Role of polarization and spin-spin correlations at ILC/LHC

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ICHEPAP 2023, SINP Kolkata

December 15, 2023

Spin and Polarization

Poincare Invariance

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$$[J_m, P_n] = i\epsilon_{mnk} P_k \ , \quad [J_i, P_0] = 0 \ , \quad [K_i, P_k] = i\eta_{ik} P_0 \ ,$$

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- Casimir operators are : $P_\mu P^\mu \equiv P^2$ and $W_\mu W^\mu \equiv W^2$ where $W_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} M^{\nu\alpha} P^\beta$

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- For particle of mass m and spin s we have $P^2|\psi\rangle = m^2|\psi\rangle$ and $W^2|\psi\rangle = m^2 \ s(s+1)|\psi\rangle$

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Scalars, Fermions and Gauge Bosons

The Scalar Field

Spin-0 particle of mass m, either scalar of psudo-scalar, is described by the Kleine-Gordan equation

$$\partial_\mu \partial^\mu \phi - m^2 \phi = 0$$

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- Interaction with gauge bosons: $A^\mu \left[(\partial_\mu \phi^\dagger)\phi - \phi^\dagger(\partial_\mu \phi) \right]$ and $A^\mu A_\mu \phi^\dagger \phi$

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$$D^\mu \phi \equiv (\partial^\mu + ieA^\mu)\phi$$

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The Standard Model

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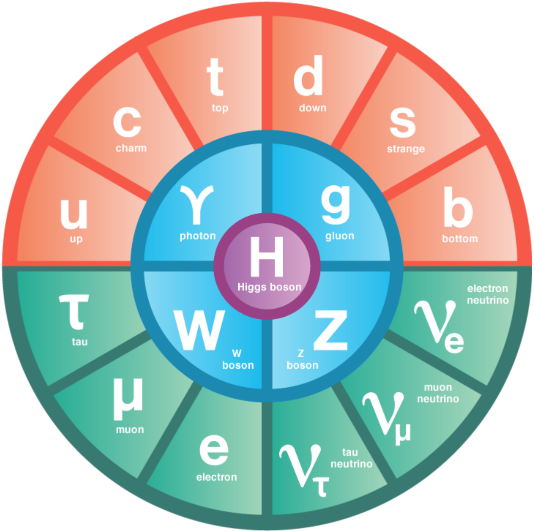

- Spin 0: H
- Spin 1/2: $u, d, c, s,$ t, b, $e, \nu_e, \mu, \nu_\mu,$ τ, ν_τ
- Spin 1: $\gamma, g,$ W, Z

The Spin Code

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Helicity amplitude for the decay $|s,\lambda\rangle \to |s_1,l_1\rangle + |s_2,l_2\rangle$ is

$$\begin{aligned} M^{s\lambda}_{l_1 l_2}(\theta,\phi) &= \sqrt{\frac{2s+1}{4\pi}} D^{s*}_{\lambda l}(\phi,\theta,-\phi)\mathcal{M}^s_{l_1,l_2} \\ &= \sqrt{\frac{2s+1}{4\pi}} e^{i(\lambda-l)\phi} d^s_{\lambda l}(\theta)\mathcal{M}^s_{l_1,l_2}, \quad l = l_1 - l_2. \end{aligned}$$

$d^s_{\lambda l}(\theta)$ is $2s$ degree polynomial in $\cos(\theta/2)$ and $\sin(\theta/2)$

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- distribution in the production angle relying on a known production mechanism.
- extracting the $(\cos\theta)^{2s}$ polar angle dependence or $\cos 2s\phi$ azimuthal angle dependence of the decay distributions

The Collider Experiment

Scattering and Cross-sections

Polarized particle production

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$$M(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}}) = M_\gamma(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}}) + M_Z(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}})$$

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$$\eta_3 = \frac{|M(\lambda_{e^-}, \lambda_{e^+}, +, \lambda_{\bar{t}})|^2 - |M(\lambda_{e^-}, \lambda_{e^+}, -, \lambda_{\bar{t}})|^2}{|M(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}})|^2}$$

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$$\eta_1 = \frac{\Re\left(M(\lambda_{e^-}, \lambda_{e^+}, +, \lambda_{\bar{t}}) M^*(\lambda_{e^-}, \lambda_{e^+}, -, \lambda_{\bar{t}})\right)}{|M(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}})|^2}$$

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\eta_1 &= \frac{\Re\left(M(\lambda_{e^-}, \lambda_{e^+}, +, \lambda_{\bar{t}}) M^*(\lambda_{e^-}, \lambda_{e^+}, -, \lambda_{\bar{t}})\right)}{|M(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}})|^2} \\
\eta_2 &= \frac{-\Im\left(M(\lambda_{e^-}, \lambda_{e^+}, +, \lambda_{\bar{t}}) M^*(\lambda_{e^-}, \lambda_{e^+}, -, \lambda_{\bar{t}})\right)}{|M(\lambda_{e^-}, \lambda_{e^+}, \lambda_t, \lambda_{\bar{t}})|^2}
\end{aligned}$$

Polarized beam collision

Polarization density matrix for fermion:

$$\rho(\lambda, \lambda') = \frac{1}{2}\vec{\eta} \cdot \vec{\sigma} = \frac{1}{2} \begin{bmatrix} 1 + \eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & 1 - \eta_3 \end{bmatrix}.$$

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With initial state e^+ and e^- polarized the matrix element is given by

$$|M|^2 = \sum_{\lambda_{e^-},\lambda'_{e^-},\lambda_{e^+},\lambda'_{e^+},\lambda_t,\lambda_{\bar{t}}} \rho_{e^-}(\lambda_{e^-},\lambda'_{e^-})\ \rho_{e^+}(\lambda_{e^+},\lambda'_{e^+}) \times M(\lambda_{e^-},\lambda_{e^+},\lambda_t,\lambda_{\bar{t}})\ M^*(\lambda'_{e^-},\lambda'_{e^+},\lambda_t,\lambda_{\bar{t}})$$

The density matrix

The process

We look at the production process $B_1 B_2 \to A\ A_1\ ...\ A_{n-1}$ followed by the decay of A as $A \to BC$. The differential cross-section is given by

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$$d\sigma = \sum_{\lambda,\lambda'} \left[\frac{(2\pi)^4}{2I} \rho(\lambda,\lambda') \delta^4 \left(k_{B_1} + k_{B_2} - p_A - \left(\sum_i^{n-1} p_i \right) \right) \right.$$
$$\left. \times \frac{d^3 p_A}{2E_A (2\pi)^3} \prod_i^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \right]$$
$$\times \left[\frac{1}{\Gamma_A} \frac{(2\pi)^4}{2m_A} \Gamma'(\lambda,\lambda') \delta^4(p_A - p_B - p_C) \frac{d^3 p_B}{2E_B (2\pi)^3} \frac{d^3 p_C}{2E_C (2\pi)^3} \right]$$

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$$\text{First bracket} = \sigma(\lambda,\lambda') = \sigma_A\ P_A(\lambda,\lambda')$$

$$\text{Second bracket} = \frac{B_{BC}(2s+1)}{4\pi}\ \Gamma_A(\lambda,\lambda') d\Omega_B$$

The angular distribution

$$\frac{1}{\sigma}\ \frac{d\sigma}{d\Omega_B} = \frac{2s+1}{4\pi} \sum_{\lambda,\lambda'} \ P_A(\lambda,\lambda') \ \ \Gamma_A(\lambda,\lambda'),$$

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- $\sigma = B_{BC}\;\sigma_A$ is the cross-section of production of A and its decay into BC.

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- $P_A(\lambda,\lambda') = \sigma(\lambda,\lambda')/\sigma_A$ is the polarization density matrix.
- $\Gamma_A(\lambda,\lambda')$ is the normalized decay density matrix in the rest frame of A.

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$$\begin{aligned}\Gamma'^{s}(\lambda,\lambda') &= \sum_{l_1,l_2} M^{s\lambda}_{l_1 l_2} M^{*s\lambda'}_{l_1 l_2} \\ &= \left(\frac{2s+1}{4\pi}\right) e^{i(\lambda-\lambda')\phi} \sum_{l_1,l_2} d^{s}_{\lambda l}(\theta) d^{s}_{\lambda' l}(\theta)\ |\mathcal{M}^{s}_{l_1,l_2}|^2\end{aligned}$$

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&= e^{i(\lambda-\lambda')\phi} \sum_{l} d^{s}_{\lambda l}(\theta) d^{s}_{\lambda' l}(\theta) \left[\sum_{l_1} \left(\frac{2s+1}{4\pi}\right) |\mathcal{M}^{s}_{l_1,l_1-l}|^2\right]
\end{aligned}$$

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&= e^{i(\lambda-\lambda')\phi} \sum_{l} d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta) \left[\sum_{l_1} \left(\frac{2s+1}{4\pi}\right) |\mathcal{M}^s_{l_1,l_1-l}|^2\right] \\
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&= e^{i(\lambda-\lambda')\phi} \sum_{l} d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta) \left[\sum_{l_1} \left(\frac{2s+1}{4\pi}\right) |\mathcal{M}^s_{l_1,l_1-l}|^2\right] \\
&= e^{i(\lambda-\lambda')\phi} \sum_{l} d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta)\; a^s_l
\end{aligned}$$

$|l_1| \le s_1, \quad |l_1 - l| \le s_2, \quad |l| \le s$ and $\mathrm{Tr}(\Gamma'^s(\lambda,\lambda')) = \sum_l a^s_l$.

The final distribution

The normalized decay density matrix is given by

$$\Gamma_A(\lambda,\lambda') = e^{i(\lambda-\lambda')\phi} \frac{\sum_l d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta) a^s_l}{\sum_l a^s_l} = e^{i(\lambda-\lambda')\phi}\, \gamma_A(\lambda,\lambda';\theta),$$

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and the final distribution is given by

$$\begin{aligned} \frac{1}{\sigma}\,\frac{d\sigma}{d\Omega_B} \quad &= \quad \frac{2s+1}{4\pi}\Bigg[\sum_{\lambda} P_A(\lambda,\lambda)\;\; \gamma_A(\lambda,\lambda) \\ &+ \sum_{\lambda\neq\lambda'} \Re[P_A(\lambda,\lambda')]\;\; \gamma_A(\lambda,\lambda')\;\cos((\lambda-\lambda')\phi) \\ &- \sum_{\lambda\neq\lambda'} \Im[P_A(\lambda,\lambda')]\;\; \gamma_A(\lambda,\lambda')\;\sin((\lambda-\lambda')\phi)\Bigg]. \end{aligned}$$

Spin-$1/2$ case

The density matrix: $s = 1/2$

For $|\frac{1}{2}, l\rangle \to |s_1, l_1\rangle + |s_2, l_2\rangle$

$$\Gamma_{\frac{1}{2}}(\lambda, \lambda') = \begin{bmatrix} \frac{1+\alpha\cos\theta}{2} & \frac{\alpha\sin\theta}{2}\; e^{i\phi} \\ \frac{\alpha\sin\theta}{2}\; e^{-i\phi} & \frac{1-\alpha\cos\theta}{2} \end{bmatrix},$$

Here $\alpha = (a^{1/2}_{1/2} - a^{1/2}_{-1/2})/(a^{1/2}_{1/2} + a^{1/2}_{-1/2})$ and

$$\begin{aligned} a^{1/2}_{1/2} &= \left(\frac{1}{2\pi}\right) \sum_{l_1} |\mathcal{M}^{1/2}_{l_1, l_1 - 1/2}|^2 \qquad |l_1| \le s_1, \;\; |l_1 - 1/2| \le s_2 \\ a^{1/2}_{-1/2} &= \left(\frac{1}{2\pi}\right) \sum_{l_1} |\mathcal{M}^{1/2}_{l_1, l_1 + 1/2}|^2 \qquad |l_1| \le s_1, \;\; |l_1 + 1/2| \le s_2. \end{aligned}$$

Angular distribution: $s = 1/2$

Polarization density matrix:

$$P_{\frac{1}{2}}(\lambda, \lambda') = \frac{1}{2} \begin{bmatrix} 1+\eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & 1-\eta_3 \end{bmatrix},$$

Thus the angular distribution becomes:

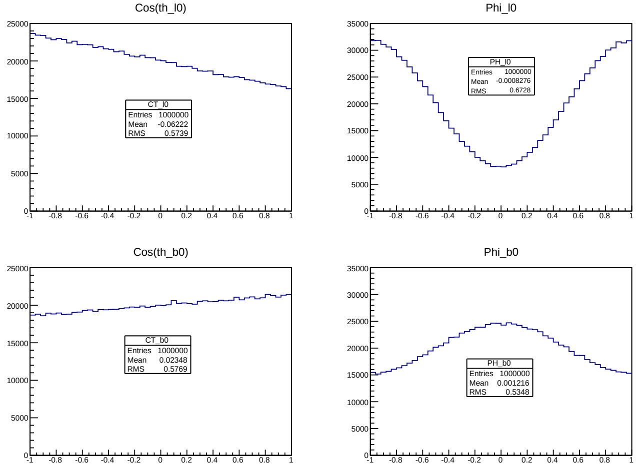
$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\Omega_B} = \frac{1}{4\pi}\left[1 + \alpha\eta_3\cos\theta + \alpha\eta_1\sin\theta\cos\phi + \alpha\eta_2\sin\theta\sin\phi\right].$$

The $\cos\theta$ averaged azimuthal distribution is given by

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\phi} = \frac{1}{2\pi}\left[1 + \frac{\alpha\eta_1\pi}{4}\cos\phi + \frac{\alpha\eta_2\pi}{4}\sin\phi\right].$$

Angular distribution: $e^+e^- \rightarrow t\bar{t}$

$$\sqrt{s} = 400\text{GeV},\ \eta_1 = -0.75,\ \eta_2 \approx 0, \eta_3 = -0.19$$



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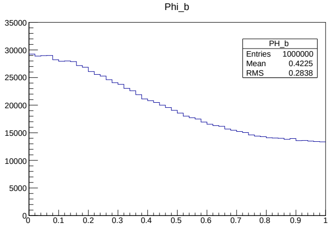
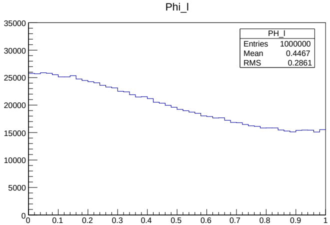




Spin-1 case

The density matrix: $s = 1$

For $|1, l\rangle \to |s_1, l_1\rangle + |s_2, l_2\rangle$,

$$\Gamma_1(l, l') = \begin{bmatrix} \frac{1+\delta+(1-3\delta)\cos^2\theta+2\alpha\cos\theta}{4} & \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}}\, e^{i\phi} & (1-3\delta)\frac{(1-\cos^2\theta)}{4}\, e^{i2\phi} \\ \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}}\, e^{-i\phi} & \delta+(1-3\delta)\frac{\sin^2\theta}{2} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}}\, e^{i\phi} \\ (1-3\delta)\frac{(1-\cos^2\theta)}{4}\, e^{-i2\phi} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}}\, e^{-i\phi} & \frac{1+\delta+(1-3\delta)\cos^2\theta-2\alpha\cos\theta}{4} \end{bmatrix},$$

where,

$$\alpha = \frac{a^1_1 - a^1_{-1}}{a^1_1 + a^1_0 + a^1_{-1}} \qquad , \qquad \delta = \frac{a^1_0}{a^1_1 + a^1_0 + a^1_{-1}}$$

and

$$a^1_1 = \left(\frac{3}{4\pi}\right)\sum_{l_1} |\mathcal{M}^1_{l_1, l_1-1}|^2 \qquad |l_1| \le s_1, \quad |l_1 - 1| \le s_2$$

$$a^1_0 = \left(\frac{3}{4\pi}\right)\sum_{l_1} |\mathcal{M}^1_{l_1, l_1}|^2 \qquad |l_1| \le \min(s_1, s_2)$$

$$a^1_{-1} = \left(\frac{3}{4\pi}\right)\sum_{l_1} |\mathcal{M}^1_{l_1, l_1+1}|^2 \qquad |l_1| \le s_1, \quad |l_1 + 1| \le s_2$$

Angular distribution: $s = 1$

Polarization density matrix:

$$P_1(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3} + \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} + \frac{T_{xz} - iT_{yz}}{\sqrt{3}} & \frac{T_{xx} - T_{yy} - 2iT_{xy}}{\sqrt{6}} \\ \frac{p_x + ip_y}{2\sqrt{2}} + \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{2T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} - \frac{T_{xz} - iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx} - T_{yy} + 2iT_{xy}}{\sqrt{6}} & \frac{p_x + ip_y}{2\sqrt{2}} - \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} \end{bmatrix},$$

The angular distribution is:

$$\begin{aligned} \frac{1}{\sigma}\frac{d\sigma}{d\Omega} \quad = \quad & \frac{3}{8\pi}\Bigg[\left(\frac{2}{3} - (1-3\delta)\ \frac{T_{zz}}{\sqrt{6}}\right) + \alpha\ p_z \cos\theta + \sqrt{\frac{3}{2}}(1-3\delta)\ T_{zz}\cos^2\theta \\ + \quad & \left(\alpha\ p_x + 2\sqrt{\frac{2}{3}}(1-3\delta)\ T_{xz}\cos\theta\right)\sin\theta\ \cos\phi \\ + \quad & \left(\alpha\ p_y + 2\sqrt{\frac{2}{3}}(1-3\delta)\ T_{yz}\cos\theta\right)\sin\theta\ \sin\phi \\ + \quad & (1-3\delta)\left(\frac{T_{xx} - T_{yy}}{\sqrt{6}}\right)\sin^2\theta\cos(2\phi) + \sqrt{\frac{2}{3}}(1-3\delta)\ T_{xy}\ \sin^2\theta\ \sin(2\phi)\Bigg] \end{aligned}$$

Polarization asymmetries: $e^+e^- \to ZZ$

Define $I(\theta, \phi) = (1/\sigma)(d\sigma/d\Omega)$, then we have

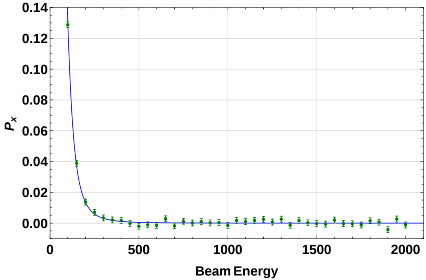
$$\begin{aligned} A_x &= \left[\int_{\theta=0}^{\pi} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} I(\theta,\phi) \sin(\theta) d\theta d\phi - \int_{\theta=0}^{\pi} \int_{\phi=\frac{\pi}{2}}^{3\frac{\pi}{2}} I(\theta,\phi) \sin(\theta) d\theta d\phi \right] \\ &= \frac{3\alpha p_x}{4} \\ &= \frac{\sigma(c_x > 0) - \sigma(c_x < 0)}{\sigma(c_x > 0) + \sigma(c_x < 0)} \end{aligned}$$

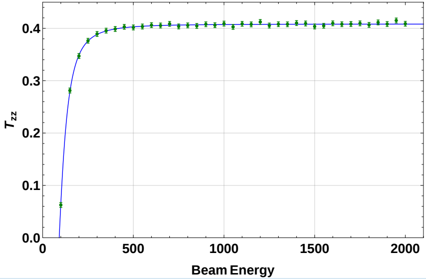
Here $c_x = \sin\theta \cos\phi$ in the rest frame of the decaying particle.

Similarly one can define asymmetries for other polarization parameters as well.

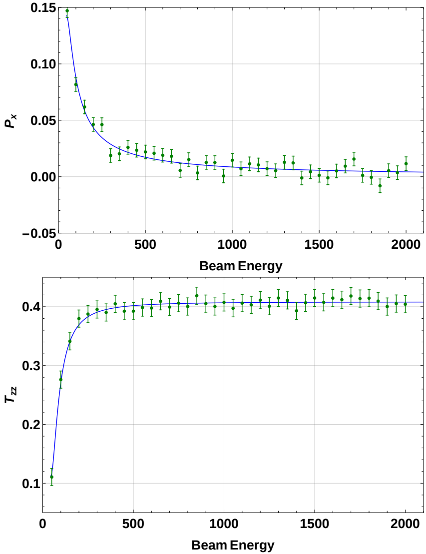
Correlations	Asymmetries
$c_x = \sin\theta\cos\phi$	$A_x = \frac{3\alpha p_x}{4}$
$c_y = \sin\theta\sin\phi$	$A_y = \frac{3\alpha p_y}{4}$
$c_z = \cos\theta$	$A_z = \frac{3\alpha p_z}{4}$
$c_{xy} = c_x c_y$	$A_{xy} = \frac{2}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)T_{xy}$
$c_{xz} = c_x c_z$	$A_{xz} = \frac{2}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)T_{xz}$
$c_{yz} = c_y c_z$	$A_{yz} = \frac{2}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)T_{yz}$
$c_{x^2-y^2} = c_x^2 - c_y^2$	$A_{yz} = \frac{1}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)(T_{xx} - T_{yy})$
$c_{yz} = \sin 3\theta$	$A_{zz} = \frac{3}{8}\sqrt{\frac{3}{2}}(1-3\delta)T_{zz}$

Polarization asymmetries: $e^+e^- \to ZZ$





Polarization asymmetries: $e^+e^- \to Z\gamma$



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With two spin-full particles in the final state we can do more.

Spin-Spin Correlations

Polarization and Correlations

For single particle spin-polarization we have:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \mathbb{I} + \vec{P}.\vec{S} + T_{ij} S_i S_j + \ldots$$

For two particle spin-polarization we have:

$$\begin{aligned}\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_a d\Omega_b} &\propto \mathbb{I}\otimes\mathbb{I} + \vec{P^A}.\vec{S}\otimes\mathbb{I} + T^A_{ij} S_i S_j \otimes \mathbb{I} \\ &+ \mathbb{I}\otimes\vec{P^B}.\vec{S} + \mathbb{I}\otimes T^B_{ij} S_i S_j \\ &+ \vec{P^A}.\vec{S}\otimes\vec{P^B}.\vec{S} + T^A_{ij} S_i S_j \otimes T^B_{ij} S_i S_j \\ &+ \vec{P^A}.\vec{S}\otimes T^B_{ij} S_i S_j + T^A_{ij} S_i S_j \otimes \vec{P^B}.\vec{S}\end{aligned}$$

[R. Rahaman, **RKS** NPB984 (2022)]

Asymmetries

σ	A_x	A_y	A_z	A_{xy}	A_{xz}	A_{yz}	$A_{x^2-y^2}$	A_{zz}
A_x	E	O	E	O	E	O	E	E
A_y	O	E	O	E	O	E	O	O
A_z	E	O	E	O	E	O	E	E
A_{xy}	O	E	O	E	O	E	O	O
A_{xz}	E	O	E	O	E	O	E	E
A_{yz}	O	E	O	E	O	E	O	O
$A_{x^2-y^2}$	E	O	E	O	E	O	E	E
A_{zz}	E	O	E	O	E	O	E	E

- 80 asymmetries + cross-section
- Asymmetries: 44 CP-even, 36 -odd
- 45 asymmetries require flavor tagging, 35 dont.

Spin-correlations @ Collider Studies

- $e^+e^- \to W^+W^-$, anomalous couplings, light flavor tagging using ANN and BDT [A. Subba, **RKS**, 2212.12973]

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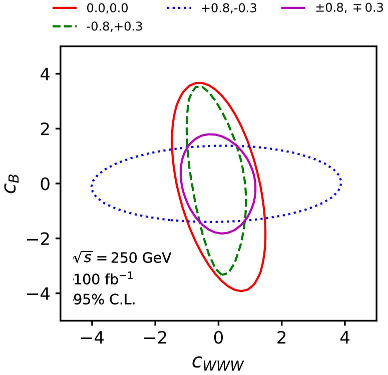
- $e^+e^- \to W^+W^-$, anomalous couplings, light flavor tagging using ANN and BDT, using beam polarizations [A. Subba, **RKS**, 2305.15106]



Beyond two-particle correlations

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- Three-body entanglement in particle physics [Sakurai, Spannowsky, 2310.01477]

Conclusions

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- Polarization and spin-correlation asymmetries are very useful in probing anomalous couplings and BSM in general.
- Study of Bell inequalities and quantum entanglements at collider is a hot topic in past two years.