Electroweak Unification and the Standard Model

Lecture 2

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For gauge invariance, we must set $M_\gamma = 0$, i.e. the photon must be massless

gauge invariance $\Leftrightarrow$ long range interactions

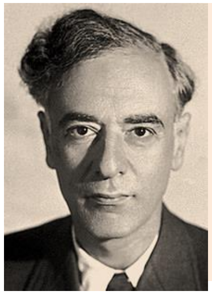
The electromagnetic interaction is not always long-range...

Somehow, the local U(1) gauge symmetry breaks down inside the superconductor...

Do we understand this phenomenon? Yes.

- It was first explained by Landau and Ginzburg in 1937 for a non-relativistic theory (which applies to superconductors).
- It was extensively applied in condensed matter systems by Philip Anderson in the 1950's
- It was worked out for a relativistic theory by Englert & Brout (1964) and independently, by Peter Higgs (1964).

The phenomenon is called spontaneous symmetry-breaking, but a better name (Coleman) is hidden symmetry...



Lev Landau (1908 – 1968) – pioneering work in QM and QFT

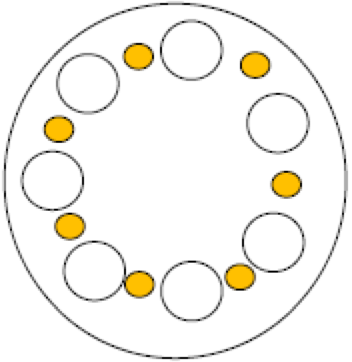
Vitaly Ginzburg (1916 – 2009) – superconductivity and plasma theory

Philip Anderson (1923 – 2020) – pioneer of condensed matter physics

Hidden symmetry

This arises when the action of a system has a particular symmetry, but the ground state does not...

Example 1: Salam's banquet

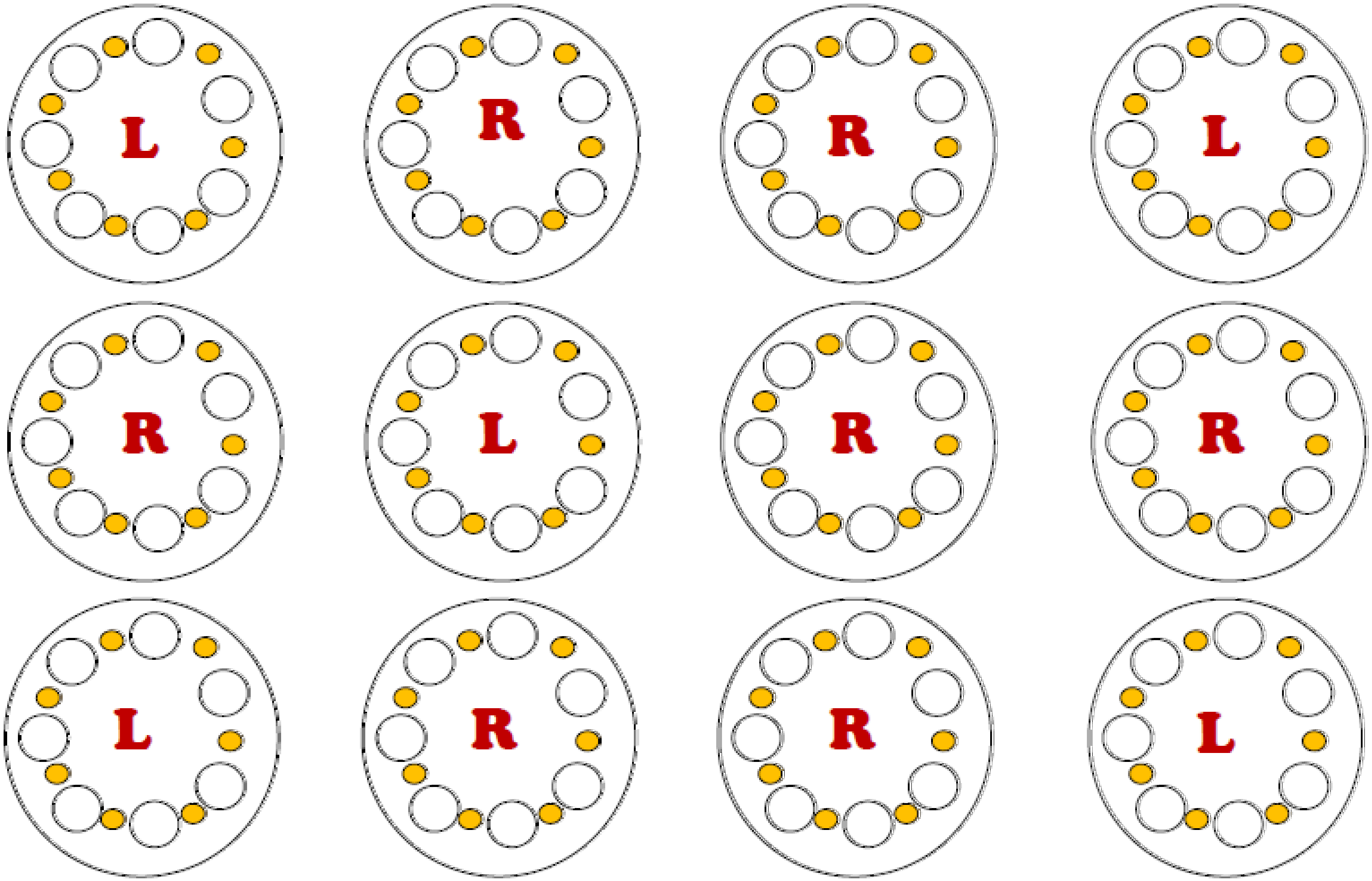


People are sitting to dinner at a round table. Each has a plate in front and a glass on either hand. Before the meal, there is perfect symmetry between left glasses and right glasses.

Abdus Salam (1926 – 1996) – electroweak unification

Has the symmetry really been destroyed?

No: if we consider an ensemble of systems...

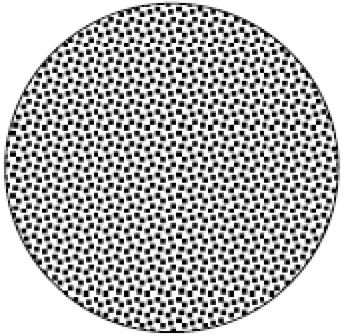


the symmetry reappears!

Hidden symmetry

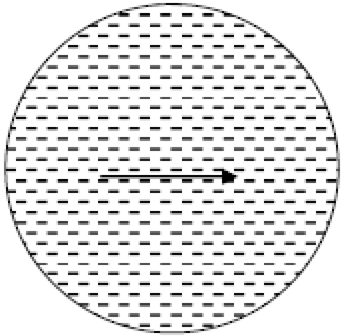
This arises when the action of a system has a particular symmetry, but the ground state does not...

Example 2: Ferromagnet below Curie temperature



Above the Curie temperature, all the domains are in random directions... obeys rotation invariance...

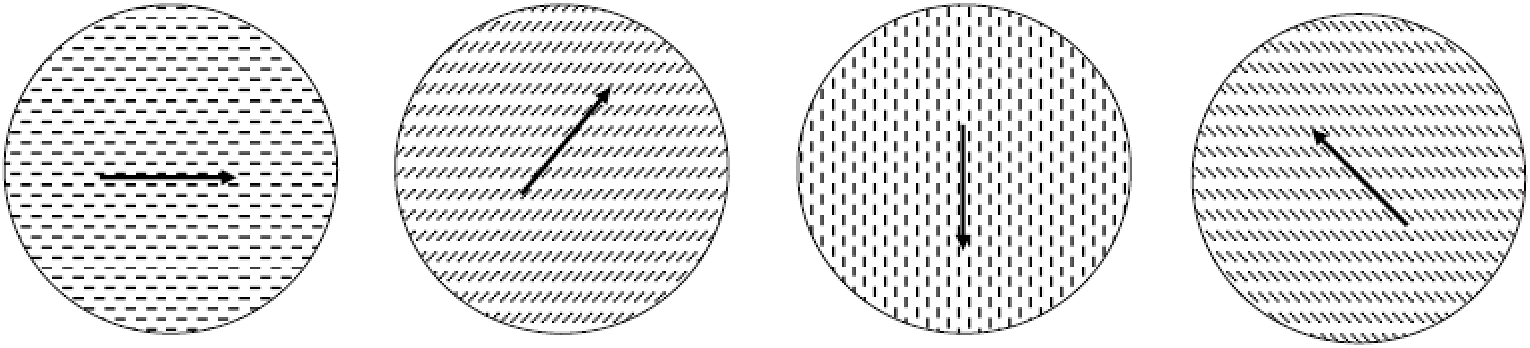
$$H = \sum_{\langle ij \rangle} J_{ij}\, \vec{S}_i . \vec{S}_j$$



Below the Curie temperature, all the domains are aligned parallel to a particular direction... magnetic field measurement will show a preferred direction, i.e. rotation invariance is lost

Has the symmetry really been destroyed?

No: if we consider an ensemble of systems... the symmetry reappears!



If we confine ourselves to the inside of a ferromagnet (Coleman's demon), then rotation invariance will always be violated...

Sidney Coleman
(1937 – 2007)

This is always associated with a phase transition:

i.e. at some high temperature, the symmetry exists
at low temperature the symmetry disappears
in between a flip occurs... critical temperature... $\Rightarrow$ phase transition

How does the superconducting phase break the electromagnetic U(1) gauge invariance?

We discuss the relativistic model, because we will apply the same idea to particle physics problems...

Imagine the interior of the superconductor to have, in addition to the electromagnetic field, a charged scalar field $\varphi(x)$. We have already seen that this leads to a Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^{\mu}\varphi]^{*}[D_{\mu}\varphi] - M^{2}\varphi^{*}\varphi$$

In addition to this, let the scalar field have a self-interaction term

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^{\mu}\varphi]^{*}[D_{\mu}\varphi] - M^{2}\varphi^{*}\varphi - \lambda(\varphi^{*}\varphi)^{2}$$

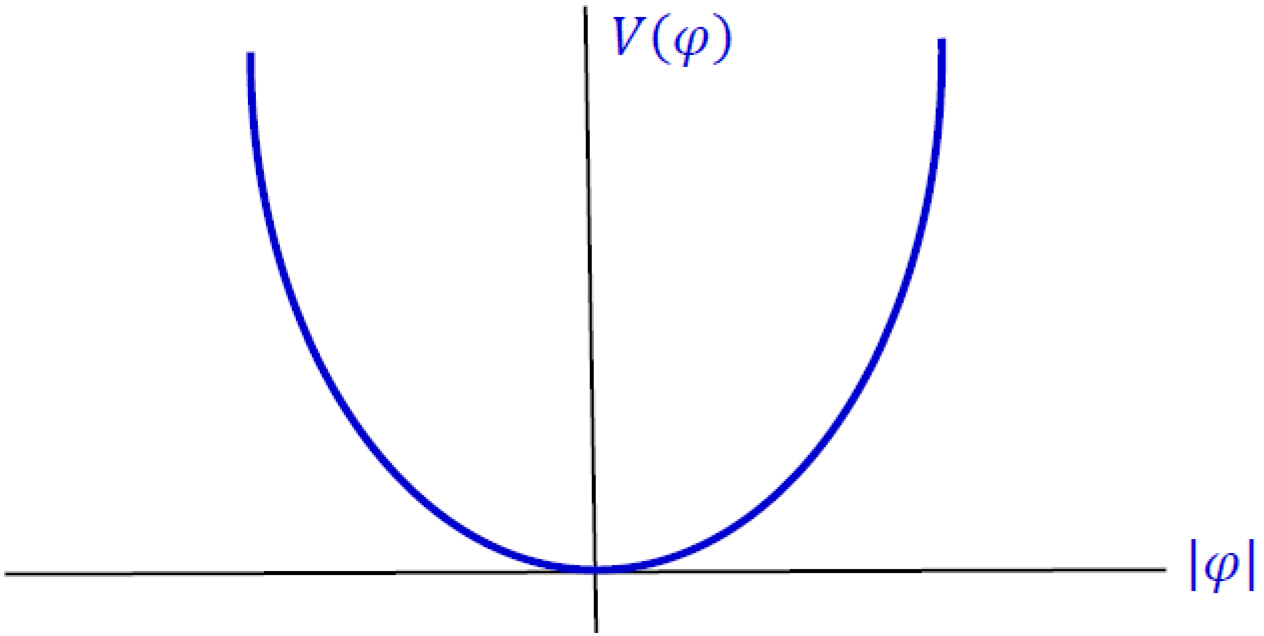
The last two terms can then be thought of as a gauge-invariant potential, i.e. we rewrite

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^{\mu}\varphi]^{*}\left[D_{\mu}\varphi\right] - V(\varphi)$$

where

$$V(\varphi) = M^2\varphi^{*}\varphi + \lambda(\varphi^{*}\varphi)^2 = M^2|\varphi|^2 + \lambda|\varphi|^4$$

If we plot this potential as a function of $|\varphi|$, we will get



with a minimum at $|\varphi| = 0$, i.e. at $\varphi = 0$. No symmetries are broken.

But now, let us consider another variant of this theory, viz.

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] + M^2 \varphi^* \varphi - \lambda(\varphi^* \varphi)^2$$

If we try to treat $+M^2 \varphi^* \varphi$ as a mass term, the scalar particle will become a tachyon. Don't try this. Just let $+M^2 \varphi^* \varphi$ be an interaction term. Now, rewrite

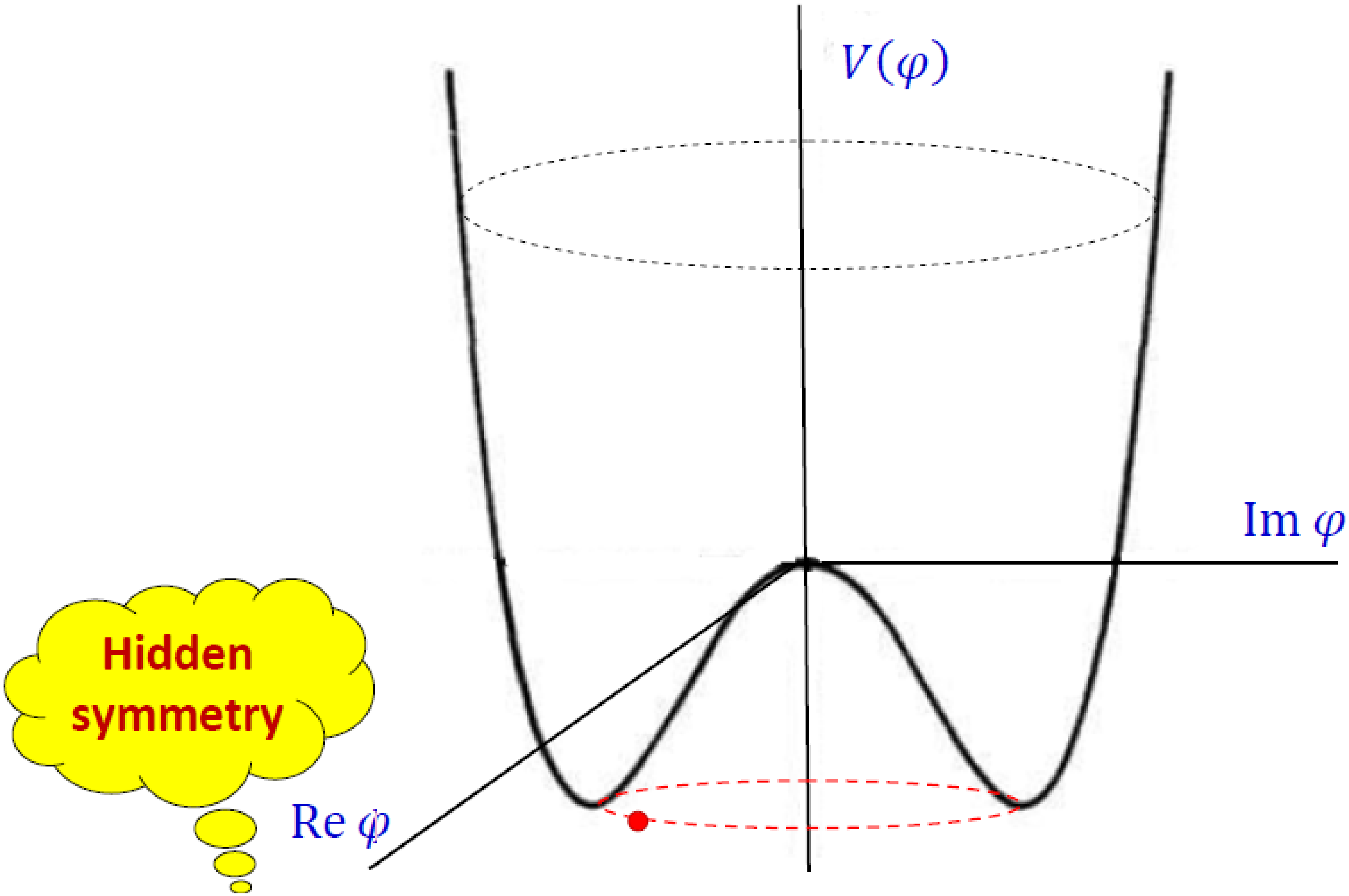
$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] - V(\varphi)$$

where $V(\varphi) = -M^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2 = -M^2 |\varphi|^2 + \lambda |\varphi|^4$

This clearly has extrema at

$$\frac{\partial V}{\partial |\varphi|} = 0 \quad \Rightarrow \quad |\varphi| = 0 \ (\max) \, , \sqrt{\frac{M^2}{2\lambda}} \ (\min) \equiv \frac{v}{\sqrt{2}}$$

Thus, there are an infinite number of possible ground states $\varphi = ve^{i\alpha}$



Only one of these can be the true ground state... as in a ferromagnet

Let us orient the axes in the complex φ plane such that the ground state (wherever it is) falls along the real axis.

(Just a convenient parametrisation – like choosing the *z*-axis along a constant magnetic field)

The ground state is now $\varphi_0 = \frac{v}{\sqrt{2}}$. To construct a viable field theory we must expand around this ground state, i.e. $\varphi(x) = \varphi_0 + \varphi'(x)$.

Rewrite the Lagrangian density in terms of this new field $\varphi'(x)$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^{\mu}(\varphi_0 + \varphi')]^{*}[D_{\mu}(\varphi_0 + \varphi')] - V(\varphi_0 + \varphi')$$

Calculate the terms one by one:

$$D_{\mu}(\varphi_0 + \varphi') = (\partial_{\mu} + ieA_{\mu})(\varphi_0 + \varphi') = \partial_{\mu}\varphi' + ieA_{\mu}\varphi' + ie\varphi_0 A_{\mu}$$

It follows that

$$[D^{\mu}(\varphi_0 + \varphi^{'})]^{*}[D_{\mu}(\varphi_0 + \varphi^{'})]$$

$$= [\partial_{\mu}\varphi^{'} + ieA^{\mu}\varphi^{'} + ie\varphi_0 A^{\mu}]^{*}[\partial_{\mu}\varphi^{'} + ieA_{\mu}\varphi^{'} + ie\varphi_0 A_{\mu}]$$

Recall $\mathcal{L}_M = \frac{1}{2} M_{\gamma}^2 A^{\nu} A_{\nu}$

Inside a superconductor with a potential as assumed here, the photon has become massive!

$$M_{\gamma} = \sqrt{2} e\varphi_0 = \sqrt{2} e \frac{v}{\sqrt{2}} = ev$$

Another miracle...

$$V(\varphi_0 + \varphi') = -M^2(\varphi_0 + \varphi')^*(\varphi_0 + \varphi') + \lambda[(\varphi_0 + \varphi')^*(\varphi_0 + \varphi')]^2$$

Goldstone theorem (1962):

To every spontaneously broken continuous global symmetry, there corresponds a massless boson

How to get rid of this massless boson (would induce new long-range interactions otherwise)?

Jeffrey Goldstone
(1933 –)

Englert & Brout (1964), Higgs (1964):

Can be done if it is a local symmetry...

Idea is very simple: parametrise $\varphi(x) = \eta(x)\, e^{i\xi(x)}$ (polar form)

Consider the unbroken (i.e. gauge invariant) Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^{\mu}\varphi]^{*}[D_{\mu}\varphi] - V(\varphi)$$

where $V(\varphi) = -M^2\varphi^*\varphi + \lambda(\varphi^*\varphi)^2 = -M^2|\varphi|^2 + \lambda|\varphi|^4$

At this level, we are free to make any gauge choice we wish...

Make a gauge transformation

$$\varphi(x) \to e^{-ig\theta(x)}\varphi(x) = \eta(x)\, e^{i[\xi(x) - g\theta(x)]}$$

We might as well choose a special gauge, since the gauge symmetry is going to be broken anyway...

Choose the gauge function $\theta(x)$ such that

$$\xi(x) - g\theta(x) = 0$$

This is called the unitary gauge.

In this gauge, $\varphi(x) = \eta(x)$ and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \eta]^* [D_\mu \eta] - V(\eta)$$

where $V(\eta) = -M^2\eta^2 + \lambda\eta^4$

The ground state is still at $v/\sqrt{2}$ so we must shift

$$\eta = \frac{v}{\sqrt{2}} + \eta'$$

This will lead to

1. $\mathcal{L}_M = \frac{1}{2} M_\gamma^2 A^\nu A_\nu$ with $M_\gamma = ev$
2. $V(\varphi_0 + \varphi') = +\frac{1}{2} 4M^2\eta^2 + \cdots$ i.e. $M_\eta = 2M$
3. and there is no Goldstone boson...

if we had kept $\xi(x)$ it would have been the Goldstone boson...

Looks like magic!!

How can a degree of freedom of the system vanish?

In the unitary gauge, the photon is massive,

i.e. it has <u>three</u> polarisations.

The extra degree of freedom (longitudinal polarisation) which appears here is at the cost of the disappearance of the Goldstone degree of freedom...

Higgs mechanism

Gerald Guralnik
(1936 – 2014)

Carl Hagen
(1937 –)

Tom Kibble
(1932 – 2016)

Why is all this relevant?

Weak Interactions

Pauli's neutrino hypothesis

Physikalisches Institut
der Eidg. Technischen Hochschule
Gloriastr.
Zürich

Zürich, 4 December 1930

Dear Radioactive Ladies and Gentlemen,
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the 'wrong' statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the 'exchange theorem' of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have the spin $\frac{1}{2}$ and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses.—The continuous β-spectrum would then become understandable by the assumption that in β-decay, a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant......

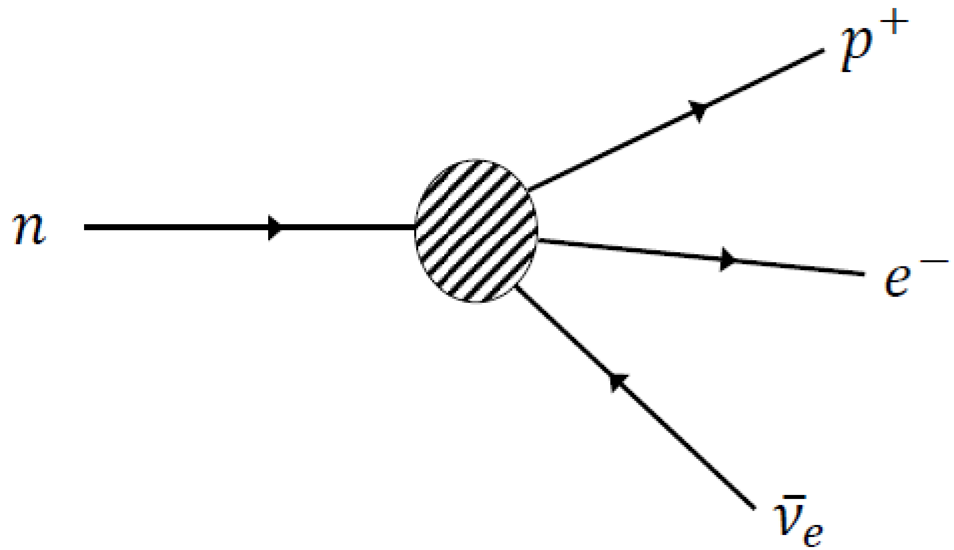
So, dear Radioactives, examine and judge it.—Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December.—With my best regards to you, and also Mr Back, your humble servant,

W Pauli

Wolfgang Pauli
(1900 – 1958)

Fermi's theory of beta decay

$$ {}^1_0n \;\rightarrow\; {}^1_1p + {}^{\;\,0}_{-1}e + {}^0_0\bar{\nu}_e $$



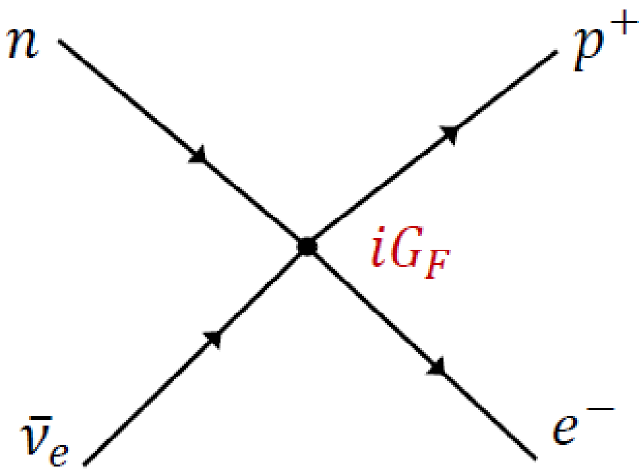
The decay must take place through weak interactions ($\tau = 887$ s).

Can we write down an interaction vertex?

First attempted by Fermi (1934)

Denote the Dirac fields: $\Psi_n = n,\;\; \Psi_p = p,\;\; \Psi_e = e$ and $\Psi_{\bar{\nu}_e} = \bar{\nu}_e$

Fermi's first attempt: try a four-fermion vertex



Weak interaction Hamiltonian:

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\,\overline{p}n\;\bar{e}\nu_e$$

dimension of G_F is M^{-2} : Fermi coupling constant
Simplest possible form of a four-fermion coupling

With this interaction, the probability for the transition, in the rest-frame of the neutron, comes out to be

$$|\mathcal{M}|^2 \approx 4G_F^2 M_n M_p E_e^2 (1 - \cos\theta_{e\overline{\nu}})$$

i.e. the electron and the antineutrino should tend to come out back-to-back...

Actual experiment showed that, instead, the electron and the antineutrino tended to come out *in the same direction*!

More as if we have $|\mathcal{M}|^2 \propto (1 + \cos\theta_{e\overline{\nu}})$

Fermi's second attempt: try a vertex modelled on e.m. interactions,

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} \overline{p}\gamma^\mu n \; \bar{e}\gamma_\mu \nu_e = \frac{G_F}{\sqrt{2}} J^\mu_{\text{had}} \, J^{\text{lep}}_\mu$$

Current-current form of the weak interaction

With this interaction, the probability for the transition, in the rest-frame of the neutron, comes out to be

$$|\mathcal{M}|^2 \approx 8 G_F^2 M_n M_p E_e^2 (1 + \cos\theta_{e\bar{\nu}})$$

Total decay width (rough estimate):

$$\Gamma_\beta \approx \frac{G_F^2 \, \Delta^5}{80\,\pi^3} \approx \frac{1}{887\text{ s}} \quad \text{where} \quad \Delta = M_n - M_p$$

From this we can estimate

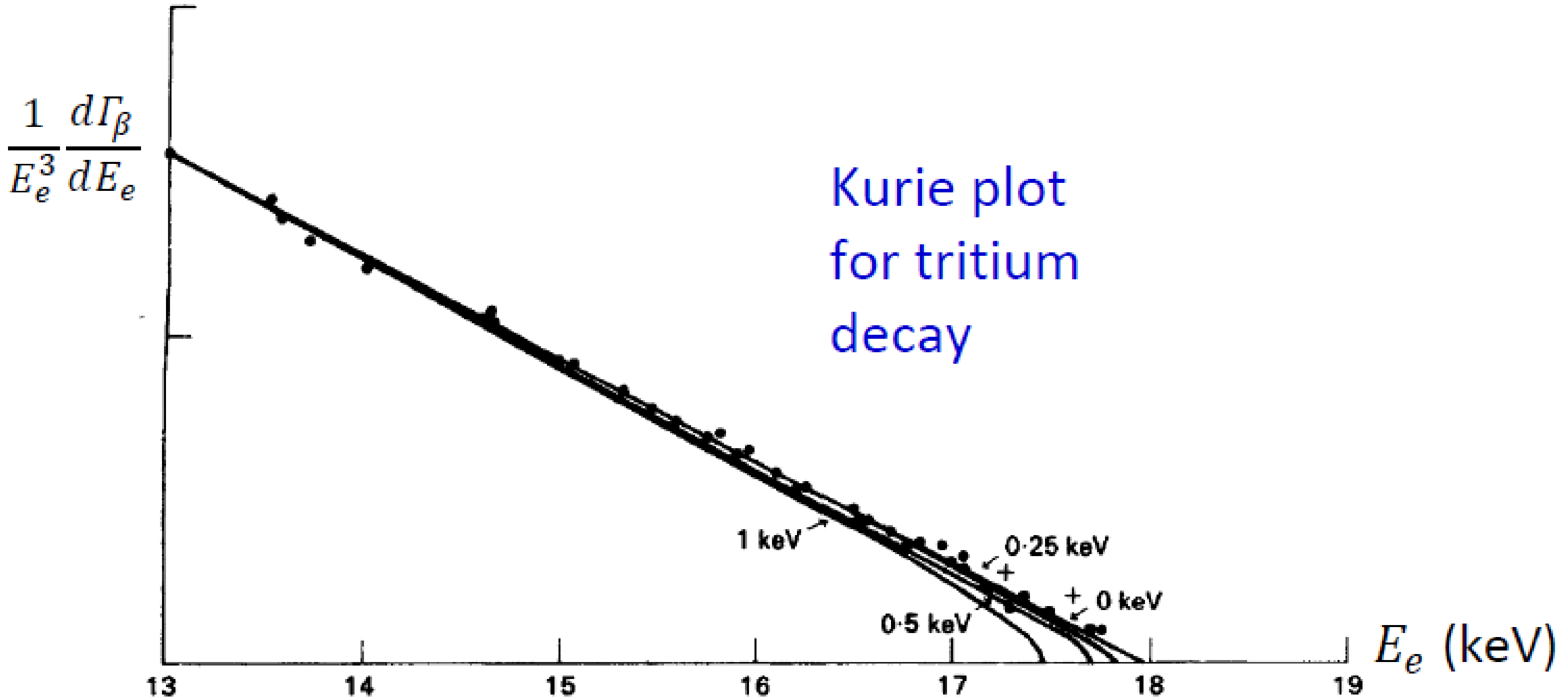
$$G_F \approx 1.8 \times 10^{-5} \text{ GeV}^{-2}$$

Given the crudeness of the approximation, this is not a bad estimate...

Current value: $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$

More important:

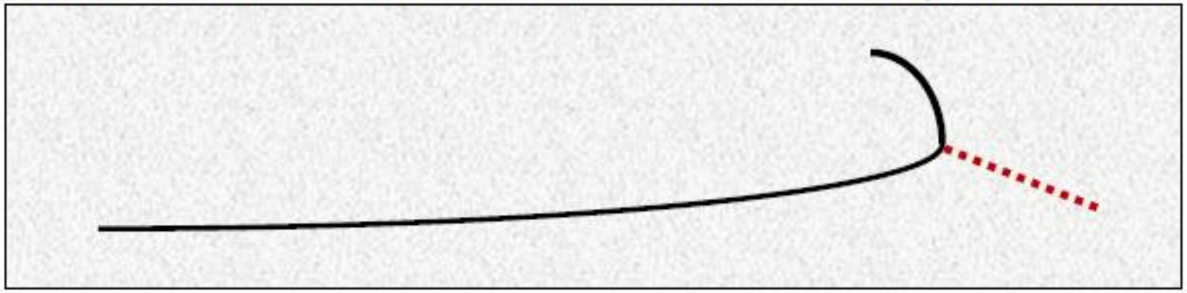
$$\frac{d\Gamma_\beta}{dE_e} \approx \frac{G_F^2}{2\pi^3}\, E_e^3\, [\Delta - E_e] \quad \Rightarrow \quad \frac{1}{E_e^3}\frac{d\Gamma_\beta}{dE_e} \approx \frac{G_F^2}{2\pi^3}\,(\Delta - E_e)$$



Kurie plot for tritium decay

Fermi's theory is spectacularly successful in explaining beta energy spectrum

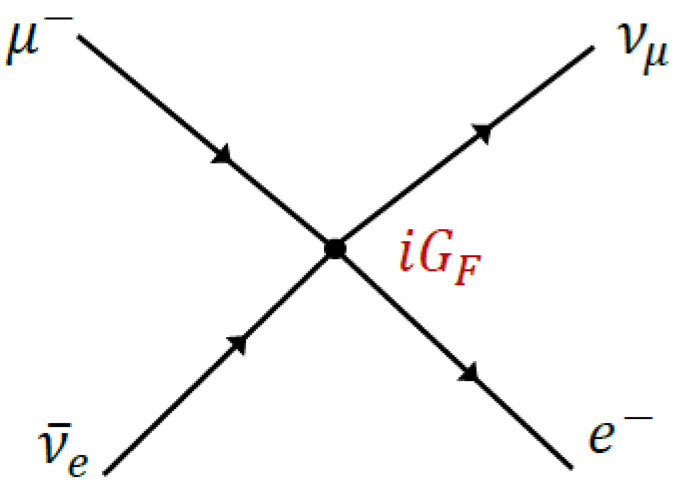
In 1937, the muon was discovered... it decays to electron...



Decay must be through weak interactions (tracks are seen)...

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

Fermi's guess: universality of weak interactions



Use this to calculate the muon lifetime:

$$\tau_\mu \approx \frac{192\,\pi^3}{G_F^2\, M_\mu^5} \approx 2.25 \times 10^{-6}\ \mathrm{s}$$

Spectacular agreement with the experimental value 2.197×10^{-6} s

Vindicates Fermi's hypothesis about universality of weak interactions... today we have many more proofs...

Interestingly, Fermi could have written several forms of the interaction, e.g.

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\,\overline{p}\gamma^{\mu}\gamma_5 n\;\; \bar{e}\gamma_{\mu}\gamma_5\nu_e \quad \text{or} \quad \mathcal{H}_I = \frac{G_F}{\sqrt{2}}\,\overline{p}\sigma^{\mu\nu} n\;\; \bar{e}\sigma_{\mu\nu}\nu_e$$

The choice of the vector-vector form turned out to be a stroke of genius, for that is exactly what we predict in the gauge theory of weak interactions – which is what the Fermi theory ultimately leads to...

Enrico Fermi (1901 – 1954) was a pioneer of quantum mechanics and is credited with the discovery of Fermi-Dirac statistics, discovering nuclear fission (without realizing it), building the first nuclear reactor and discovering the origin of high energy cosmic rays. He also coined the word 'neutrino'.