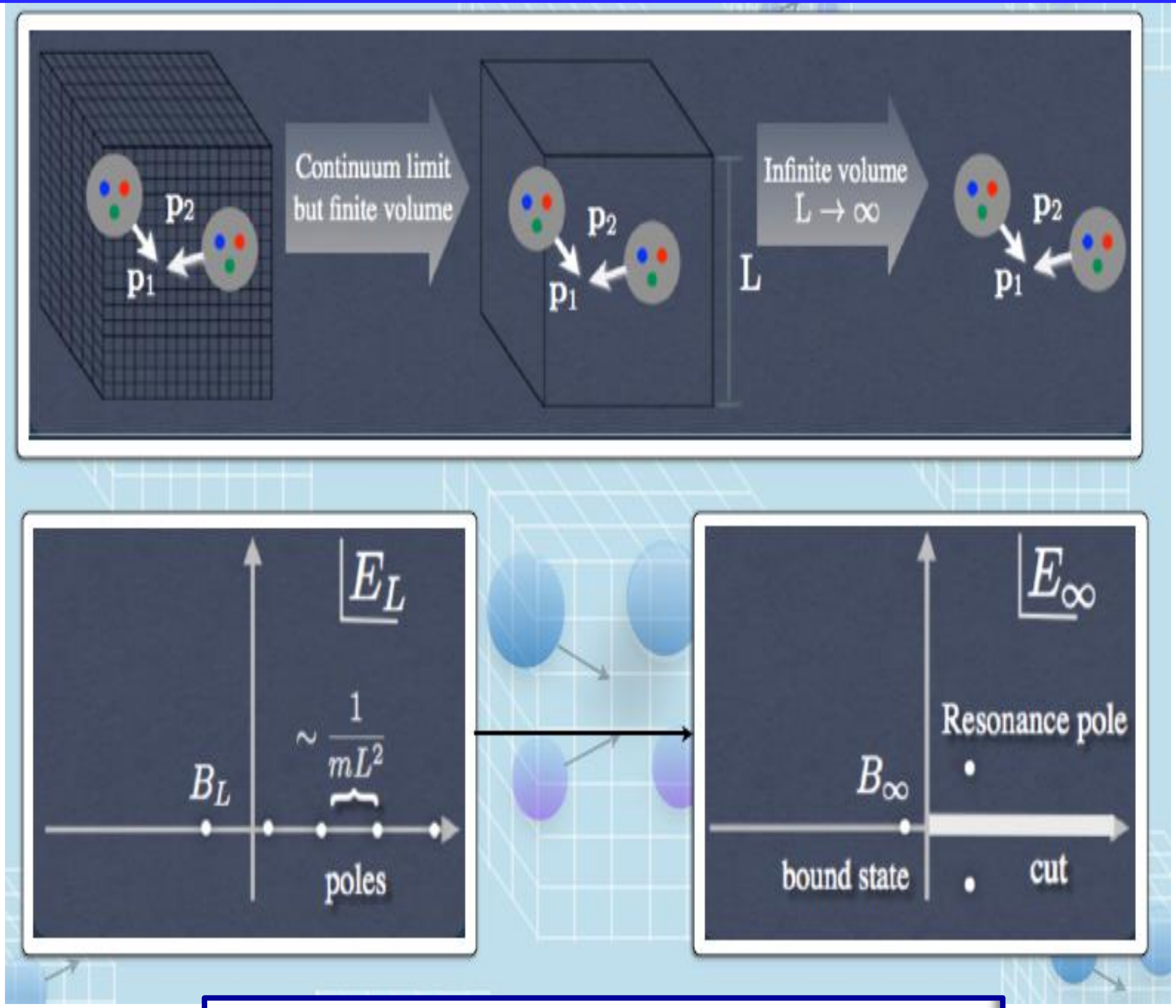
- HQ-symmetry inspired and non-chiral models: mostly unbound or very weekly bound
- QCD sum rule, chiral models: a bound state (both for 0 and 1-isospins) with binding over a wide range ~ 20-400 MeV !

Hudspith et al, Phys. Rev. D102, 114506 (2020)



B. Colquhoun et al, Rev. Mex. Fis. Suppl. 3 (2022) 3, 0308044

Resonance structure from finite volume study



Luscher:
CMP 105,153 (1986); NPB 354, 531(1991)

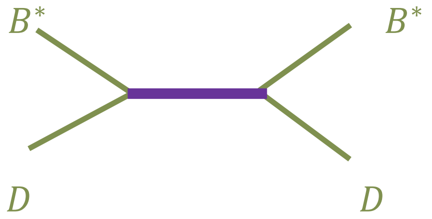
$$\det\left[\delta G^V(E) + M^{-1}(E)\right]$$

Finite volume spectra plus boundary condition

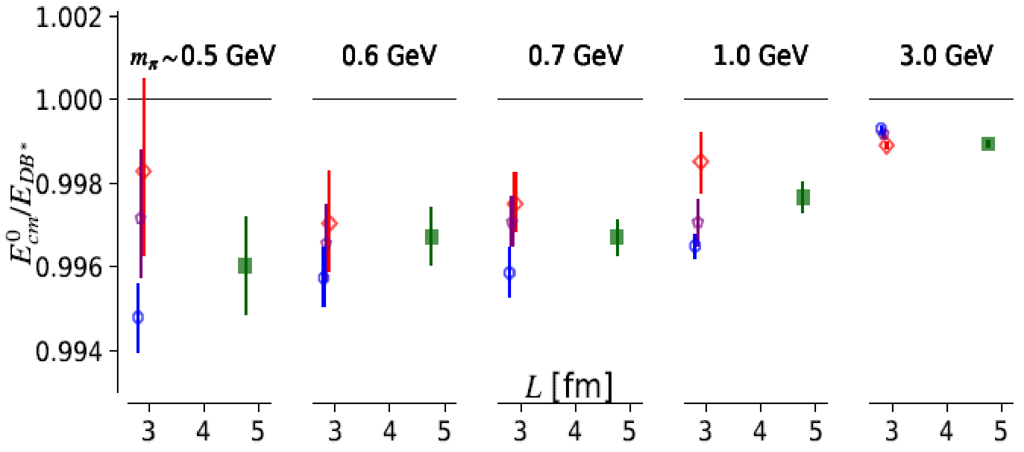
Infinite volume scattering amplitude

Z. Davoudi@Lat18

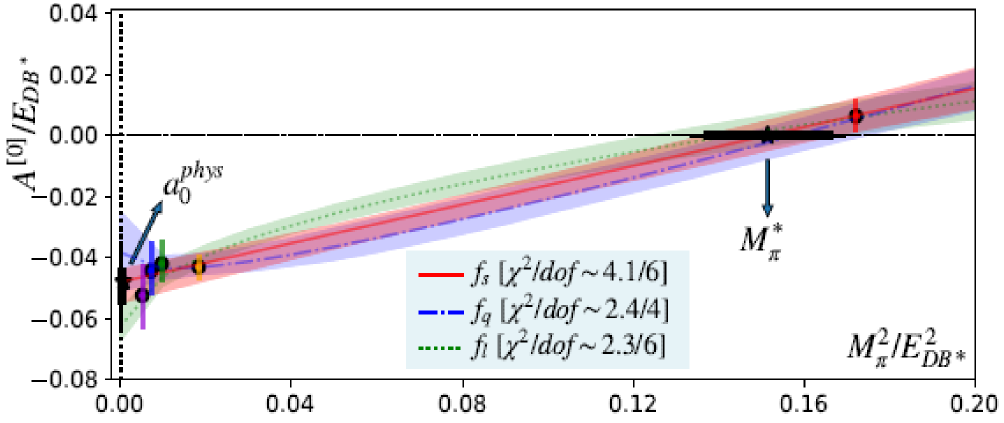
$bc\bar{u}\bar{d}\ \ 0(1^+)$



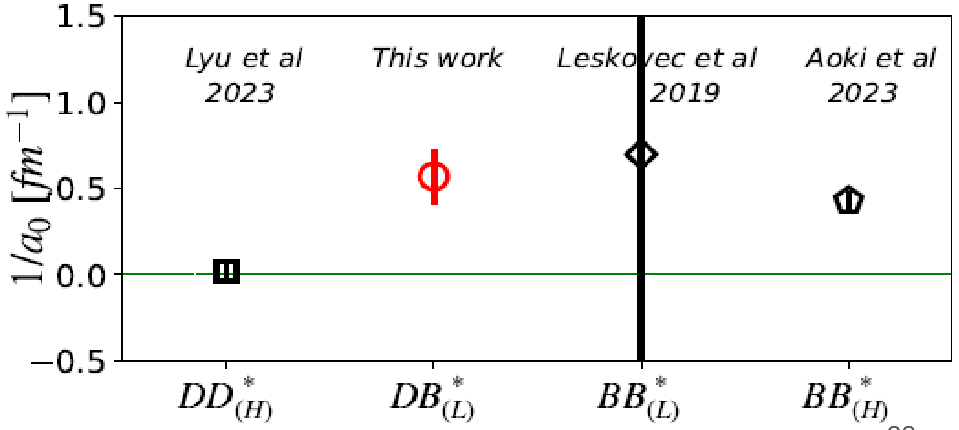
Presence of a bound state



M Padmanath, A Radhakrishnan, N Mathur
arXiv:2307.14128



Strong indication of a bound state of about 20-40 MeV binding energy



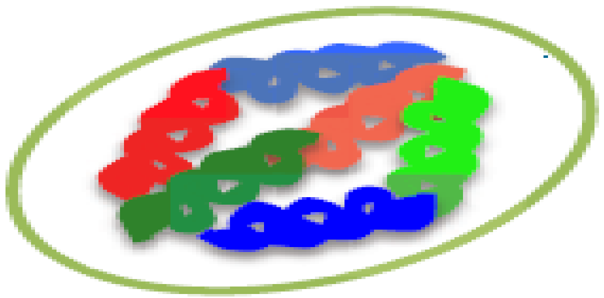
Tetraquarks with charm and bottom may be accessible to experiments - theoretical predictions can help in such searches!

Bound or virtual bound state within a few MeV or less below threshold

arXiv: 2312.0292

Glueball

- A glueball is a gluonic bound state.



- In the theory of QCD, gluon self coupling admits the existence of such a state.

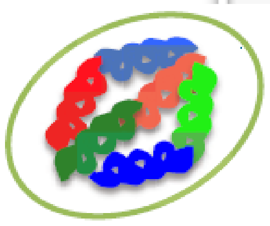
$8 \otimes 8$	$1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27$

- No conclusive experimental evidence of glueball as yet though the f_o states are indicative. Difficult to detect due to mixing but the searches are ongoing

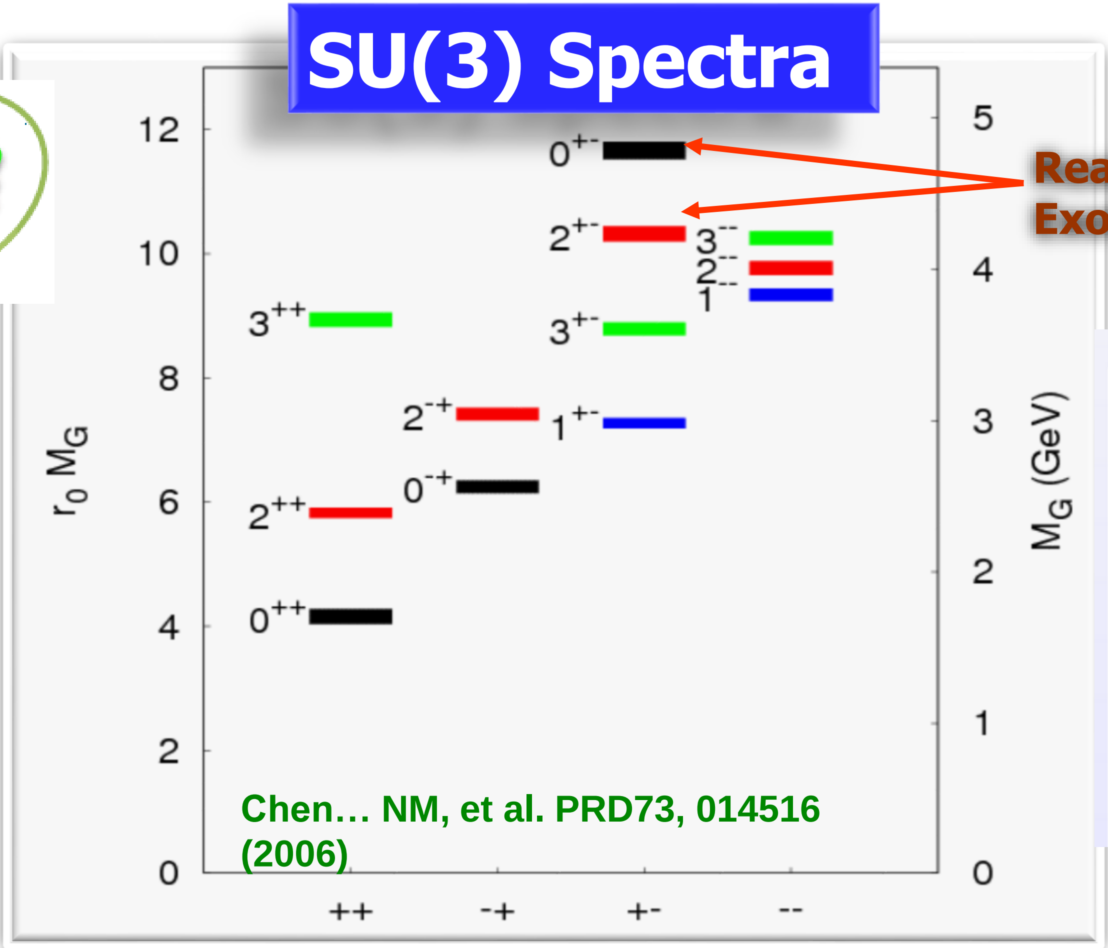
- However, lattice QCD calculations can tell us about glueball spectra

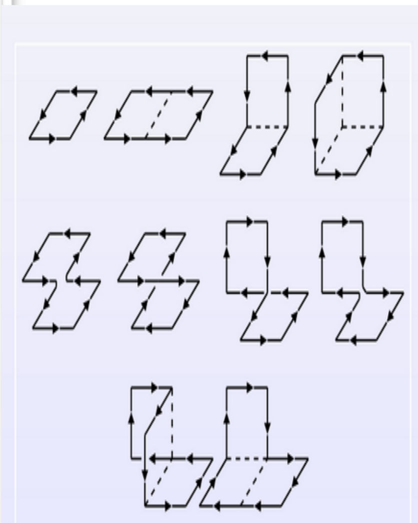
Glueball

- ➢ A glueball is a gluonic bound state.



- ➢ In the theory of QCD gluon self coupling admits the existence of such a state.
- ➢ No conclusive experimental evidence of glueball as yet though the f_o states are indicative. Difficult to detect due to mixing but the searches are ongoing





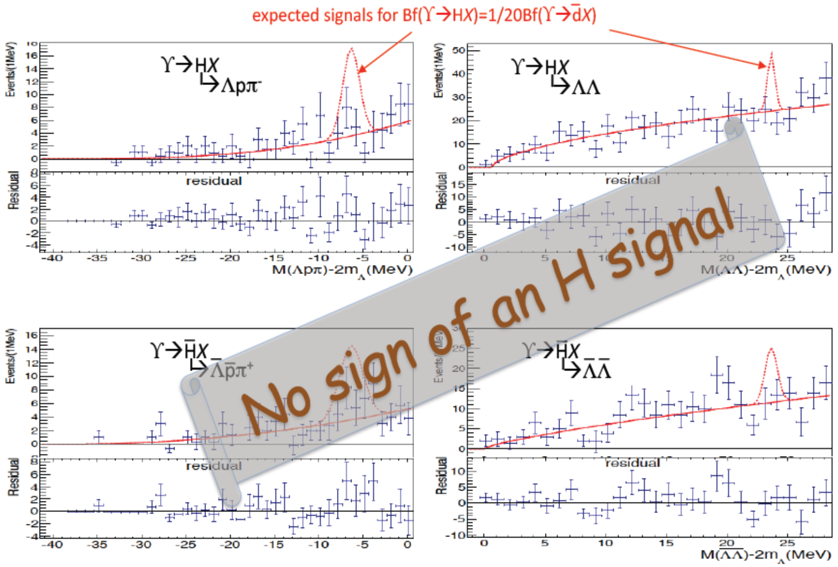
- ▪ Signal-to-noise ratios in lattice glueball correlation functions with dynamical quarks are still very poor.
- ▪ Multiple channels with glueball, two-quarks and four-quarks with the same quantum numbers need to be addressed together
- ▪ **Precise calculations with dynamical quarks including mixing effects are necessary**

H Dibaryon

Bound state of two Λ $\Lambda\Lambda\ (udssud)$ Proposed by Jaffe (1976)

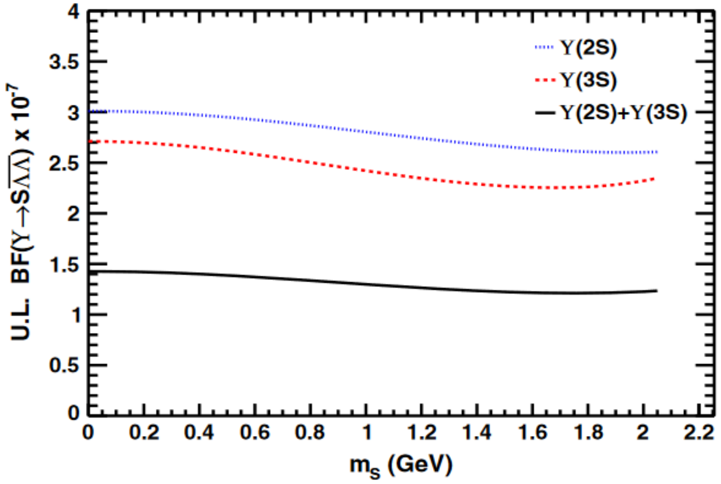
- Has to be below the two proton threshold. Then it will be bound
- If it exists it is extremely stable and could be a candidate for SM dark matter? (May not be as oxygen may not exist with that!)

No H dibaryon

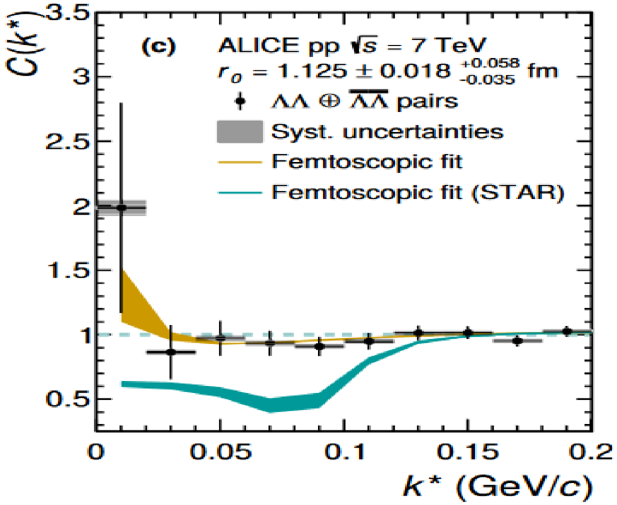


$M(\Lambda p\pi^-)-2m_\Lambda$ B.H. Kim *et al* (Belle) PRL 110, 222002 (2013) $M(\Lambda\Lambda)-2m_\Lambda$

Belle: PRL 110,222002(2013)

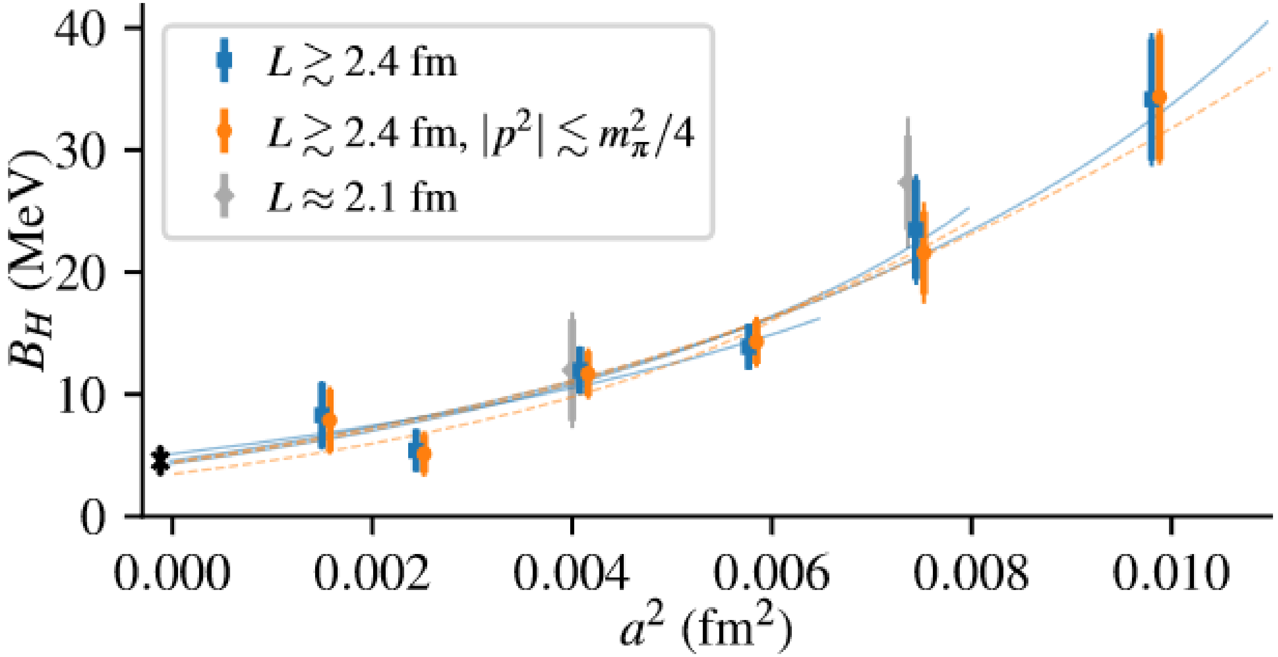


BABAR Collaboration:
Phys. Rev. Lett. 122, 072002 (2019)
No signal is observed
in Υ **decays** (90% confidence limit)



Alice: Phys. Rev. C 99, 024001 (2019)

H-dibaryon at $\mathrm{SU(3)_F}$ symmetric point

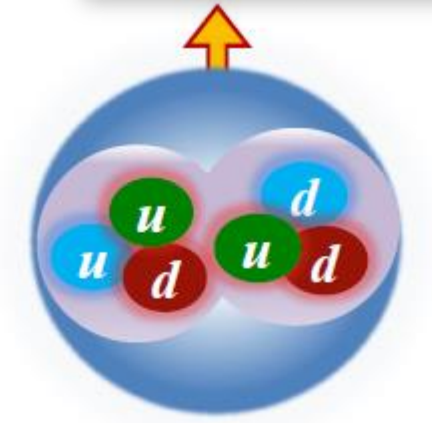


$$\breve{B}_H^{\mathrm{SU(3)_f}} = 4.56 \pm 1.13_{\mathrm{stat}} \pm 0.63_{\mathrm{syst}}\ \mathrm{MeV}.$$

Green et al : *Phys. Rev. Lett.* 127 (2021) 24, 242003

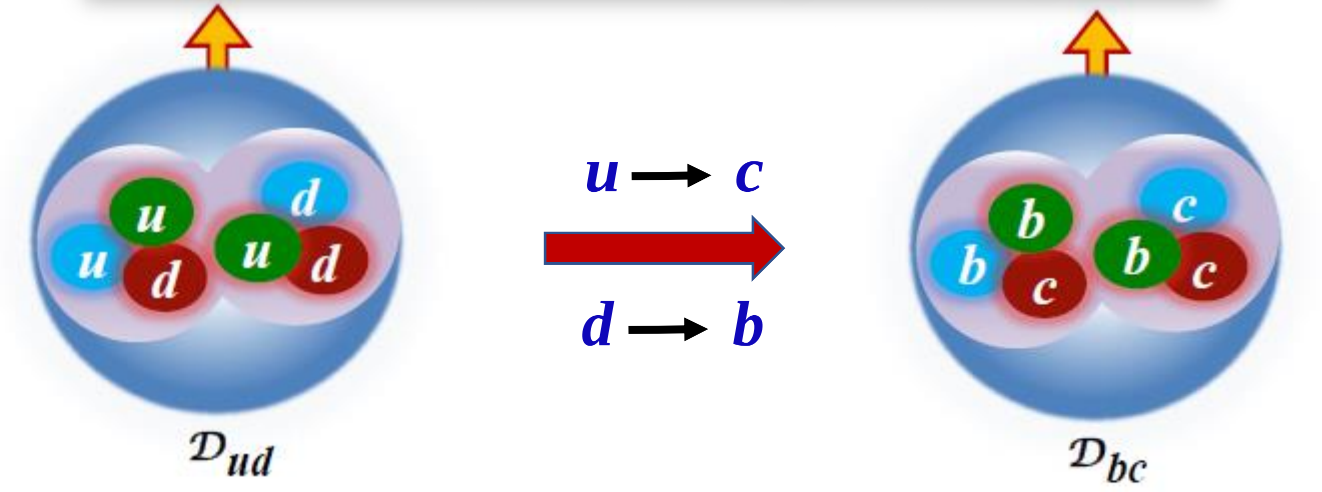
Are there heavy dibaryons?

Deuteron-like heavy dibaryons

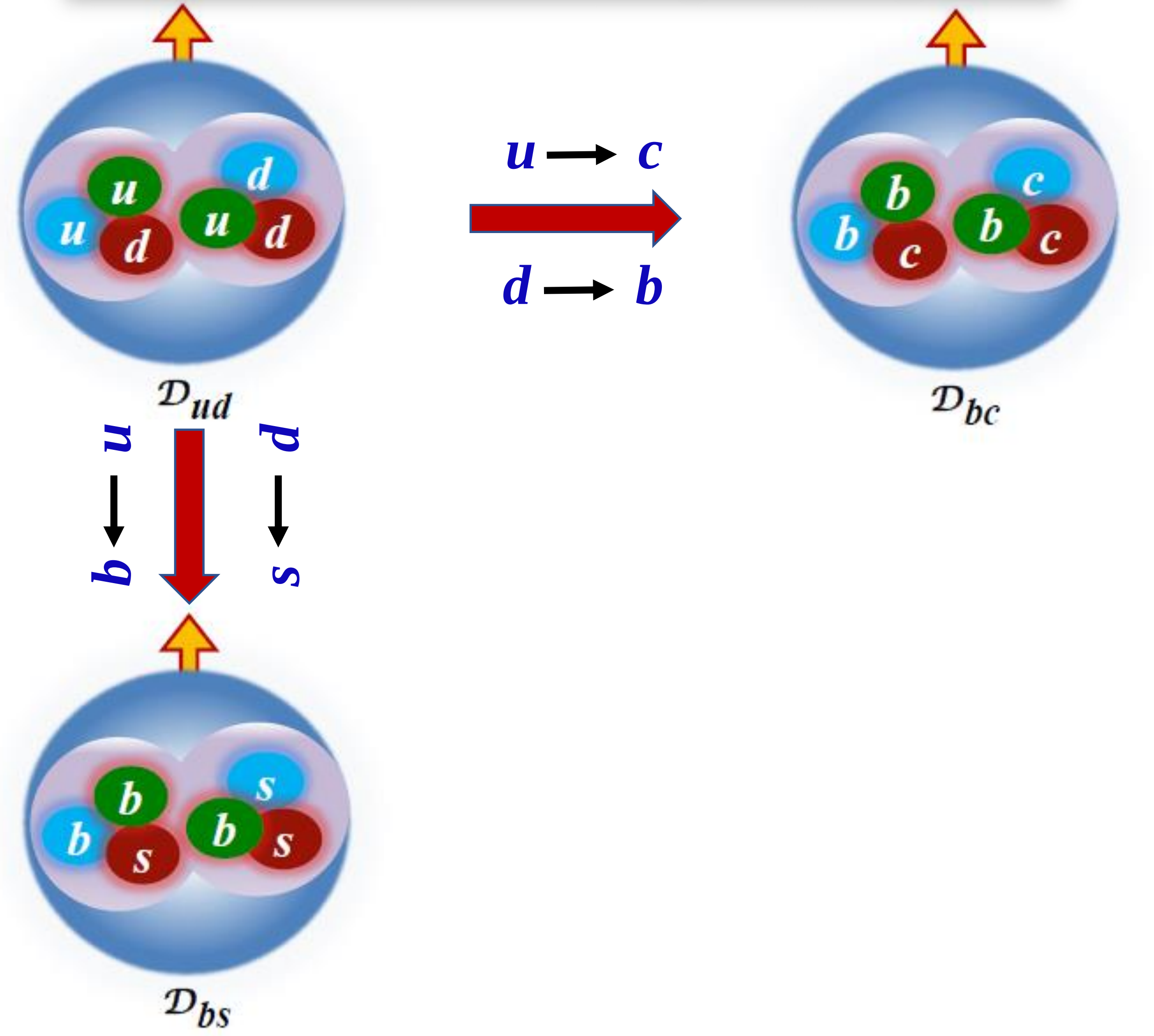


$\mathcal{D}_{ud}$

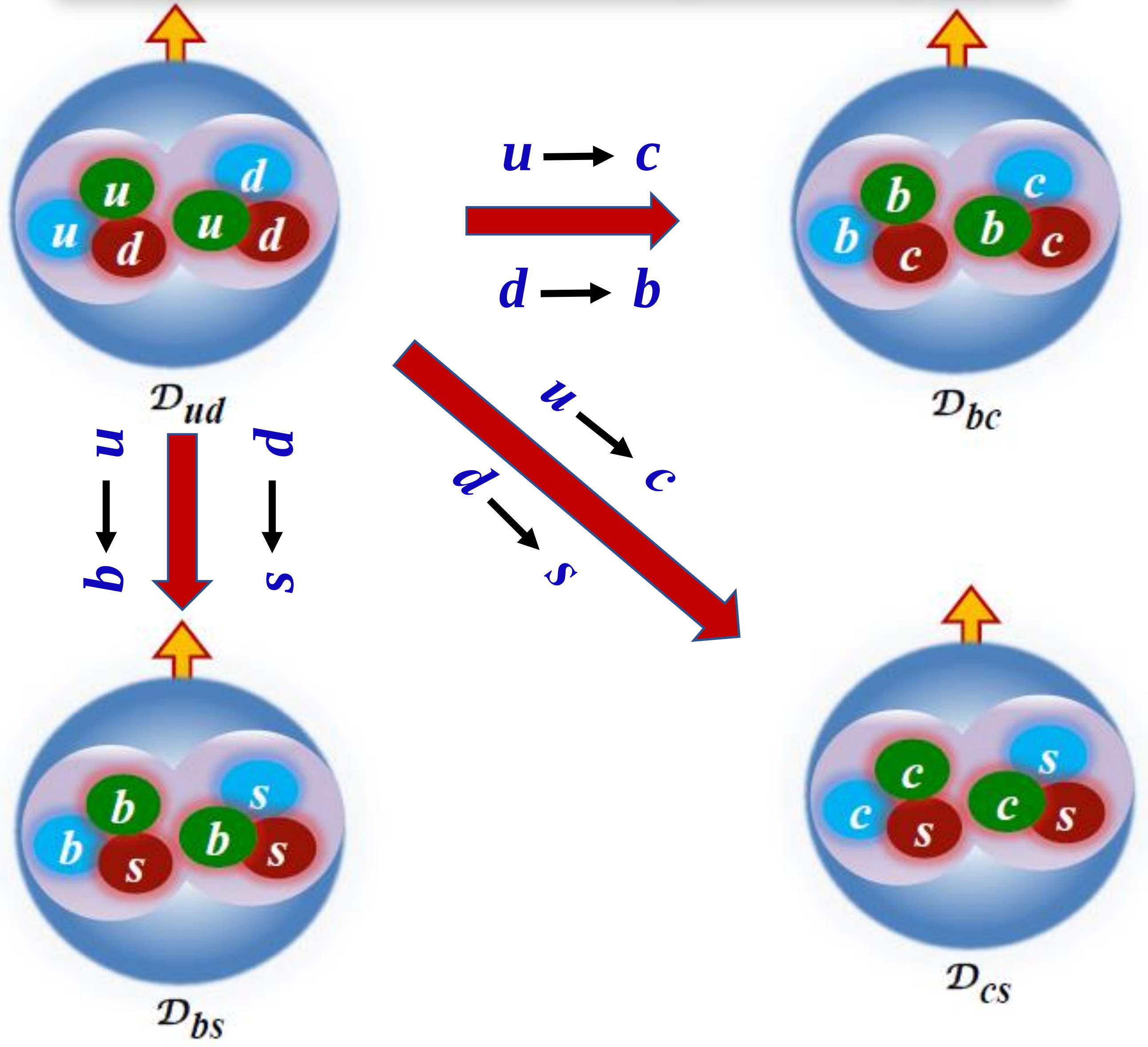
Deuteron-like heavy dibaryons



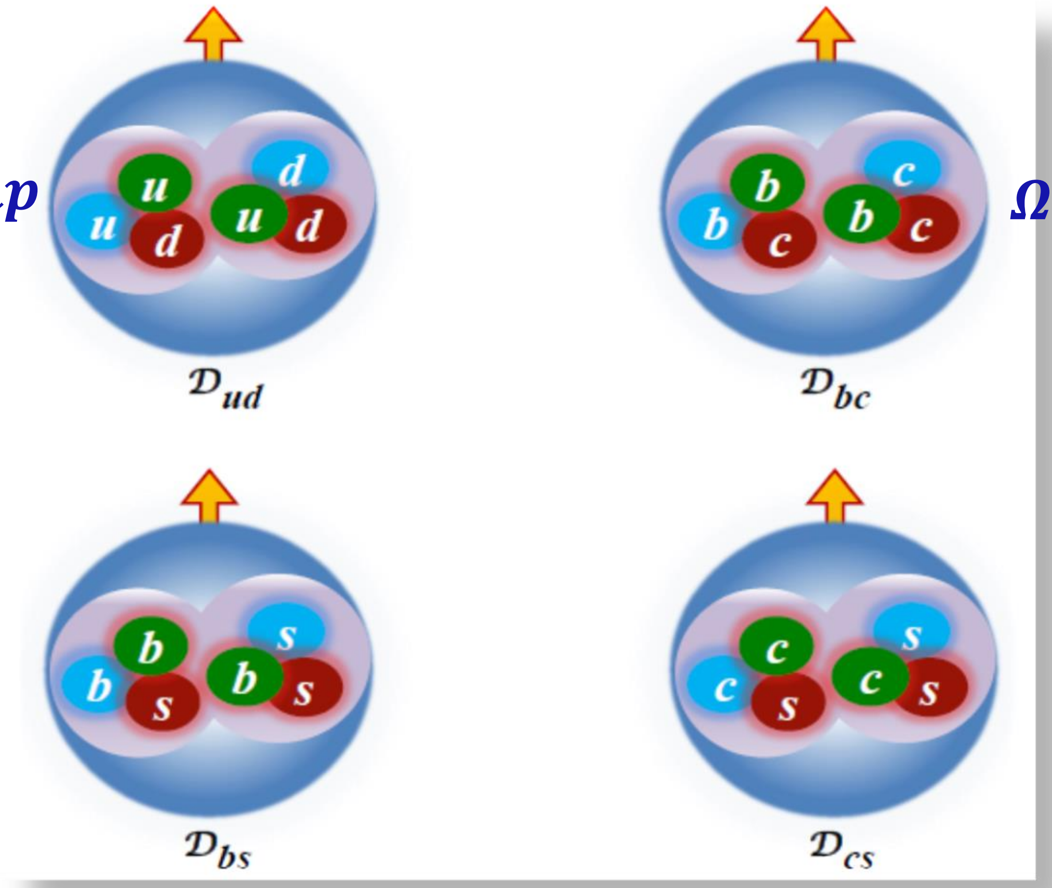
Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons



np

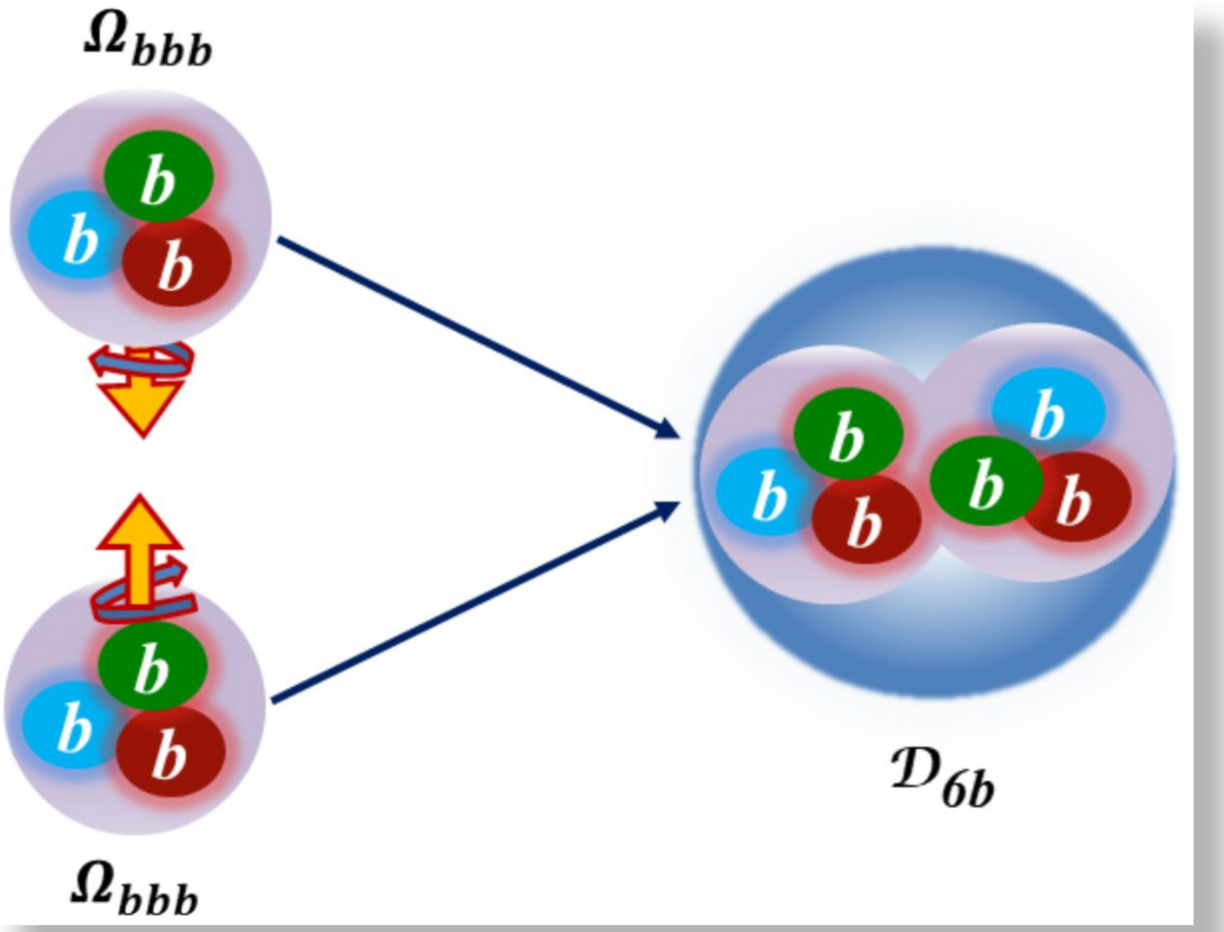
$\Omega_{ccb}\Omega_{cbb}$

$\Omega_b\Omega_{bb}$

$\Omega_c\Omega_{cc}$

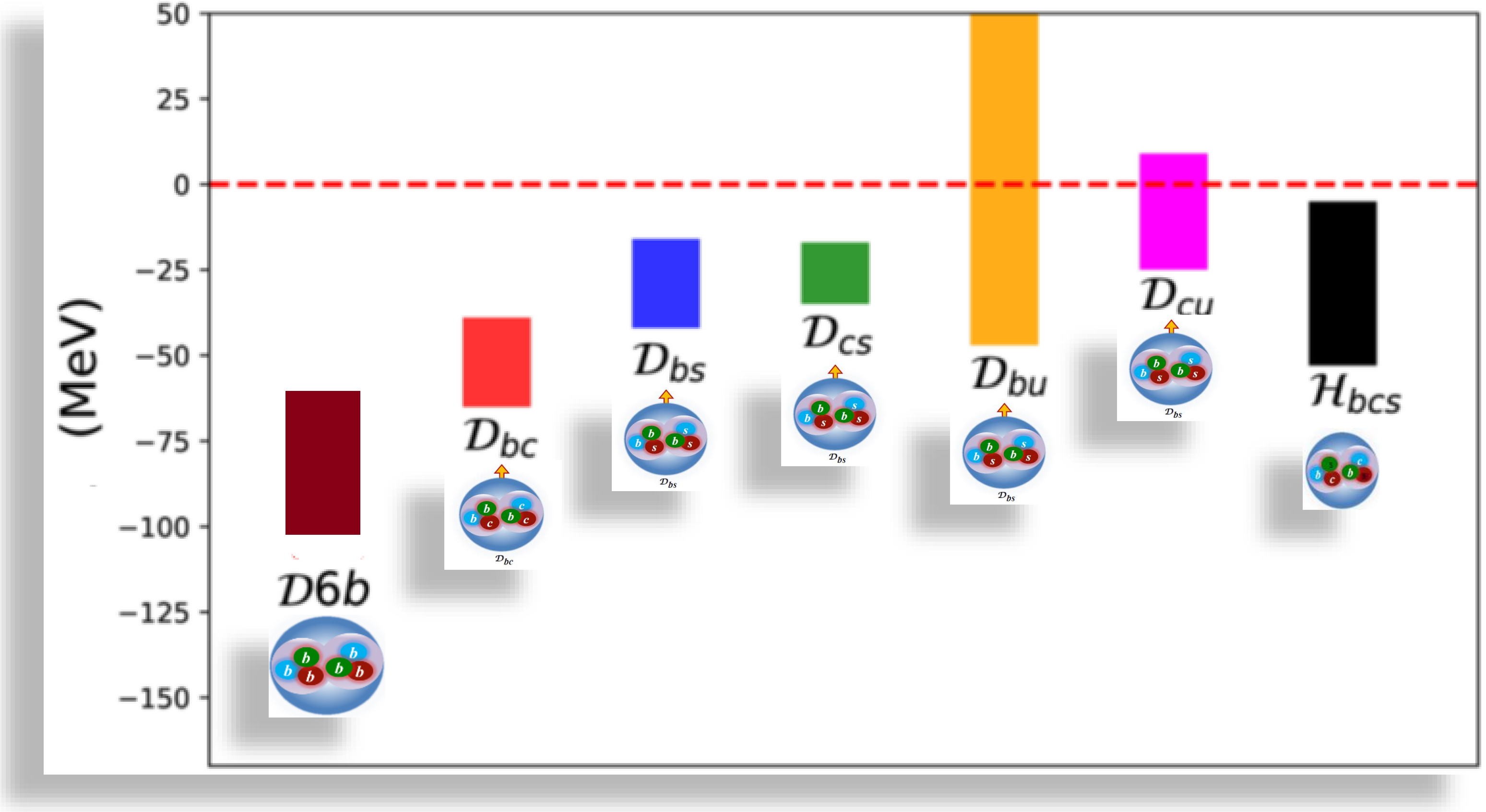
Junnarkar and NM : Phys. Rev. Lett. 123, 162003(2019)

Most beautiful dibaryons!



NM, Padmanath and Chakraborty: PRL 130, 111901 (2023)

Heavy Dibaryon Candidates?



PRL 123,162003 (2019): Junnarkar, NM

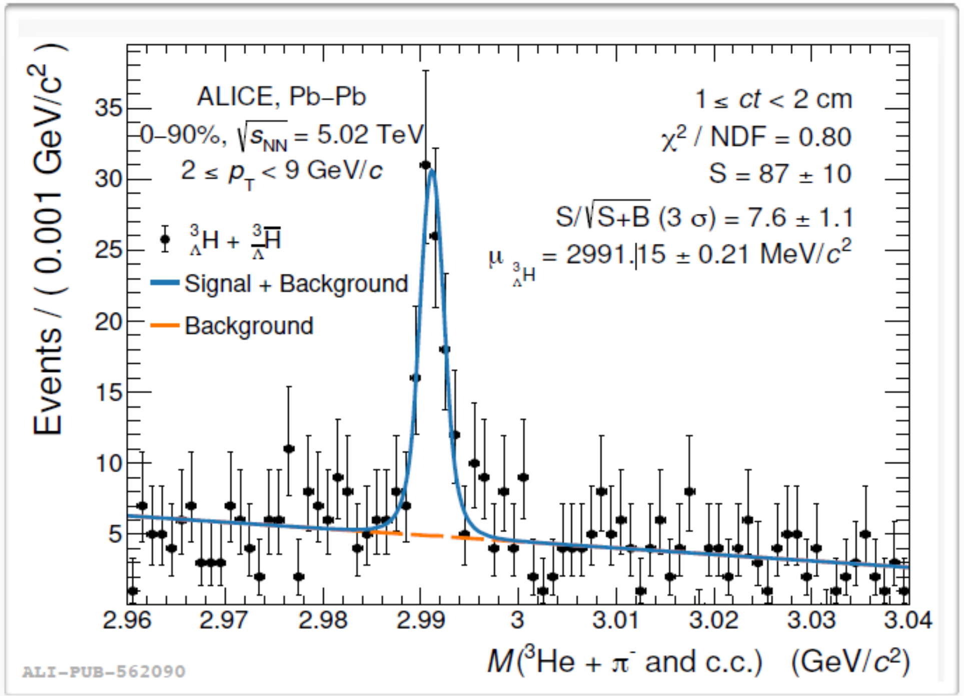
PRL 130, 111901 (2023): NM, Padmanath and Chakraborty

PRD 106, 054511 (2019): Junnarkar, NM

Hypernuclei

Nuclei with one or more hyperon

$$^{3}_{\Lambda}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \pi \; (\mathrm{B.R.} \approx 25\%)$$



$$\tau = (253 \pm 11(\mathrm{stat}) \pm 6(\mathrm{syst}))\ \mathrm{ps}$$

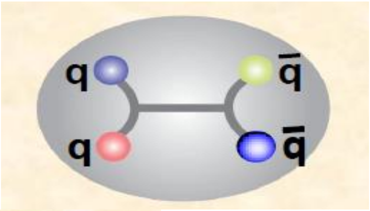
$$B_\Lambda = (102 \pm 63(\mathrm{stat}) \pm 67(\mathrm{syst})\ \mathrm{keV}$$

ALICE: Phys Rev Lett 131, 102302 (2023)

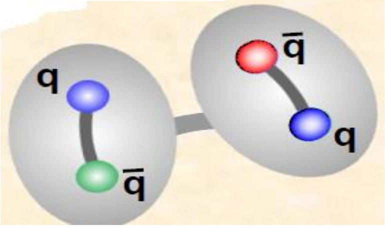
Models for Exotics

Exciting discovery :

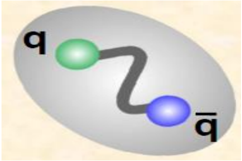
- Compact Tetraquark



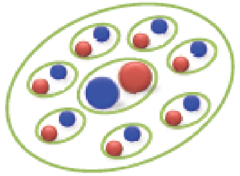
- Molecules
  (like deuteron!)



- Hybrids



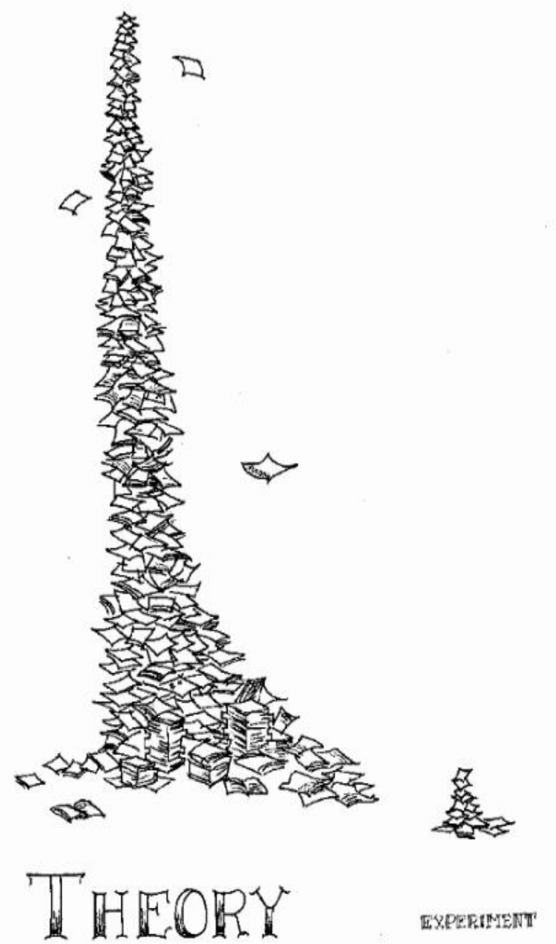
- Hadro-quarkonium



- Glueballs

Experimental effects

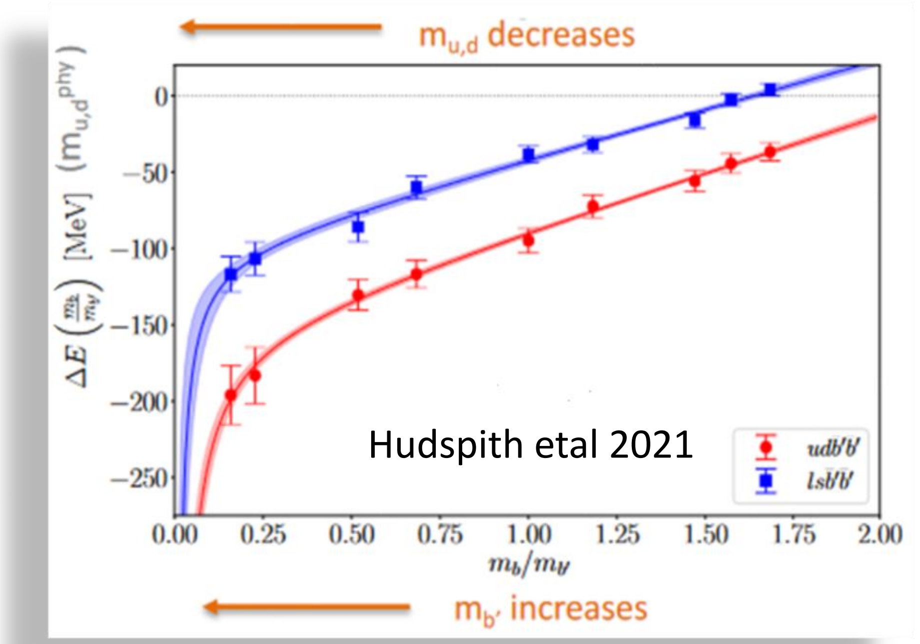
- Cusp/rescattering effects



Conclusions and Outlooks

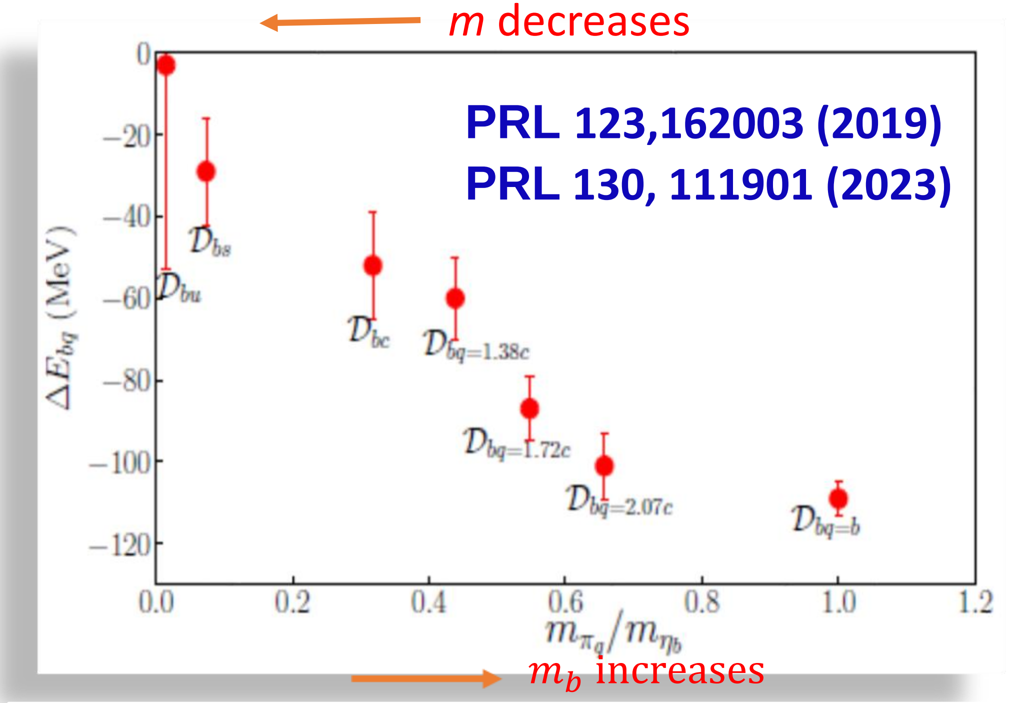
- They (Exotic hadrons) are here (in Nature): Experiments have discovered Exotic hadrons.
- Many more will be discovered soon: Good time for experimentalists in spectroscopy.
- No one has any clue about their formation and structures: Good time for theoreticians.
- Lattice QCD provides a rigorous approach to hadron spectroscopy. Lattice QCD calculations have predicted and postdicted some of these exotic hadrons.
- Lattice QCD calculations are essential :
    - To predict the unknown exotic hadrons, hence aiding their discovery,
    - To understand the structures and properties of exotic hadrons

Four-quark hadrons $(Q_1Q_2\bar{q}_1\bar{q}_2)$



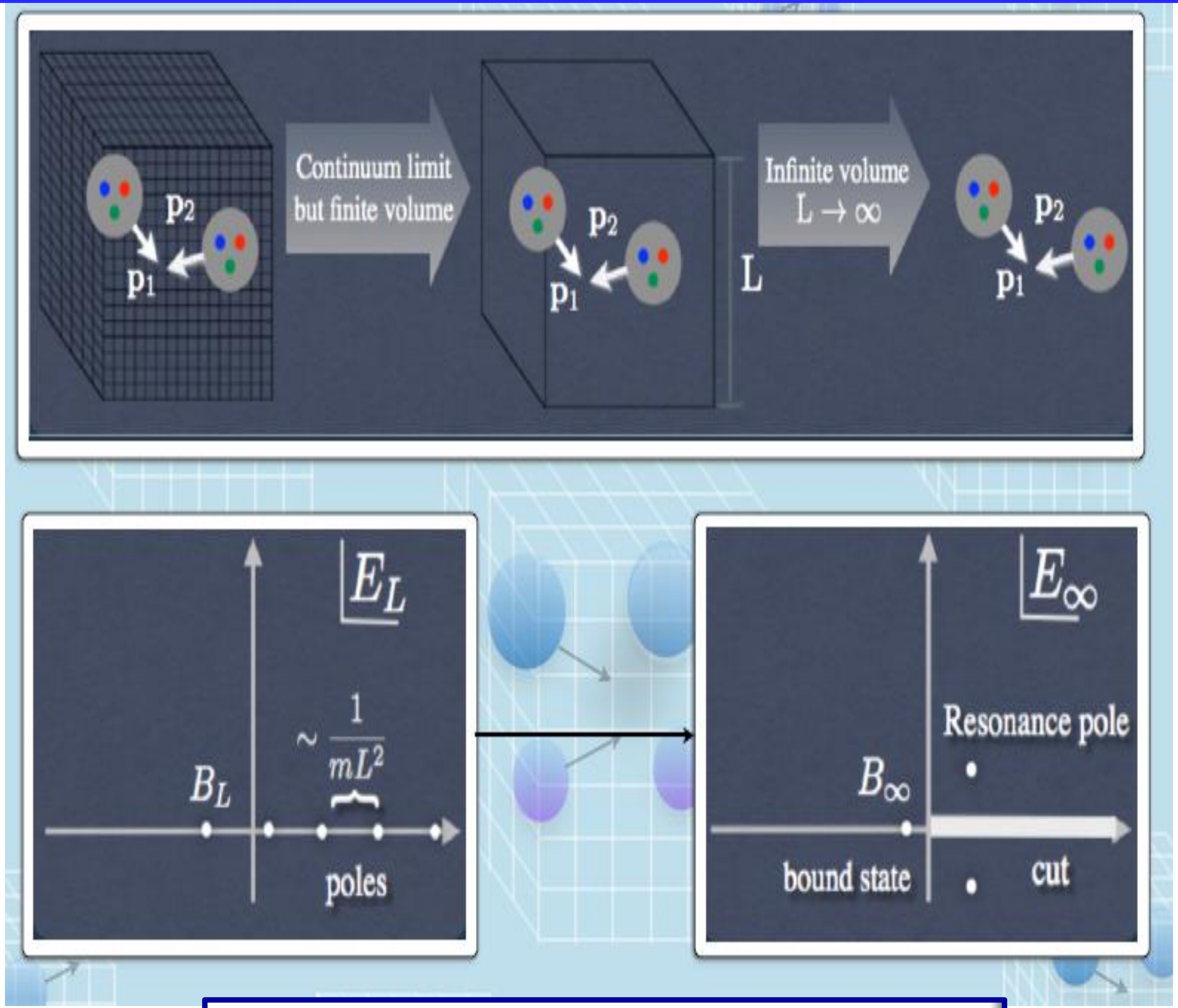
- Heavier the heavy quark masses, stronger the binding
- Lighter the light quark masses, stronger the binding

Six-quark hadrons $(Q_1Q_2q_1Q_1Q_2q_2)$



- Heavier the heavy quark masses, stronger the binding
- Heavier the light quark masses, deeper the binding

Resonance structure from finite volume study



Luscher:
CMP 105,153 (1986); NPB 354, 531(1991)

$$\det\left[\delta G^V(E) + M^{-1}(E)\right]$$

Finite volume spectra plus boundary condition

Infinite volume scattering amplitude

Z. Davoudi@Lat18

Example of an Exotic

- States with quantum number : $\mathbf{1^{-+}}$
- It is not possible to write an interpolating field for this state with a form : $\bar{q}\Gamma q$
- Possible operators :

$$\bar{q}^a \gamma_4 E_j^{ab} q^b,$$
$$i\varepsilon_{jkl}\bar{q}^a \gamma_k B_l^{ab} q^b \Rightarrow \rho \otimes B$$
$$i\varepsilon_{jkl}\bar{q}^a \gamma_4 \gamma_k B_l^{ab} q^b$$
$$\varepsilon_{jkl}\bar{q}^a \gamma_5 \gamma_4 \gamma_k E_l^{ab} q^b$$
$$\bar{q}\gamma_4 \overleftrightarrow{D} q$$
$$\bar{q}_\alpha^a \gamma_5 q_\beta^a \bar{q}_\beta^b \gamma_5 \gamma_i q_\lambda^b \Rightarrow \pi \otimes a_1$$
$$\varepsilon_{ijk}\bar{q}\gamma_5 \gamma_4 \gamma_j \overleftrightarrow{D}_k q$$
$$\varepsilon_{ijk}\bar{q}\gamma_j \vec{B}_k q, \quad \vec{B}_i = \varepsilon_{ijk}\vec{D}_j \vec{D}_k$$
$$\varepsilon_{ijk}\bar{q}\gamma_4 \gamma_j \vec{B}_k q$$

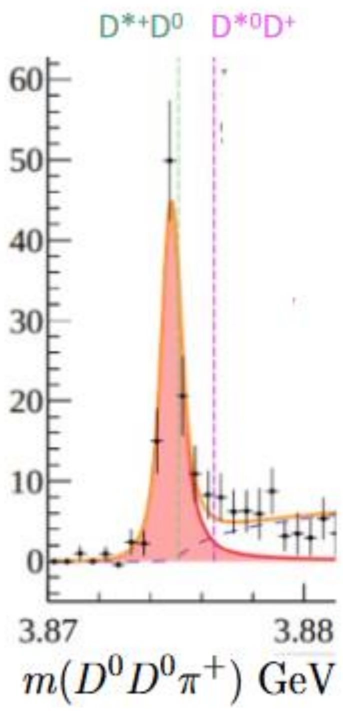
$$\begin{aligned}\mathbb{B}_i &= \epsilon_{ijk}\overrightarrow{D}_j\overrightarrow{D}_k \\ &= \epsilon_{ijk}\tfrac{1}{2}\left([\overrightarrow{D}_j,\overrightarrow{D}_k] + \{\overrightarrow{D}_j,\overrightarrow{D}_k\}\right) \\ &= \epsilon_{ijk}\tfrac{1}{2}[\overrightarrow{D}_j,\overrightarrow{D}_k] \\ &= -\tfrac{i}{2}\epsilon_{ijk}F^{jk}\end{aligned}$$

$$\mathbb{E}_i = \mathbb{Q}_{ijk}\overleftrightarrow{D}_j\overleftrightarrow{D}_k$$

Hadspec (2008)

T_{cc}

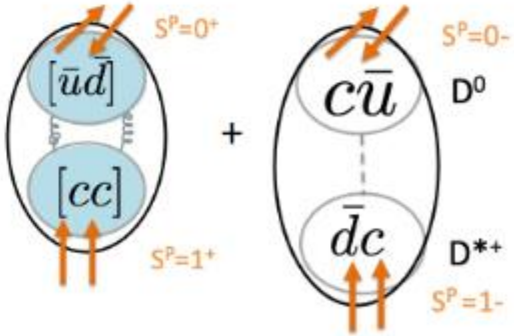
Doubly charm tetraquark T_{cc} $cc\bar{d}\bar{u}$

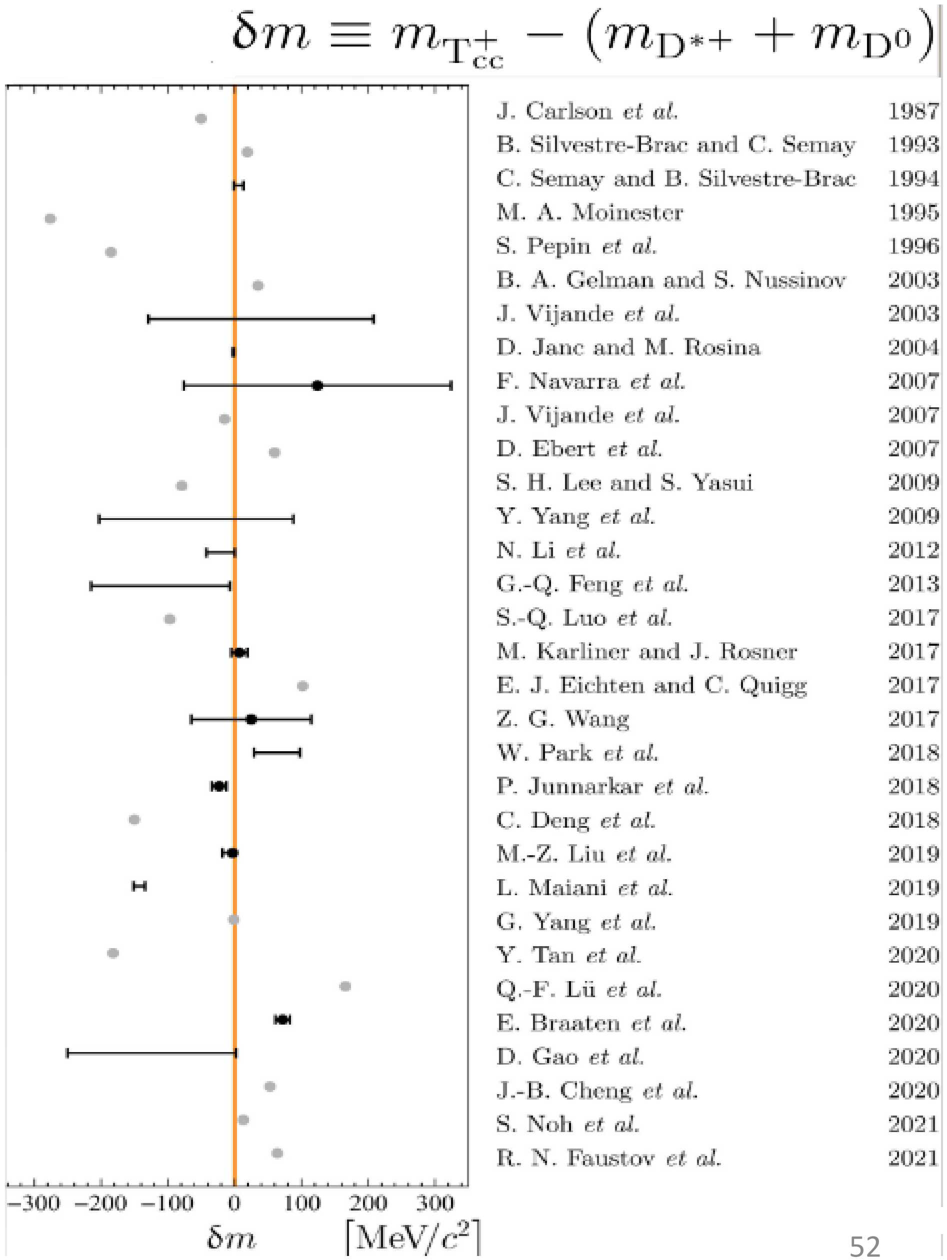


$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056

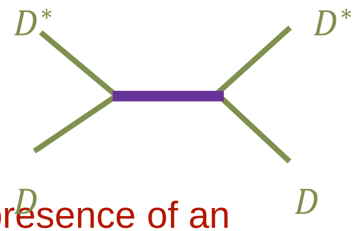




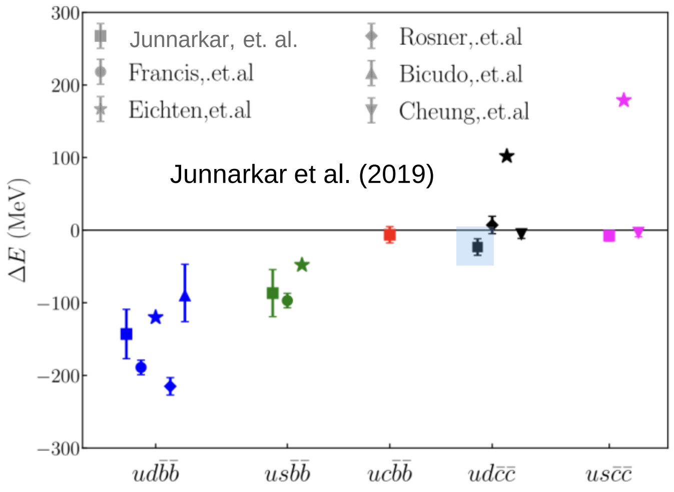
What does LQCD tell us?

States near threshold

$cc\bar{u}\bar{d} \quad 0(1^+)$



A lattice calculation had seen presence of an energy level below lowest threshold- **later discovered by LHCb!**



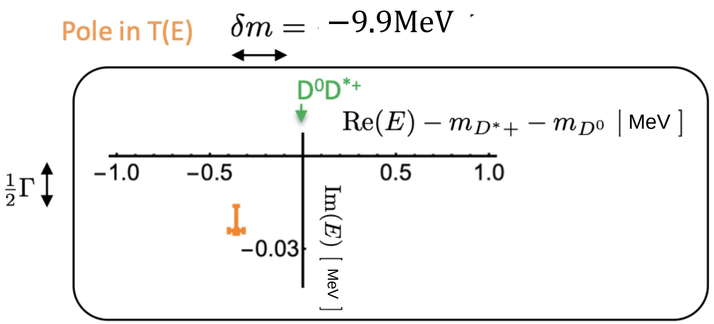
Exploring tetraquarks with bottom and charm may be accessible to experiments

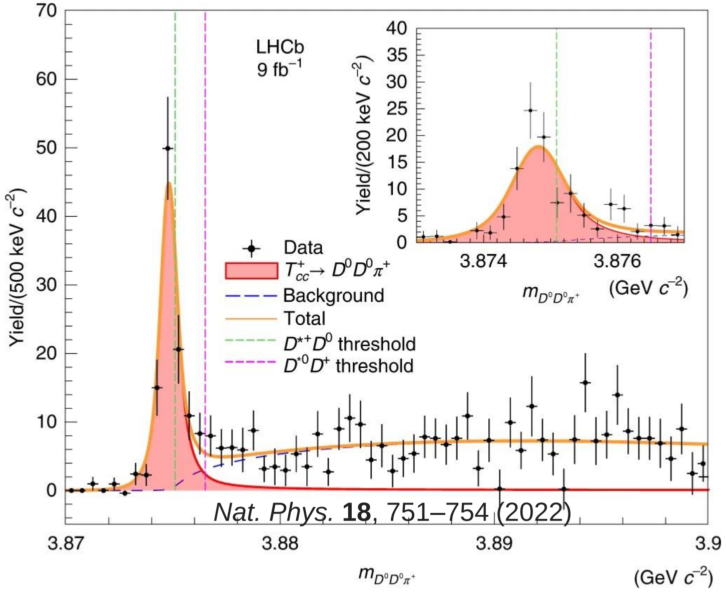
Motivates the study of $bc\bar{u}\bar{d}$

but significant finite volume effects possible - close to threshold

Need to find the poles in the scattering amplitude to extract (virtual) bound poles:

Padmanath et al. Phys.Rev.Lett. 129 (2022)



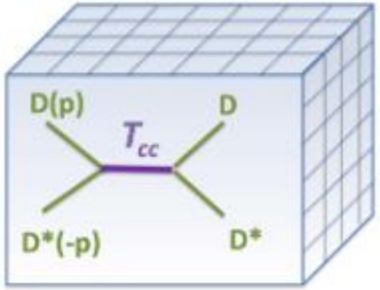


Nat. Phys. **18**, 751–754 (2022)

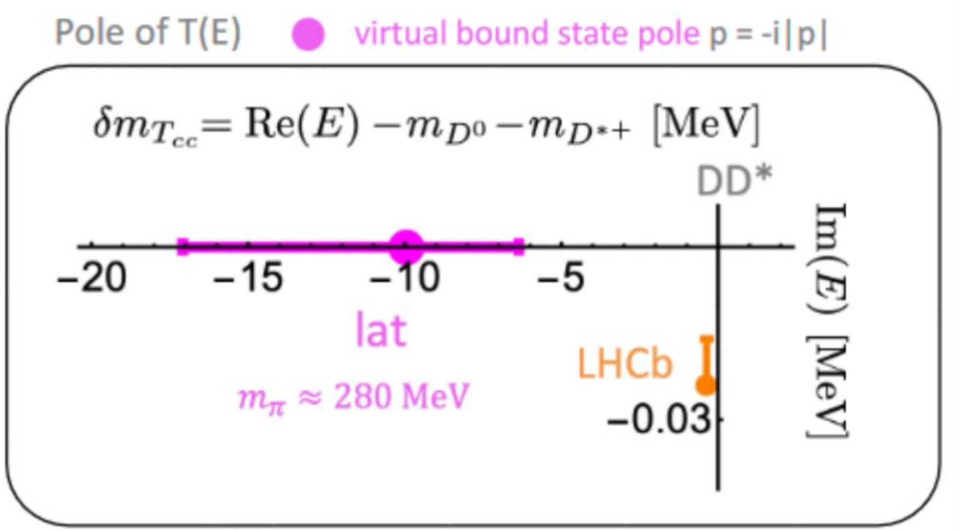
Other recent lattice studies confirm the presence the bound state:

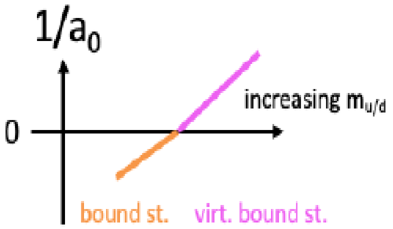
Lyu et al. arXiv.2302.04505
Chen et al. j.physletb.2022.137391

T_{cc} on lattice



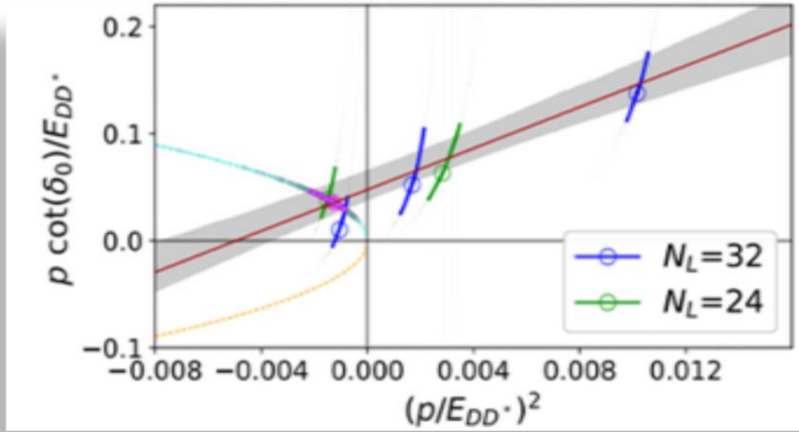
PRL 129, 032002 (2022)
Padmanath and Prelovsek





$$t_l^{(J)} = \frac{E_{\rm cm}}{2}\frac{1}{p\cot\delta_l^{(J)} - ip}, \qquad p^{2l+1}\cot\delta_l^{(J)} = \frac{1}{a_l^{(J)}} + \frac{r_l^{(J)}}{2}p^2,$$

$$p\cot\delta_{l=0}^{(J=1)} = \frac{1}{a_0^{(1)}} + \frac{1}{2}r_0^{(1)}p^2,$$



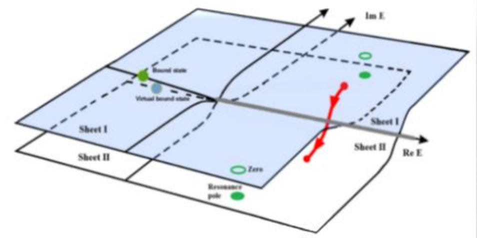
$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	−0.36(4)	bound st.

$$\delta m = E^p_{cm} - E_{th}$$

$$T \propto (p\cot\delta_0 - ip)^{-1}$$

- Bound state: $p = i|p| \rightarrow e^{ipr} = e^{-|p|r}$
- Virtual bound state $p = -i|p| \rightarrow e^{ipr} = e^{|p|r}$ like the spin-singlet dineutron



Slide thanks: Padmanath and Sasa