Select puzzles in hadronic B decays

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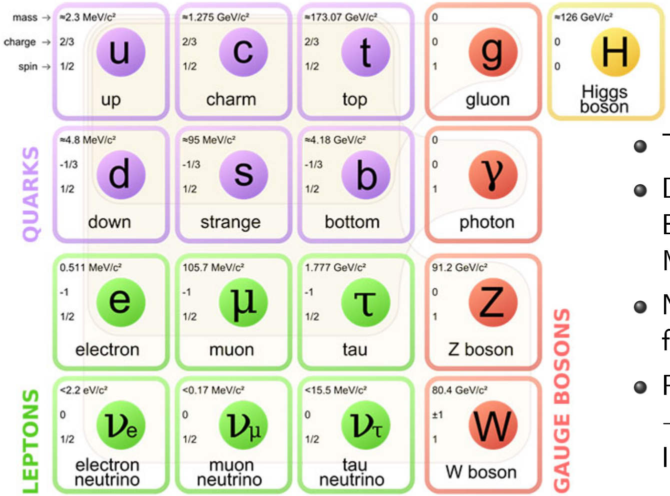
bit.ly/ichepap23in

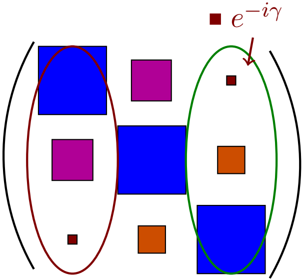
The Standard Model and beyond



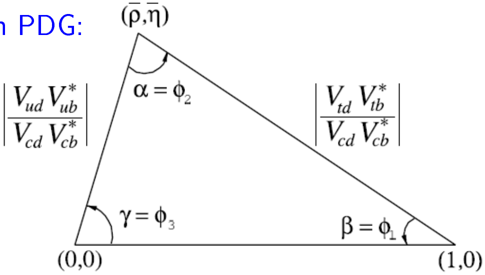
- The Standard Model is incomplete!
- Dark Matter/Dark energy
  Baryon-asymmetry problem
  May require new particles/symmetry
- New physics may be beyond energy frontier reach
- Puzzles/Anomalies
  $\rightarrow$ SM prediction $\neq$ Expt.
  Intensity frontier $\Leftrightarrow$ Energy frontier

CKM Unitarity and the angle γ/ϕ_3

$$V_{\rm CKM} = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{matrix} d & s & b \end{matrix} \\ \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{array}$$

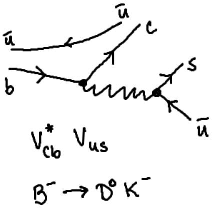


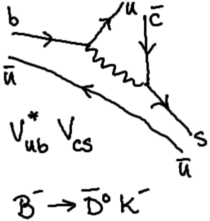
- V_{CKM} is Unitary $\Rightarrow V_{CKM}^\dagger V_{CKM} = 1$
- Empirically close to diagonal
  $\rightarrow$ smaller elements farther from diagonal
- $V_{ij}^* V_{ik} = \delta_{jk} \qquad \sum_i |V_{ij}|^2 = 1$
- $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$
- Taken from PDG:



Direct measurement of γ: methods

- Consider $B^- \to D^0 K^-$ and $B^- \to \overline{D}^0 K^-$ with $D^0 \to f$
- Anti-decays are $B^+ \to \overline{D}^0 K^-$ and $B^+ \to D^0 K^+$; No CP Violation in D decays
- Total number of observables in the B decays: $\Gamma, \overline{\Gamma}$ for each decay $\to$ 4





Only tree-level contributions in the SM

Highly-suppressed Loop (box diagrams)

Brod and Zupan (2013)

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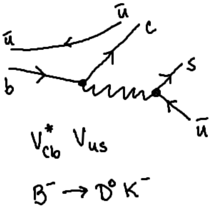
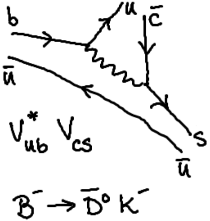




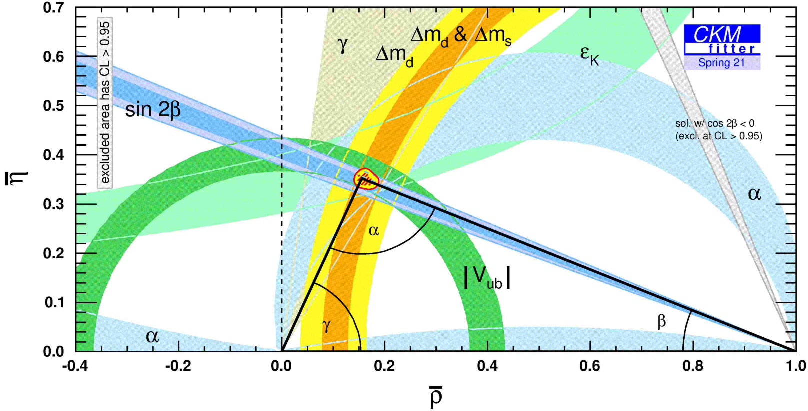
Only tree-level contributions in the SM

Highly-suppressed Loop (box diagrams)

Brod and Zupan (2013)

- GLW method (f_{CP} such as $\pi^+\pi^-$)
- 3 theory parameters: $|r_B|, \delta, \gamma$
- Extract γ from a fit
- ~~Theory calculations of non-p amplitudes?~~
- ADS method (f such as $K^+\pi^-$)
- GGSZ method (f such as $K_S\pi\pi$)
  $\leftarrow$ k bins in 3-body Dalitz analysis
  $\Rightarrow 2k+3$ parameters, $4k$ observables
  $\Rightarrow$ Analysis works for $k \geq 2$

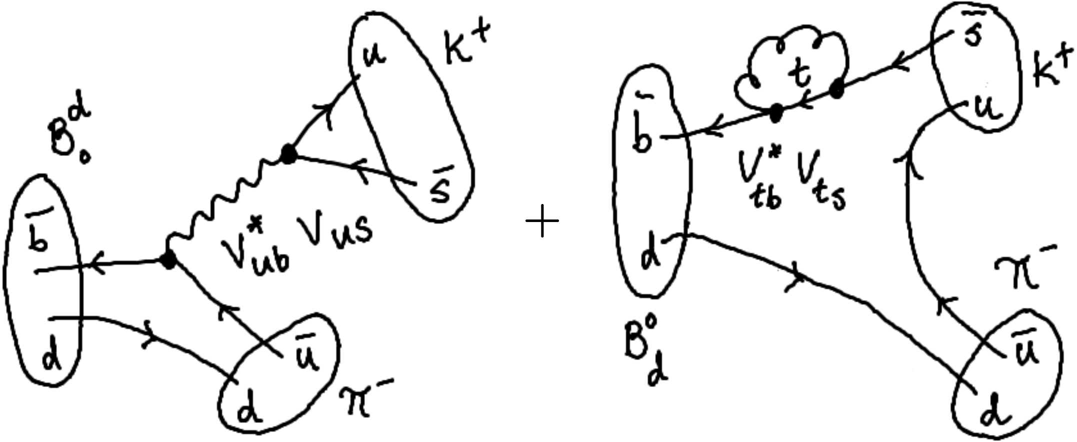
CKM Unitarity: experiments



- Direct γ measurement is statistics limited
- Current $\Delta\gamma \sim 7^\circ$ (LHCb-CONF-2022-003)
- Long term LHCb target $\Delta\gamma \sim 1 - 2^\circ \leftarrow$ discrepancy possible

Alternative methods: decays with tree + loop

- Consider the decay $B_d^0 \to \pi^- K^+$: $\mathcal{A}(B_d^0 \to \pi^- K^+) = -T' e^{i\gamma} + P'_{tc}$



Weak-phase information from B decays with tree + loop

- $\mathcal{A}(B \to f) = |a| + |b|e^{i\phi}e^{i\delta} \quad \to \quad \Gamma \propto |\mathcal{A}|^2$

  $\bar{\mathcal{A}}(\bar{B} \to \bar{f}) = |a| + |b|e^{-i\phi}e^{i\delta} \quad \to \quad \bar{\Gamma} \propto |\bar{\mathcal{A}}|^2$

  – 4 parameters: 2 magnitudes ($|a|, |b|$), 1 rel. strong phase (δ), 1 rel. weak phase (ϕ)
- 2 Observables: $\mathcal{B}_{\rm CP} = \dfrac{\Gamma + \bar{\Gamma}}{2\Gamma_B}$, $C_{\rm CP} = \dfrac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$ (direct CP asymmetry)

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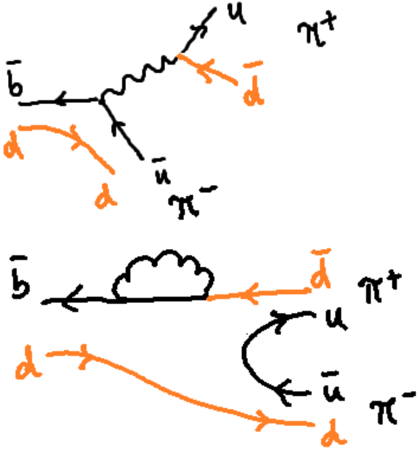
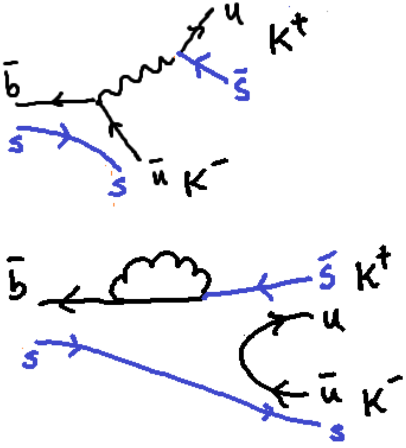
- 2 Observables: $\mathcal{B}_{\rm CP} = \dfrac{\Gamma + \bar{\Gamma}}{2\Gamma_B}$, $\; C_{\rm CP} = \dfrac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$ (direct CP asymmetry)
- For $B^0 \to f$ with $f = \bar{f}$ additional observable $S_{\rm CP}$ (indirect CP asymmetry)

  B-mixing: $|B\rangle_{\rm mass} = p\,|B\rangle + q\,|\bar{B}\rangle$ with $\lambda = \dfrac{q}{p}\dfrac{\bar{\mathcal{A}}}{\mathcal{A}} \Rightarrow S_f = \dfrac{2{\rm Im}[\lambda]}{1 + |\lambda|^2}$
- Information about q/p comes from $B - \bar{B}$ mixing (independent source)
- For B_s, additional observable $A^{\Delta\Gamma} = \dfrac{-2{\rm Re}[\lambda]}{1 + |\lambda|^2}$ (since $\Delta\Gamma_s$ is sizable)
- $C_{\rm CP} = \dfrac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad \Rightarrow$ Identity: $(C_{\rm CP})^2 + (S_{\rm CP})^2 + (A^{\Delta\Gamma})^2 = 1$ (LHCb)

U-spin in hadronic B decays



$B_d^0 \to \pi^+\pi^-$



$B_s^0 \to K^+K^-$

Weak-phase info using U-spin

- R. Fleischer, hep-ph/9903456: Compare $B_s \to K^+K^-$ with $B_d \to \pi^+\pi^-$
- 4 observables: $C_{KK}, S_{KK}, C_{\pi\pi}, S_{\pi\pi}$
- $|q/p| \approx 1$ for $B^0_{d,s}$ (can check from semileptonic B decays);
  $\arg(q_s/p_s) \approx 2\beta_s$ $\to$ from $B_s \to J/\Psi\phi$
- Hadronic parameters same for both decays: $(|b/a|, \delta) \leftarrow$ 2 parameters
- Weak decay parameters: $\gamma, \beta_d \leftarrow$ Up to 2 parameters
- $C_{\pi\pi}, C_{KK}, S_{KK}$ sufficient to determine γ + 2 hadronic parameters
- Use $S_{\pi\pi}$ to also get β_d
- Data unavailable at the time

> The strategies proposed in this paper are very interesting for "second-generation" B-physics experiments performed at hadron machines, for example LHCb, where the very

3-body B Decays: Fully-antisymmetric state under $SU(3)_F$

- Work completed with LTU student Andrea Houck: 2308.16240 (*PRD Accepted*)
  - ✓ Find flavor-SU(3) representations of $\langle B| H |PPP\rangle_{\rm FA}$

    $B \to (P_1P_2P_3)_{\rm FA}$ with $|P_1P_2P_3\rangle = -|P_2P_1P_3\rangle$.

Decay Amplitude	$V^*_{cb}V_{cs}$ $B_1^{(FA)}$	$B^{(FA)}$	$V^*_{ub}V_{us}$ $A_1^{(FA)}$	$A^{(FA)}$	$R_8^{(FA)}$	$R_{10}^{(FA)}$	$P_8^{(FA)}$	$P_{10^*}^{(FA)}$	$P_{27}^{(FA)}$
$A(B^+\to K^+\pi^+\pi^-)$	0	$\frac{1}{\sqrt5}$	0	$\frac{1}{\sqrt5}$	$\frac{1}{\sqrt{15}}$	$-\frac{1}{\sqrt3}$	$-\frac{3}{5}$	0	$\frac{3\sqrt6}{5}$
$A(B^+\to K^0\pi^+\pi^0)$	0	$\sqrt{\frac{2}{5}}$	0	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{15}}$	$\frac{1}{\sqrt6}$	$-\frac{3\sqrt2}{5}$	0	$\frac{\sqrt3}{5}$
$A(B^0\to K^0\pi^+\pi^-)$	0	$-\frac{1}{\sqrt5}$	0	$-\frac{1}{\sqrt5}$	$\frac{1}{\sqrt{15}}$	$-\frac{1}{\sqrt3}$	$-\frac{1}{5}$	0	$\frac{\sqrt6}{5}$
$A(B^0\to K^+\pi^0\pi^-)$	0	$\sqrt{\frac{2}{5}}$	0	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{15}}$	$-\frac{1}{\sqrt6}$	$\frac{\sqrt2}{5}$	0	$\frac{3\sqrt3}{5}$
$A(B^+\to K^+K^0\bar{K}^0)$	0	$-\frac{1}{\sqrt5}$	0	$-\frac{1}{\sqrt5}$	$-\frac{1}{\sqrt{15}}$	0	$\frac{3}{5}$	0	$\frac{2\sqrt6}{5}$
$A(B^0\to K^0K^+K^-)$	0	$-\frac{1}{\sqrt5}$	0	$-\frac{1}{\sqrt5}$	$\frac{1}{\sqrt{15}}$	0	$-\frac{1}{5}$	$\sqrt2$	$\frac{\sqrt6}{5}$
$\sqrt2 A(B_s^0\to \pi^0K^+K^-)$	$-\frac{1}{2\sqrt6}$	0	$-\frac{1}{2\sqrt6}$	0	$\frac{2}{\sqrt{15}}$	$\frac{1}{2\sqrt3}$	$\frac{4}{5}$	$-\frac{1}{\sqrt2}$	$-\frac{\sqrt{\frac{3}{2}}}{10}$
$\sqrt2 A(B_s^0\to \pi^0K^0\bar{K}^0)$	$-\frac{1}{2\sqrt6}$	0	$-\frac{1}{2\sqrt6}$	0	$-\frac{2}{\sqrt{15}}$	$-\frac{1}{2\sqrt3}$	$-\frac{4}{5}$	$\frac{1}{\sqrt2}$	$-\frac{9\sqrt{\frac{3}{2}}}{10}$
$A(B_s^0\to \pi^-K^+\bar{K}^0)$	$\frac{1}{2\sqrt6}$	0	$\frac{1}{2\sqrt6}$	0	0	$-\frac{1}{2\sqrt3}$	0	$-\frac{1}{\sqrt2}$	$\frac{\sqrt{\frac{3}{2}}}{2}$
$A(B_s^0\to \pi^+K^-K^0)$	$-\frac{1}{2\sqrt6}$	0	$-\frac{1}{2\sqrt6}$	0	0	$-\frac{1}{2\sqrt3}$	0	$-\frac{1}{\sqrt2}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
$\sqrt2 A(B_s^0\to \pi^0\pi^+\pi^-)$	$-\frac{1}{\sqrt6}$	$\frac{2}{\sqrt5}$	$-\frac{1}{\sqrt6}$	$\frac{2}{\sqrt5}$	0	0	$\frac{6}{5}$	0	$\frac{3\sqrt{\frac{3}{2}}}{5}$

→ $|PPP\rangle_{\rm FA} \equiv (\mathbf{8}\times\mathbf{8}\times\mathbf{8})_{\rm FA}$
$= \mathbf{27}_{\rm FA} + \mathbf{10}_{\rm FA} + \mathbf{10^*}_{\rm FA} + \mathbf{8}_{\rm FA} + \mathbf{1}$

→ $H \to \bar{\mathbf{3}}\times\mathbf{3}\times\bar{\mathbf{3}}$ has $\bar{\mathbf{3}}, \mathbf{6}, \overline{\mathbf{15}}$

→ Find reduced set of SU(3) amplitudes

→ Establish γ extraction method

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$A(B^+ \to K^0\pi^+\pi^0)$	0	$\sqrt{\frac{2}{5}}$	0	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{15}}$	$\frac{1}{\sqrt{6}}$	$-\frac{3\sqrt{2}}{5}$	0	$\frac{\sqrt{3}}{5}$
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$A(B^+ \to K^+K^0\bar{K}^0)$	0	$-\frac{1}{\sqrt{5}}$	0	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{\sqrt{15}}$	0	$\frac{3}{5}$	0	$\frac{2\sqrt{6}}{5}$
$A(B^0 \to K^0K^+K^-)$	0	$-\frac{1}{\sqrt{5}}$	0	$-\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{15}}$	0	$-\frac{1}{5}$	$\sqrt{2}$	$\frac{\sqrt{6}}{5}$
$\sqrt{2}A(B_s^0 \to \pi^0K^+K^-)$	$-\frac{1}{2\sqrt{6}}$	0	$-\frac{1}{2\sqrt{6}}$	0	$\frac{2}{\sqrt{15}}$	$\frac{1}{2\sqrt{3}}$	$\frac{4}{5}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{\frac{3}{2}}}{10}$
$\sqrt{2}A(B_s^0 \to \pi^0K^0\bar{K}^0)$	$-\frac{1}{2\sqrt{6}}$	0	$-\frac{1}{2\sqrt{6}}$	0	$-\frac{2}{\sqrt{15}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$-\frac{9\sqrt{\frac{3}{2}}}{10}$
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$A(B_s^0 \to \pi^+K^-K^0)$	$-\frac{1}{2\sqrt{6}}$	0	$-\frac{1}{2\sqrt{6}}$	0	0	$-\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{\frac{3}{2}}}{2}$
$\sqrt{2}A(B_s^0 \to \pi^0\pi^+\pi^-)$	$-\frac{1}{\sqrt{6}}$	$\frac{2}{\sqrt{5}}$	$-\frac{1}{\sqrt{6}}$	$\frac{2}{\sqrt{5}}$	0	0	$\frac{6}{5}$	0	$\frac{3\sqrt{\frac{3}{2}}}{5}$

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→ Find reduced set of SU(3) amplitudes

→ Establish γ extraction method

→ $\gamma_{\rm FS} \neq \gamma_{\rm FA} \Rightarrow B \to K\pi\pi$ puzzle?

→ 5 decays have 12 observables

→ 5 decays depend on 11 hadronic parameters

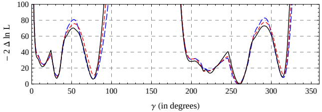
and γ

3-body B Decays: Fully-symmetric state under $SU(3)_F$

A result

γ from three-body decays

- 3-body final state under SU(3) : $B \to K\pi\pi, K\overline{K}K$
  - $\rightarrow$ 6 final state symmetries : permutations of 3 particles
- Fully-symmetric state (Rey-Le Lorier, London, 1109.0881)
  - $\rightarrow$ More observables than unknowns $\Rightarrow$ γ can be extracted
  - $\rightarrow$ BB, Imbeault, London, 1303.0846



$\rightarrow$ SM-like : 77°

$\rightarrow$ 32°, 259°, 315°

$K\pi\pi - K\overline{K}K$ puzzle ?

David London's talk in this session!

- Group theory analysis : I-spin, U-spin, SU(3) relations
  - $\rightarrow$ BB, Gronau, Imbeault, London, Rosner, 1402.2909

$$
\begin{aligned}
2A(B^0 \to K^+\pi^0\pi^-)_{\rm fs} &= be^{i\gamma} - \kappa c \,, \\
\sqrt{2}A(B^0 \to K^0\pi^+\pi^-)_{\rm fs} &= -de^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa d \,, \\
\sqrt{2}A(B^+ \to K^+\pi^+\pi^-)_{\rm fs} &= -ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b \,, \\
\sqrt{2}A(B^0 \to K^+K^0K^-)_{\rm fs} &= \alpha_{SU(3)}(-ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b) \\
A(B^0 \to K^0K^0\bar{K}^0)_{\rm fs} &= \alpha_{SU(3)}(\tilde{P}'_{uc}e^{i\gamma} + a) \,,
\end{aligned}
$$

- $SU(3)_F$: ignore m_u, m_d, m_s
- $\equiv$ diagrams 1402.2909
- BB+ data fit, 1303.0846
- Updated: Bertholet et al., 1812.06194
- N Dalitz points
  $\Rightarrow$ 8N hadronic parameters + γ
- 11N observables
  $\Rightarrow$ γ can be extracted

A puzzle in $B \to \pi K$ decays

* Amplitudes: $\mathcal{A} \;=\; A_1 + A_2\, e^{i\phi}\, e^{i\delta}$ and $\bar{\mathcal{A}} \;=\; A_1 + A_2\, e^{-i\phi}\, e^{i\delta}$

  $\Rightarrow$ CP Asymmetry: $A_{\rm CP} \;=\; \dfrac{|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2} \propto \sin(\phi)\sin(\delta)$

* Consider processes:

  $B^+ \to \pi^0 K^+ \qquad \mathcal{A}^{0+} \;=\; -T'\, e^{i\gamma} + P'_{tc} - P'_{EW} \qquad (P'_{EW} \propto T')$

  $B_d^0 \to \pi^- K^+ \qquad \mathcal{A}^{-+} \;=\; -T'\, e^{i\gamma} + P'_{tc}$

  $\Rightarrow$ $\boxed{A_{\rm CP}(B^+ \to \pi^0 K^+) \;=\; A_{\rm CP}(B_d^0 \to \pi^- K^+)}$ in *Theory*!

* Experiment:

  $A_{\rm CP}^{0+} = 0.025 \pm 0.016$ 2012.12789

  $A_{\rm CP}^{-+} = -0.084 \pm 0.004$ 1805.06759 $\sim 6.5\sigma$ discrepancy!

The complete $B \to \pi K$ puzzle

Decay	BR	$A_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+ K^0$	✓	✓	
$B^+ \to \pi^0 K^+$	✓	✓	
$B_d^0 \to \pi^- K^+$	✓	✓	
$B_d^0 \to \pi^0 K^0$	✓	✓	✓

- 4 $B \to K\pi$ processes with 9 observables
- Hadronic parameters + γ fits prefer

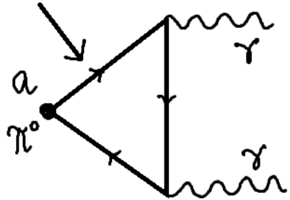
  A) Large C/T or B) $\gamma \neq \gamma_{\rm tree}$
- Expected $C/T \sim 0.2 - 0.3$

$$
\begin{aligned}
A^{+0} &= -P'_{tc} + P'_{uc} e^{i\gamma} - \frac{1}{3} P'^{C}_{EW} , \\
\sqrt{2} A^{0+} &= -T' e^{i\gamma} - C' e^{i\gamma} + P'_{tc} - P'_{uc} e^{i\gamma} \\
&\quad - P'_{EW} - \frac{2}{3} P'^{C}_{EW} , \\
A^{-+} &= -T' e^{i\gamma} + P'_{tc} - P'_{uc} e^{i\gamma} - \frac{2}{3} P'^{C}_{EW} , \\
\sqrt{2} A^{00} &= -C' e^{i\gamma} - P'_{tc} + P'_{uc} e^{i\gamma} \\
&\quad - P'_{EW} - \frac{1}{3} P'^{C}_{EW} .
\end{aligned}
$$

The $B \to \pi K$ puzzle: A solution!

* Consider an *Axionlike Particle (ALP)* – 2104.03947

$$\mathcal{L} \supset -i \sum_{f=u,d,l} \eta_f \frac{m_f}{f_a} \bar{f} \gamma_5 f a + \frac{1}{4} \kappa a F^{\mu\nu} \tilde{F}_{\mu\nu}$$



- → $m_a \simeq m_{\pi^0}$ and ALP promptly decays to $\gamma\gamma$
- → Mixes with the π^0: $|a\rangle = \sin\theta \, |\pi^0\rangle_{\rm phys} + \cos\theta \, |a\rangle_{\rm phys}$
- → $B \to \pi^0 K$ processes get new contribution: $\mathcal{A} = |\mathcal{A}| e^{i\pi/2}$
  $\sqrt{2}\mathcal{A}^{0+} = \ldots + \mathcal{A}$; $\sqrt{2}\mathcal{A}^{00} = \ldots + \mathcal{A}$
- → Processes not involving a π^0 unaltered
  $\mathcal{A}^{+0}$ and $\mathcal{A}^{-+}$ stay the same
- → Leads to a good fit with $|\mathcal{A}| \sim P'_{EW}$
- → Constraint from $B \to Ka$ ($B \to K$ + invis): $\mathcal{B} \sim 10^{-5} \Rightarrow \sin\theta \sim$ 0.1 – 0.2

* Work in progress: How to detect an ALP with mass close to m_{π^0} in other flavor processes.

The U-spin puzzle

- LHCb measurement of CP Asymmetries in $B_{s(d)} \to K^+K^-(\pi^+\pi^-)$: 1805.06759, 2012.05319
- Theory investigation of U-spin in $KK, \pi\pi$: Nir, Savoray, and Viernik, 2201.03573
- Other U-spin related decays: BB with others, 2211.06994
- 6 decays possible: 3 decays each $\Delta S = 0(b \to d), 1(b \to s)$; 4 U-spin RMEs

Decay	Representation	$\mathcal{B}_{\text{CP}}$	C_{CP}	S_{CP}
$B_d^0 \to \pi^+\pi^-$	$M_{1d}^{1/2} + M_{0d}^{1/2}$	$\sim 10^{-6}$	✓	✓
$B_d^0 \to K^+K^-$	$M_{1d}^{1/2} - M_{0d}^{1/2}$	$\sim 10^{-8}$	?	?
$B_s^0 \to \pi^+K^-$	$2\ M_{1d}^{1/2}$	$\sim 10^{-6}$	✓	
$B_s^0 \to K^+K^-$	$M_{1s}^{1/2} + M_{0s}^{1/2}$	$\sim 10^{-5}$	✓	✓
$B_s^0 \to \pi^+\pi^-$	$M_{1s}^{1/2} - M_{0s}^{1/2}$	$\sim 10^{-7}$	?	?
$B_d^0 \to K^+\pi^-$	$2\ M_{1s}^{1/2}$	$\sim 10^{-5}$	✓	

- Each $M_{xq}^{1/2}$ has two parts
- $M_{xq}^{1/2} = V_{ub}^* V_{uq} T_q^x + V_{cb}^* V_{cq} P_q^x$
- 12 measurements, 7 parameters
- Allow 5 breaking parameters
- Large SU(3) breaking found
- Need $\dfrac{T_s^0}{T_d^0} - 1 \sim \mathcal{O}(100\%)$

$B \to PP$ data by transition

- $\Delta S = 0$: $\bar{b} \to \bar{d}$ transitions
- 15 measurements available
- 7 RMEs $\to$ 13 hadronic parameters
- $\chi^2_{\rm min}/{\rm dof} = 0.35/2$; $p \sim 0.8$ good fit

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to K^+ \overline{K}^0$	✓	✓	
$B^+ \to \pi^+ \pi^0$	✓	✓	
$B^0 \to K^0 \overline{K}^0$	✓	✓	✓
$B^0 \to \pi^+ \pi^-$	✓	✓	✓
$B^0 \to \pi^0 \pi^0$	✓	✓	?
$B^0 \to K^+ K^-$	✓	?	?
$B^0_s \to \pi^+ K^-$	✓	✓	
$B^0_s \to \pi^0 \overline{K}^0$	?	?	?

- $\Delta S = 1$: $\bar{b} \to \bar{s}$ transitions
- 15 measurements available
- 7 RMEs $\to$ 13 hadronic parameters
- $\chi^2_{\rm min}/{\rm dof} = 1.7/2$; $p \sim 0.4$ good fit

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+ K^0$	✓	✓	
$B^+ \to \pi^0 K^+$	✓	✓	
$B^0 \to \pi^- K^+$	✓	✓	
$B^0 \to \pi^0 K^0$	✓	✓	✓
$B^0_s \to K^+ K^-$	✓	✓	✓
$B^0_s \to K^0 \overline{K}^0$	✓	?	?
$B^0_s \to \pi^+ \pi^-$	✓	?	?
$B^0_s \to \pi^0 \pi^0$	✓	?	?

$B \to PP$ data: Separate fits

- Both $b \to d$ ($\Delta S = 0$) and $b \to s$ ($\Delta S = 1$) fits are good
- $C/T \sim 0.2 - 0.3$ is expected
- $\Delta S = 0 : C/T \sim 1.7 \pm 0.3$; $\Delta S = 1 : C/T \sim 0.9 \pm 0.4$
- C/T differs largely from standard QCD expectations
- Comparison of parameters ($'$ indicates $b \to s$)

$\|T'/T\|$	$\|C'/C\|$	$\|P'_{uc}/P_{uc}\|$	$\|A'/A\|$	$\|PA'_{uc}/PA_{uc}\|$	$\|P'_{tc}/P_{tc}\|$	$\|PA'_{tc}/PA_{tc}\|$
12.3 ± 3.6	6.4 ± 2.2	16 ± 22	14 ± 13	10 ± 13	0.95 ± 0.52	1.2 ± 2.4

- Ratios expected to be 1 in SU(3)$_F$ $\to$ large breaking observed in T and C
- SU(3)$_F$ breaking is much larger compared to f_K/f_π or $m_s/\Lambda_{\rm QCD}$

$B \to PP$ data: Combined fit and anomaly

- BB with others in arxiv:2311.18011: fit the entire set of $B \to PP$ data
- SU(3) hypothesis: 30 observables, 13 parameters: fit gives $\chi^2_{\rm min}/{\rm dof} \sim 43/17$
- 3.5σ deviation from the SM flavor-SU(3) hypothesis
- Fit with QCDf-inspired constraint $|C/T| = 0.2$
  - $\to$ $\Delta S = 1$ fit: $\chi^2_{\rm min}/{\rm dof} \sim 6.3/3$, $p \sim 0.1$
  - $\to$ $\Delta S = 0$ fit: $\chi^2_{\rm min}/{\rm dof} \sim 18.8/3$, $p \sim 3 \times 10^{-4}$ or 3.6σ away from SM SU(3)$_F$
  - $\to$ Combined fit: $\chi^2_{\rm min}/{\rm dof} \sim 55.5/18$, $p \sim 10^{-5}$ or 4.4σ away from SM SU(3)$_F$
- Both fits find deviations in $B^0_s \to K^+K^-$ observables
- Deviations also in $B^+ \to \pi^0 K^+, B^0 \to \pi^- K^+, \pi^0 K^0, K^0 \overline{K}^0$

Summary

- Sign of anomalies in hadronic B decays
- Large U-spin breaking needed to explain U-spin related $B^0_{(s)} \to DD$ (D = Doublet)
- Puzzle appears also in SU(3)$_F$ related $B \to PP$ (P = pseudoscalar)
- Puzzles appear to involve $B^0_s \to K^+K^-$
- U-spin puzzle needs unusually large T^0_s/T^0_d
- Emerging cracks in the fabric of flavor symmetries
- Lack of QCD understanding or hint for new physics in $b \to s$?
- Lots of data from LHCb, Belle II, and other experiments in the next decade
- The future is bright!

Thanks!

- UG students: A. Jean, N. Payot (UdeM), A. Houck (LTU)
- Grad students: J. Waite (UMiss)
- Postdocs: S. Kumbhakar (UdeM)
- Faculty: D. London (UdeM), A. Datta (UMiss)
- Support: LTU,
  US National Science Foundation (PHY-2013984)

Back-up Slides

U-spin: LHCb measurement and theory progress

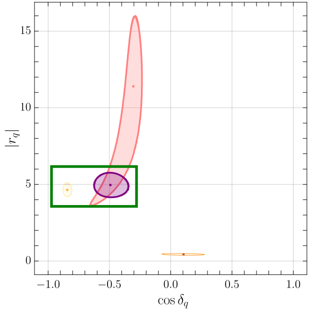
- LHCb measurement of CP Asymmetries in $B_{s(d)} \to K^+K^-(\pi^+\pi^-)$: 1805.06759, 2012.05319
- Theory investigation of U-spin: Nir, Savoray, and Viernik, 2201.03573
- $C_{KK} = 0.172 \pm 0.031$, $S_{KK} = 0.139 \pm 0.032$, $C_{\pi\pi} = -0.32 \pm 0.04$, $S_{\pi\pi} = -0.64 \pm 0.04$
- Use β_d $(B_d \to J/\Psi K_s)$, β_s $(B_s \to J/\Psi \phi)$

  γ $(B \to DK)$
- Find hadronic parameters for both decays

  $\to$ test U-spin
- $\dfrac{|b_s/a_s|}{|b_d/a_d|} = 1.07$, $|a_s/a_d| = 1.26$

  $\to$ (0 - 30%) U-spin breaking

  $(\mathcal{O}(m_s/\Lambda_{\rm QCD}) \sim 30\%, f_K/f_\pi - 1 \sim 20\%)$
- Result: NP + different orders of breaking at play



3-body B Decays: Fully-antisymmetric state under $\mathrm{SU}(3)_F$

- $|P_1 P_2 P_3\rangle \to \mathbf{8} \times \mathbf{8} \times \mathbf{8} = 512$

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- $H = \mathcal{O}(\bar{b} \to \bar{c}c\bar{s}) + \mathcal{O}(\bar{b} \to \bar{u}u\bar{s})$: $V_{cb}^* V_{cs}\ \mathbf{3}^* + V_{ub}^* V_{us} (\mathbf{3}^* \times \mathbf{3} \times \mathbf{3}^*)$

$$= V_{cb}^* V_{cs}\ \mathbf{3}^* + V_{ub}^* V_{us} (\mathbf{3}^* + \mathbf{6} + \mathbf{15}^*)$$

- $H\,|B\rangle$: $(\mathbf{3}^* + \mathbf{6} + \mathbf{15}^*) \times \mathbf{3} \to (\mathbf{1} + \mathbf{8}) + (\mathbf{8} + \mathbf{10}) + (\mathbf{8} + \mathbf{10}^* + \mathbf{27})$

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- Decay amplitude $= \langle B|\,H\,|PPP\rangle_{\mathrm{FA}} = \sum_{i=1}^{7} C_i^u \langle \mathbf{3}|\,\mathbf{3}^*, \mathbf{6}, \mathbf{15}^*\,|56\rangle_i\ \ V_{ub}^* V_{us}$

$$+ \sum_{i=1}^{2} C_i^c \langle \mathbf{3}|\,\mathbf{3}^*\,|56\rangle_i\ \ V_{cb}^* V_{cs}$$

Constructing the fully-antisymmetric state

- $|[P_1P_2P_3]\rangle_{\rm FA} = (|[P_1P_2]P_3\rangle + |[P_2P_3]P_1\rangle + |[P_3P_1]P_2\rangle)/\sqrt{3}$
- $|[P_1P_2]P_3\rangle = (|P_1P_2P_3\rangle - |P_2P_1P_3\rangle)/\sqrt{2} = -|[P_2P_1]P_3\rangle$

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- Example state: $|K^+\rangle = \left|\mathbf{8}, 1, \tfrac{1}{2}, \tfrac{1}{2}\right\rangle, |\pi^+\rangle = |\mathbf{8}, 0, 1, 1\rangle, |\pi^-\rangle = |\mathbf{8}, 0, 1, -1\rangle$

$$|K^+\pi^+\pi^-\rangle_{\rm FA} = \frac{1}{\sqrt{6}}\left|\mathbf{27}, 1, \tfrac{3}{2}, \tfrac{1}{2}\right\rangle + \frac{\sqrt{2}}{\sqrt{15}}\left|\mathbf{27}, 1, \tfrac{1}{2}, \tfrac{1}{2}\right\rangle + \frac{1}{\sqrt{2}}\left|\mathbf{10}, 1, \tfrac{3}{2}, \tfrac{1}{2}\right\rangle + \frac{1}{\sqrt{5}}\left|\mathbf{8}, 1, \tfrac{1}{2}, \tfrac{1}{2}\right\rangle$$

SU(2) Clebsch-Gordan coefficient $\times$ SU(3) isoscalar factor

$SU(3)_F$ amplitudes and diagrams: equivalence

- $\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FA} = \frac{V_{cb}^* V_{cs}}{\sqrt{5}} B^{fa} + V_{ub}^* V_{us}\left[\frac{A^{fa}}{\sqrt{5}} + \frac{1}{\sqrt{5}}(R_8^{fa} + \sqrt{5}R_{10}^{fa}) - \frac{3}{5}(P_8^{fa} - 3\sqrt{6}P_{27}^{fa})\right]$
$= \tilde{P}'_{ct} + \tilde{P}'_{ut} - C'_1 + T'_2 - A'$

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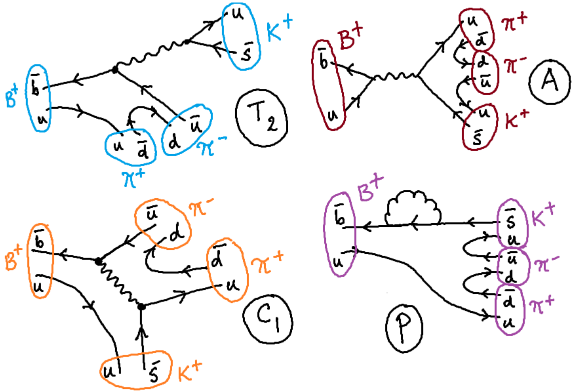

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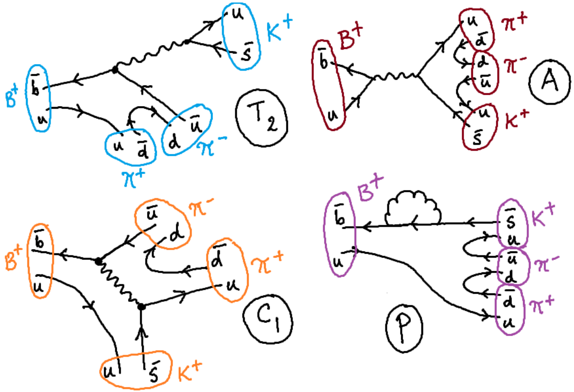

- $V_{cb}^* V_{cs} B_1^{fa} = 2\sqrt{6}(\tilde{P}'_{ct} - PA'_{ct})$
- $V_{cb}^* V_{cs} B^{fa} = \sqrt{5}\tilde{P}'_{ct}$
- $V_{ub}^* V_{us} R_{10}^{fa} = \frac{\sqrt{3}}{2}(T'_1 + T'_2 - C'_1 + C'_2)$
- $V_{ub}^* V_{us} P_{10^*}^{fa} = -\frac{1}{2\sqrt{2}}(T'_1 + T'_2 + C'_1 - C'_2)$
- $V_{ub}^* V_{us} P_{27}^{fa} = -\frac{1}{2\sqrt{6}}(T'_1 - T'_2 + C'_1 + C'_2)$
- Similar relations for

$$A_1^{fa}, A^{fa}, R_8^{fa}, P_8^{fa}$$

Observables for the FA states

- Construct a fully-antisymmetric amplitude under momentum interchanges

$$\mathcal{M}_{\rm FA}(s_{12}, s_{13}) = \frac{1}{\sqrt{6}}\left[\mathcal{M}(s_{12}, s_{13}) - \mathcal{M}(s_{13}, s_{12}) - \mathcal{M}(s_{12}, s_{23}) + \mathcal{M}(s_{23}, s_{12}) - \mathcal{M}(s_{23}, s_{13}) + \mathcal{M}(s_{13}, s_{23})\right]$$

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- Construct the following Dalitz plot observables

$\mathcal{X}(s_{12},s_{13}) = |\mathcal{M}_{\rm FA}(s_{12},s_{13})|^2 + |\overline{\mathcal{M}}_{\rm FA}(s_{12},s_{13})|^2 \rightarrow$ available for all decays

$\mathcal{Y}(s_{12},s_{13}) = |\mathcal{M}_{\rm FA}(s_{12},s_{13})|^2 - |\overline{\mathcal{M}}_{\rm FA}(s_{12},s_{13})|^2 \rightarrow$ available *in principle* for all decays

$\mathcal{Z}(s_{12},s_{13}) = {\rm Im}\left[\mathcal{M}^*_{\rm FA}(s_{12},s_{13})\overline{\mathcal{M}}_{\rm FA}(s_{12},s_{13})\right] \rightarrow$ only available for *flavor neutral* final states

Example final states for $\mathcal{Z}$: $K_S K^+ K^-, \quad K_S \pi^+ \pi^-$

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- For $N = 8$: 36 symmetric + 28 antisymmetric

  In SU(3): $\mathbf{8} \times \mathbf{8} = (\mathbf{27} + \mathbf{8} + \mathbf{1})_{\rm sym} + (\mathbf{10} + \mathbf{10}^* + \mathbf{8})_{\rm antisym}$

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- FS/FA more complicated – check symmetry by explicit construction

  FS $\supset (\mathbf{27} + \mathbf{8} + \mathbf{1}) \times \mathbf{8} \to \mathbf{64} + \mathbf{27} + \mathbf{10} + \mathbf{10}^* + \mathbf{8} + \mathbf{1}$

  FA $\supset (\mathbf{10} + \mathbf{10}^* + \mathbf{8}) \times \mathbf{8} \to \mathbf{27} + \mathbf{10} + \mathbf{10}^* + \mathbf{8} + \mathbf{1}$

5 decays depend on 6 RMEs

Decay	$V_{cb}^*V_{cs}$		$V_{ub}^*V_{us}$						
Amplitude	$B_1^{(fa)}$	$B^{(fa)}$	$A_1^{(fa)}$	$A^{(fa)}$	$R_8^{(fa)}$	$R_{10}^{(fa)}$	$P_8^{(fa)}$	$P_{10^*}^{(fa)}$	$P_{27}^{(fa)}$
$\sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FA}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$-\frac{3\sqrt{2}}{5}$	0	$\frac{6\sqrt{3}}{5}$
$\mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$-\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{1}{\sqrt{6}}$	$\frac{3\sqrt{2}}{5}$	0	$-\frac{\sqrt{3}}{5}$
$\sqrt{2}\mathcal{A}(B^0 \to K^0\pi^+\pi^-)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$-\frac{\sqrt{2}}{5}$	0	$\frac{2\sqrt{3}}{5}$
$\mathcal{A}(B^0 \to K^+\pi^0\pi^-)_{\rm FA}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	$-\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{1}{\sqrt{6}}$	$\frac{\sqrt{2}}{5}$	0	$\frac{3\sqrt{3}}{5}$
$\sqrt{2}\mathcal{A}(B^0 \to K^0K^+K^-)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	0	$-\frac{\sqrt{2}}{5}$	2	$\frac{2\sqrt{3}}{5}$

- 4 reduced matrix elements:
- $B^{fa}, R_{10}^{fa}, P_{10^*}^{fa}, P_{27}^{fa}$

5 decays depend on 6 RMEs

Decay	$V_{cb}^* V_{cs}$		$V_{ub}^* V_{us}$						
Amplitude	$B_1^{(fa)}$	$B^{(fa)}$	$A_1^{(fa)}$	$A^{(fa)}$	$R_8^{(fa)}$	$R_{10}^{(fa)}$	$P_8^{(fa)}$	$P_{10*}^{(fa)}$	$P_{27}^{(fa)}$
$\sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FA}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$-\frac{3\sqrt{2}}{5}$	0	$\frac{6\sqrt{3}}{5}$
$\mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$-\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{1}{\sqrt{6}}$	$\frac{3\sqrt{2}}{5}$	0	$-\frac{\sqrt{3}}{5}$
$\sqrt{2}\mathcal{A}(B^0 \to K^0\pi^+\pi^-)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$-\frac{\sqrt{2}}{5}$	0	$\frac{2\sqrt{3}}{5}$
$\mathcal{A}(B^0 \to K^+\pi^0\pi^-)_{\rm FA}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	$-\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{1}{\sqrt{6}}$	$\frac{\sqrt{2}}{5}$	0	$\frac{3\sqrt{3}}{5}$
$\sqrt{2}\mathcal{A}(B^0 \to K^0K^+K^-)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	0	$-\frac{\sqrt{2}}{5}$	2	$\frac{2\sqrt{3}}{5}$

- 4 reduced matrix elements:
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$\mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$-\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{1}{\sqrt{6}}$	$\frac{3\sqrt{2}}{5}$	0	$-\frac{\sqrt{3}}{5}$
$\sqrt{2}\mathcal{A}(B^0 \to K^0\pi^+\pi^-)_{\rm FA}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	0	$-\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	$\frac{\sqrt{2}}{\sqrt{3}}$	$-\frac{\sqrt{2}}{5}$	0	$\frac{2\sqrt{3}}{5}$
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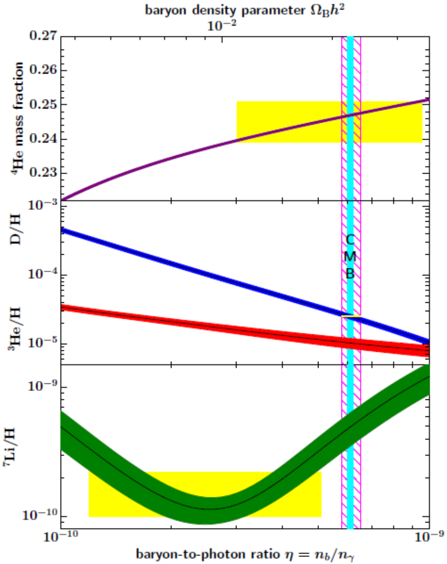
- Remaining RMEs combine
- 6 total combinations of RMEs

- 2 combinations of 3 remaining RMEs:
- $\frac{\sqrt{2}}{\sqrt{5}} A^{fa} + \frac{\sqrt{2}}{\sqrt{15}} R_8^{fa} - \frac{3\sqrt{2}}{5} P_8^{fa}$
- $\frac{\sqrt{2}}{\sqrt{5}} A^{fa} - \frac{\sqrt{2}}{\sqrt{15}} R_8^{fa} + \frac{\sqrt{2}}{5} P_8^{fa}$

A grand puzzle



- Figure taken from the PDG review on BBN
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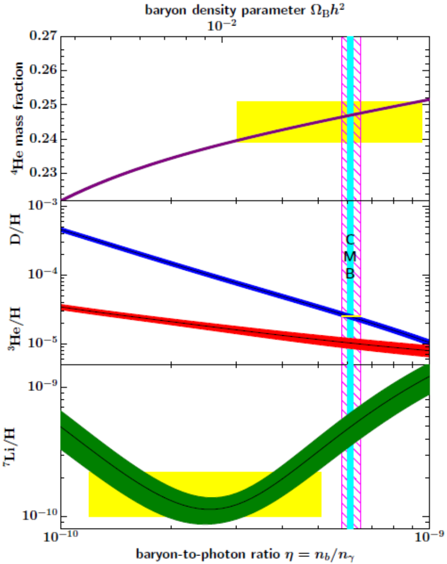

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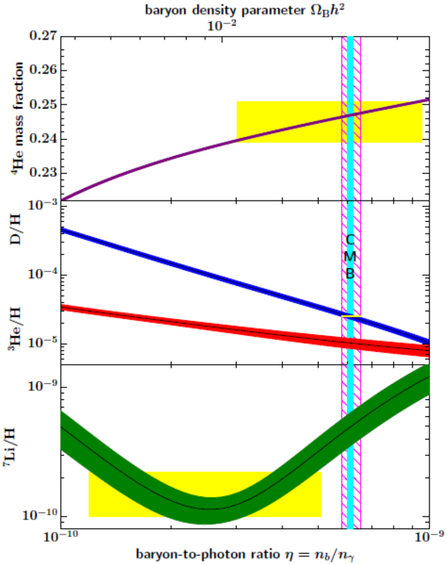

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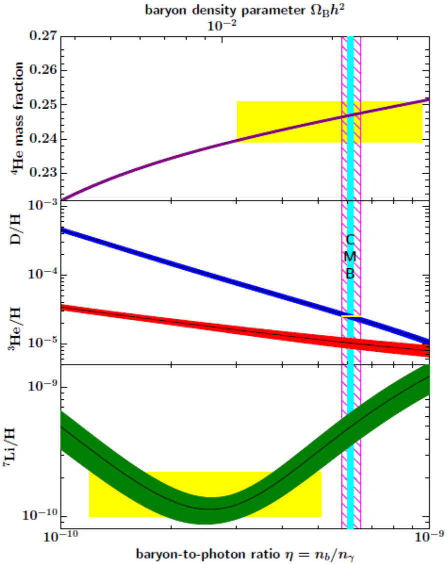

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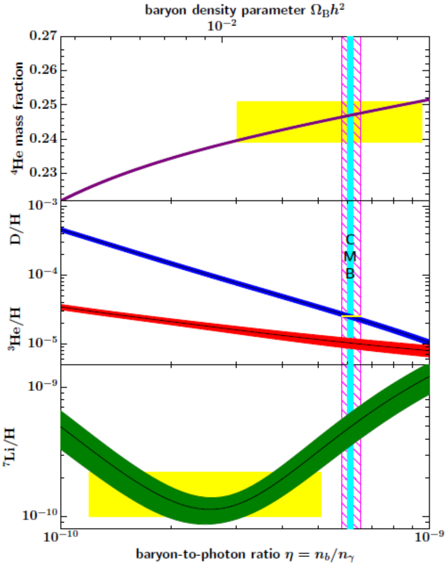

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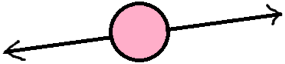
- Observed BAU » theory $\rightarrow$ Baryon Asymmetry puzzle
- *How can we address this puzzle?*
  - Initial condition: fine tuning $\leftrightarrow$ Wash out during inflation
  - Need dynamical mechanism satisfying Sakharov Criteria:
    - ★ Out of equilibrium transitions
    - ★ B violation
    - ★ **C and CP Violation**

Three-body B decays

- $|PP\rangle$ state is momentum independent



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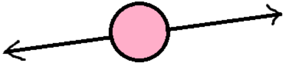
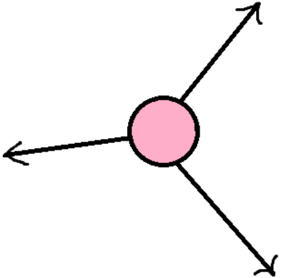
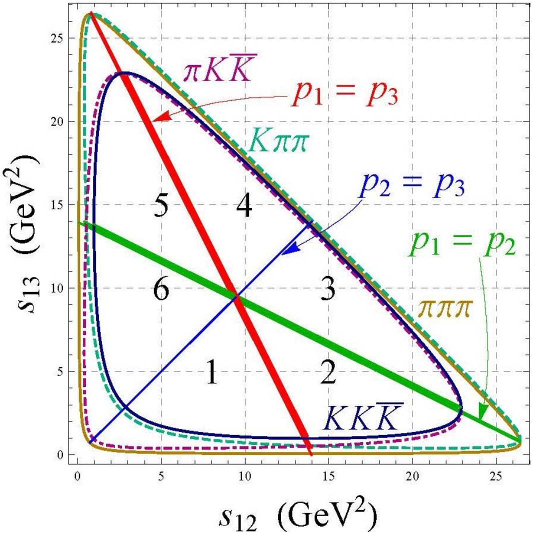


- $|P_1(\vec{p}_1)P_2(\vec{p}_2)P_3(\vec{p}_3)\rangle$

  depends on final-state momenta





Flavor-SU(3) symmetry: SU(3)$_F$

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- Decay amplitude $= \langle B|\, H\, |PPP\rangle_{\rm FS} = \sum_i C_i \langle \mathbf{3}|\, \mathbf{3}^*, \mathbf{6}, \mathbf{15}^*\, |120\rangle_i$

  C_i contains *SU(2) Clebsch-Gordan Coefficients* and *SU(3) isoscalar factors*