Electroweak Unification and the Standard Model

Lecture 5

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The $SU(2)_W x U(1)_Y$ gauge theory produces vector currents of the form

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}}\bar{\psi}_A\gamma^\mu\psi_B W_\mu^+ + \mathrm{H.c.}$$
$$+eq_A\,\bar{\psi}_A\gamma^\mu\psi_A A_\mu$$
$$+eq_B\,\bar{\psi}_B\gamma^\mu\psi_B A_\mu$$
$$+\frac{g}{\cos\theta_W}\left(\frac{t_{3A}}{2} - q_A\sin^2\theta_W\right)\bar{\psi}_A\gamma^\mu\psi_A Z_\mu$$
$$+\frac{g}{\cos\theta_W}\left(\frac{t_{3B}}{2} - q_B\sin^2\theta_W\right)\bar{\psi}_B\gamma^\mu\psi_B Z_\mu$$

We know from the experiment of Goldhaber et al that the right handed charged current term does not couple to the W boson...

We must keep both chiralities of the fermions A and B, but couple them in such a way that the W couples only to the left-chiral components...

- If the left-chiral fields ψ_{AL} and ψ_{BL} couple to the W boson, they must belong to a doublet of weak isospin, i.e. they must have $t = ½$;
- If the right-chiral fields ψ_{AR} and ψ_{BR} do not couple to the W boson, they must be singlets of weak isospin, i.e. they must have $t = 0$.

The hypercharges must be assigned to match with the Gell-Mann-Nishijima relation $q_i = t_{3i} + \frac{y_i}{2}$, thus

particle	A_L	B_L	A_R	B_R
q	q_A	q_B	q_A	q_B
t_3	$½$	$-½$	0	0
y	$2q_A - 1$	$2q_B + 1$	$2q_A$	$2q_B$

Note that $q_A - q_B = 1$ implies that

$$y(\mathrm{B_L}) = 2q_B + 1 = 2(q_A - 1) + 1 = 2q_A - 1 = y(\mathrm{A_L}) \equiv y_L$$

i.e. the left-handed doublet has a common hypercharge. In terms of this, we can write

$$y(\mathrm{A_R}) = y_L + 1$$

$$y(\mathrm{B_R}) = y_L - 1$$

All the interaction terms written above are now invariant under the gauge symmetries $SU(2)_W x U(1)_y$ as well as $U(1)_{em}$, when the symmetry is unbroken.

However, if we try to write mass terms for the fermions, i.e.

$$\mathcal{L}_m = m_A \bar{\psi}_A \psi_A + m_B \bar{\psi}_B \psi_B$$

which can be written in terms of chiral components as

$$\mathcal{L}_m = m_A \bar{\psi}_{AL} \psi_{AR} + m_A \bar{\psi}_{AR} \psi_{AL} + m_B \bar{\psi}_{BL} \psi_{BR} + m_B \bar{\psi}_{BL} \psi_{BR}$$

These terms are not gauge invariant, e.g. for the first term $\bar{\psi}_{AL} \psi_{AR}$ we have

$$t_3(\bar{\psi}_{AL} \psi_{AR}) = t_3(\bar{\psi}_{AL}) + t_3(\psi_{AR}) = -½ + 0 = -½$$

$$y(\bar{\psi}_{AL} \psi_{AR}) = y(\bar{\psi}_{AL}) + y(\psi_{AR}) = -2q_A + 1 + 2q_A = 1$$

by referring to the table. This is not invariant under $SU(2)_W$ or $U(1)_y$.

particle	A_L	B_L	A_R	B_R
q	q_A	q_B	q_A	q_B
t_3	½	$-½$	0	0
y	$2q_A - 1$	$2q_B + 1$	$2q_A$	$2q_B$

Thus, the only way to retain both gauge invariance and maximal parity violation is to set $m_A = m_B = 0$

i.e. in the phase with electroweak symmetry unbroken, all fermions must be massless, irrespective of their electric charge.

But fermions have masses!

$$m_e \simeq 511 \text{ keV} \qquad m_\mu \simeq 105 \text{ MeV} \quad \ldots$$

Since fermion masses break the gauge symmetry, the solution must come from spontaneous symmetry-breaking.

In addition to the gauge interactions, the fermions will also interact with the scalar doublet.

These are called Yukawa interactions.

Typical Yukawa interaction: write $\psi_A = A,\ \psi_B = B$

$$\mathcal{L}_{\text{Yuk}} = f_B\, \overline{\Psi}_L \Phi\, B_R + \text{H.c.}$$

where

$$\Psi_L = \begin{pmatrix} A_L \\ B_L \end{pmatrix} \qquad\qquad \Phi = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$$

After symmetry-breaking : $\Phi = \langle\Phi\rangle + \Phi'$

i.e.

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} &= f_B\, \overline{\Psi}_L (\langle\Phi\rangle + \Phi')\, B_R + \text{H.c.} \\ &= \underbrace{f_B\, \overline{\Psi}_L \langle\Phi\rangle\, B_R}_{\text{mass term!}} + f_Y\, \overline{\Psi}_L \Phi'\, B_R + \text{H.c.} \end{aligned}$$

Written out in full

$$\begin{aligned}
\mathcal{L}_{\text{mass}} &= f_B \,(\bar{A}_L \quad \bar{B}_L) \begin{pmatrix} 0 \\ \dfrac{v}{\sqrt{2}} \end{pmatrix} B_R + \text{H. c.} \\
&= f_B \frac{v}{\sqrt{2}} \,\bar{B}_L B_R \;+ \text{H. c.} \\
&= \frac{f_B v}{\sqrt{2}} \,(\bar{B}_L B_R \;+ \bar{B}_R B_L) \\
&= m_B \,\bar{B} B
\end{aligned}$$

where

$$m_B = \frac{f_B v}{\sqrt{2}}$$

What about the A fermion?

Construct the charge-conjugated scalar doublet

$$\tilde{\Phi} \equiv i\sigma_2 \Phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \varphi^- \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \varphi^0 \\ -\varphi^- \end{pmatrix}$$

This will be shifted by

$$\tilde{\Phi} = \langle \tilde{\Phi} \rangle + \tilde{\Phi}'$$

where

$$\langle \tilde{\tilde{\Phi}} \rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Now, write the Yukawa term as

$$\mathcal{L}_{\text{Yuk}} = f_A\, \overline{\Psi}_L \widetilde{\Phi}\, A_R + f_B\, \overline{\Psi}_L \Phi\, B_R + \text{H.c.}$$

Note that the hypercharge assignments match

$$y(\widetilde{\Phi}) = -y(\Phi) \qquad\qquad y(A_R) = y(B_R) + 2y(\Phi)$$

and so this term is also gauge invariant.

For the A term, after symmetry breaking

$$f_A\, \overline{\Psi}_L \widetilde{\Phi}\, A_R + \text{H.c.} \to f_A\, \overline{\Psi}_L \left(\langle\widetilde{\Phi}\rangle + \widetilde{\Phi}'\right) A_R + \text{H.c.}$$

$$= \underbrace{f_A\, \overline{\Psi}_L \langle\widetilde{\Phi}\rangle\, A_R}_{\text{mass term!}} + f_A\, \overline{\Psi}_L \widetilde{\Phi}'\, A_R + \text{H.c.}$$

Written out in full

$$\mathcal{L}^{A}_{\text{mass}} = f_A\,(\bar{A}_L \quad \bar{B}_L)\begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} A_R + \text{H.c.}$$

$$= f_A \frac{v}{\sqrt{2}} A_L A_R + \text{H.c.}$$

$$= \frac{f_A v}{\sqrt{2}} (\bar{A}_L A_R + \bar{A}_R A_L)$$

$$= m_A\,\bar{A}A$$

where

$$m_A = \frac{f_A v}{\sqrt{2}}$$

Given the fermion content of the Standard Model —

For one generation, i.e. $(\nu_e \quad e) \quad (u \quad d)$

$$\mathcal{L}_{\text{Yuk}} = f_e \, \bar{L}_L \Phi \, e_R + f_\nu \bar{L}_L \tilde{\Phi} \, \nu_R + f_d \, \bar{Q}_L \Phi \, d_R + f_u \bar{Q}_L \tilde{\Phi} \, u_R + \text{H.c.}$$

where

$$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

Abdus Salam

The masses become

$$m_e = \frac{f_e v}{\sqrt{2}} \qquad m_\nu = \frac{f_\nu v}{\sqrt{2}}$$

$$m_d = \frac{f_d v}{\sqrt{2}} \qquad m_u = \frac{f_u v}{\sqrt{2}}$$

The ν_R is a particle without any gauge charge, i.e. $t_3 = 0, y = 0, q = 0$

Given the fermion content of the Standard Model —

For one generation, i.e. $(\nu_e \quad e) \quad (u \quad d)$

$$\mathcal{L}_{\text{Yuk}} = f_e\, \bar{L}_L \Phi\, e_R \qquad + f_d\, \bar{Q}_L \Phi\, d_R + f_u \bar{Q}_L \tilde{\Phi}\, u_R + \text{H.c.}$$

where

$$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

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The masses become

$$m_e = \frac{f_e v}{\sqrt{2}} \qquad m_\nu = 0$$

$$m_d = \frac{f_d v}{\sqrt{2}} \qquad m_u = \frac{f_u v}{\sqrt{2}}$$

The ν_R is a particle without any gauge charge, i.e. $t_3 = 0, y = 0, q = 0$

Flavour mixing

There are three generations of fermions

Leptons $L_L = \begin{pmatrix} \nu_1 = \nu_e \\ e_1 = e \end{pmatrix}$ $\begin{pmatrix} \nu_2 = \nu_\mu \\ e_2 = \mu \end{pmatrix}$ $\begin{pmatrix} \nu_3 = \nu_\tau \\ e_3 = \tau \end{pmatrix}$ $I = \frac{1}{2}$

Quarks $Q_L = \begin{pmatrix} u_1 = u \\ d_1 = d \end{pmatrix}$ $\begin{pmatrix} u_2 = c \\ d_2 = s \end{pmatrix}$ $\begin{pmatrix} u_3 = t \\ d_3 = b \end{pmatrix}$ $I = \frac{1}{2}$

$$\mathcal{L}_{\text{Yuk}}^{(1)} = f_e^{(1)} \bar{L}_L^{(1)} \Phi\, e_R^{(1)} + f_d^{(1)} \bar{Q}_L^{(1)} \Phi\, d_R^{(1)} + f_u^{(1)} \bar{Q}_L^{(1)} \tilde{\Phi}\, u_R^{(1)} + \text{H.c.}$$

$$\mathcal{L}_{\text{Yuk}}^{(2)} = f_e^{(2)} \bar{L}_L^{(2)} \Phi\, e_R^{(2)} + f_d^{(2)} \bar{Q}_L^{(2)} \Phi\, d_R^{(2)} + f_u^{(2)} \bar{Q}_L^{(2)} \tilde{\Phi}\, u_R^{(2)} + \text{H.c.}$$

$$\mathcal{L}_{\text{Yuk}}^{(3)} = f_e^{(3)} \bar{L}_L^{(3)} \Phi\, e_R^{(3)} + f_d^{(3)} \bar{Q}_L^{(3)} \Phi\, d_R^{(3)} + f_u^{(3)} \bar{Q}_L^{(3)} \tilde{\Phi}\, u_R^{(3)} + \text{H.c.}$$

With all possible mixings:

$$\mathcal{L}_{\rm Yuk} = \sum_{i,j=1}^{3} f_e^{(ij)}\, \bar{L}_L^{(i)} \Phi\, e_R^{(j)} + f_d^{(ij)}\, \bar{Q}_L^{(i)} \Phi\, d_R^{(j)} + f_u^{(ij)} \bar{Q}_L^{(i)} \tilde{\Phi}\, u_R^{(j)} + \text{H. c.}$$

After symmetry-breaking

$$\Phi \to \Phi' + \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad\qquad \tilde{\Phi} \to \tilde{\Phi}' + \begin{pmatrix} \frac{v}{\sqrt{2}} & 0 \end{pmatrix}$$

$$\mathcal{L}_{\rm mass} = \sum_{i,j=1}^{3} \underbrace{\frac{v}{\sqrt{2}} f_e^{(ij)}}_{M_e^{(ij)}} \bar{e}_L^{(i)}\, e_R^{(j)} + \underbrace{\frac{v}{\sqrt{2}} f_d^{(ij)}}_{M_d^{(ij)}} \bar{d}_L^{(i)} d_R^{(j)} + \underbrace{\frac{v}{\sqrt{2}} f_u^{(ij)}}_{M_u^{(ij)}} \bar{u}_L^{(i)} u_R^{(j)} + \text{H. c.}$$

Fermion mass matrices

Consider the d-type quarks

$$\mathcal{L}_{\text{mass}}^{(\text{d})} = \sum_{i,j=1}^{3} M_d^{(ij)}\, \bar{d}_L^{\,(i)} d_R^{(j)} + \text{H.c.} = \bar{\mathcal{D}}_L \mathbb{M}_d \mathcal{D}_R + \text{H.c.}$$

where

$$\bar{\mathcal{D}}_L = (\bar{d}_L \quad \bar{s}_L \quad \bar{b}_L) \qquad\qquad \mathcal{D}_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

Now as in the case of gauge bosons, we cannot have bilinears like, e.g. $\bar{d}_L s_R$ or $\bar{b}_L d_R$ and so we must diagonalise the mass matrix $\mathbb{M}_d$, i.e.

$$\mathcal{D}_L \to \hat{\mathcal{D}}_L = \mathbb{V}_L^d \mathcal{D}_L$$

$$\mathcal{D}_R \to \hat{\mathcal{D}}_R = \mathbb{V}_R^d \mathcal{D}_R$$

If $\mathbb{M}_d$ is Hermitian, then $\mathbb{V}_L^d = \mathbb{V}_R^d$, but in general they are different so that

$$\begin{aligned}
\mathcal{L}^{(\mathrm{d})}_{\mathrm{mass}} &= \bar{\mathcal{D}}_L \mathbb{M}_d \mathcal{D}_R + \mathrm{H.c.} \\
&= \bar{\mathcal{D}}_L \left(\mathbb{V}_L^{d\dagger} \mathbb{V}_L^d\right) \mathbb{M}_d \left(\mathbb{V}_R^{d\dagger} \mathbb{V}_R^d\right) \mathcal{D}_R + \mathrm{H.c.} \\
&= \bar{\widehat{\mathcal{D}}}_L \left(\mathbb{V}_L^d \mathbb{M}_d \mathbb{V}_R^{d\dagger}\right) \widehat{\mathcal{D}}_R + \mathrm{H.c.}
\end{aligned}$$

We can always choose

$$\mathbb{V}_L^d \mathbb{M}_d \mathbb{V}_R^{d\dagger} = \mathbb{M}_d^{\mathrm{diag}} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

so that

$$\begin{aligned}
\mathcal{L}^{(\mathrm{d})}_{\mathrm{mass}} &= \bar{\widehat{\mathcal{D}}}_L \mathbb{M}_d^{\mathrm{diag}} \widehat{\mathcal{D}}_R + \mathrm{H.c.} \\
&= m_d \bar{d}_L d_R + m_s \bar{s}_L s_R + m_b \bar{b}_L b_R + \mathrm{H.c.} \\
&= m_d \bar{d} d + m_s \bar{s} s + m_b \bar{b} b
\end{aligned}$$

For the u-type quarks, we will similarly have

$$\mathcal{L}_{\rm mass}^{(\rm u)} = \sum_{i,j=1}^{3} M_u^{(ij)} \, \bar{u}_L{}^{(i)} u_R^{(j)} + \text{H.c.} = \bar{\mathcal{U}}_L \mathbb{M}_u \mathcal{U}_R + \text{H.c.}$$

where $\quad \bar{\mathcal{U}}_L = (\bar{u}_L \quad \bar{c}_L \quad \bar{t}_L) \qquad \mathcal{U}_R = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$

Once again, we define $\mathcal{U}_L \to \hat{\mathcal{U}}_L = \mathbb{V}_L^u \mathcal{U}_L \qquad \mathcal{U}_R \to \hat{\mathcal{U}}_R = \mathbb{V}_R^u \mathcal{U}_R$

such that

$$\mathbb{V}_L^u \mathbb{M}_u \mathbb{V}_R^{u\dagger} = \mathbb{M}_u^{\rm diag} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

and this leads to

$$\mathcal{L}_{\rm mass}^{(\rm u)} = \bar{\hat{\mathcal{U}}}_L \mathbb{M}_u^{\rm diag} \hat{\mathcal{U}}_R + \text{H.c.} = m_u \bar{u} u + m_c \bar{c} c + m_t \bar{t} t$$

Similarly, for the charged leptons, we will get

$$\mathcal{L}_{\text{mass}}^{(e)} = \sum_{i,j=1}^{3} M_e^{(ij)} \, \bar{e}_L{}^{(i)} e_R^{(j)} + \text{H.c.} = \bar{\mathcal{E}}_L \mathbb{M}_e \mathcal{E}_R + \text{H.c.}$$

where $\bar{\mathcal{E}}_L = (\bar{e}_L \quad \bar{\mu}_L \quad \bar{\tau}_L)$ $\qquad \mathcal{E}_R = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$

Once again, we define $\mathcal{E}_L \to \hat{\mathcal{E}}_L = \mathbb{V}_L^e \mathcal{E}_L$ $\qquad \mathcal{E}_R \to \hat{\mathcal{E}}_R = \mathbb{V}_R^e \mathcal{E}_R$

such that

$$\mathbb{V}_L^e \mathbb{M}_e \mathbb{V}_R^{e\dagger} = \mathbb{M}_e^{\text{diag}} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}}^{(\text{e})} = \bar{\hat{\mathcal{E}}}_L \mathbb{M}_e^{\text{diag}} \hat{\mathcal{E}}_R + \text{H.c.} = m_e \bar{e} e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau$$

Now let us consider the c.c. interactions of the quarks

$$\mathcal{L}_{\rm cc}^{(\rm q)} = \frac{g}{2\sqrt{2}} \quad \bar{\mathcal{U}}_L \quad \gamma^\mu \quad \mathcal{D}_L \quad W_\mu^+ + \text{H. c.}$$

$$= \frac{g}{2\sqrt{2}} \left(\hat{\bar{\mathcal{U}}}_L \mathbb{V}_L^u\right) \gamma^\mu \left(\mathbb{V}_L^{d\dagger} \hat{\mathcal{D}}_L\right) W_\mu^+ + \text{H. c.}$$

$$= \frac{g}{2\sqrt{2}} \quad \hat{\bar{\mathcal{U}}}_L \; \gamma^\mu \left(\mathbb{V}_L^u \mathbb{V}_L^{d\dagger}\right) \hat{\mathcal{D}}_L W_\mu^+ + \text{H. c.}$$

$$= \frac{g}{2\sqrt{2}} \quad \hat{\bar{\mathcal{U}}}_L \; \gamma^\mu \quad \mathbb{K} \quad \hat{\mathcal{D}}_L \, W_\mu^+ + \text{H. c.}$$

where $\mathbb{K} = \mathbb{V}_L^u \mathbb{V}_L^{d\dagger}$ is a unitary mixing matrix (not diagonal!)

Cabibbo-Kobayashi-Maskawa (CKM) matrix

What about neutral and e.m. currents? They have the form

$$\begin{aligned}
\mathcal{L}_{\text{nc}}^{(\text{q})} &= \bar{\mathcal{U}}_L \gamma^\mu \mathcal{U}_L Z_\mu \quad \oplus \quad \bar{\mathcal{D}}_L \gamma^\mu \mathcal{D}_L Z_\mu \\
&= \left(\bar{\hat{\mathcal{U}}}_L \mathbb{V}_L^u\right) \gamma^\mu \left(\mathbb{V}_L^{u\dagger} \hat{\mathcal{U}}_L\right) Z_\mu \quad \oplus \quad \left(\bar{\hat{\mathcal{D}}}_L \mathbb{V}_L^d\right) \gamma^\mu \left(\mathbb{V}_L^{d\dagger} \hat{\mathcal{D}}_L\right) Z_\mu \\
&= \bar{\hat{\mathcal{U}}}_L \gamma^\mu \hat{\mathcal{U}}_L Z_\mu \quad \oplus \quad \bar{\hat{\mathcal{D}}}_L \gamma^\mu \hat{\mathcal{D}}_L Z_\mu
\end{aligned}$$

i.e. no flavour-changing neutral currents (FCNC)

Similar for the e.m. current

Hence, flavour changing is seen only in c.c. interactions.

$$\mathcal{L}_{cc}^{(q)} = \frac{g}{2\sqrt{2}} \left(\bar{\hat{u}}_L \quad \bar{\hat{c}}_L \quad \bar{\hat{t}}_L\right) \gamma^\mu \begin{pmatrix} K_{ud} & K_{us} & K_{ub} \\ K_{cd} & K_{cs} & K_{cb} \\ K_{td} & K_{ts} & K_{tb} \end{pmatrix} \begin{pmatrix} \hat{d}_L \\ \hat{s}_L \\ \hat{b}_L \end{pmatrix} W_\mu^+ + \text{H.c.}$$

Observed pattern of masses and mixings

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} = \begin{pmatrix} 0.0005 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1.77 \end{pmatrix}$$

$$\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} = \begin{pmatrix} 0.009 & 0 & 0 \\ 0 & 0.093 & 0 \\ 0 & 0 & 4.3 \end{pmatrix}$$

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} = \begin{pmatrix} 0.005 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 172.3 \end{pmatrix}$$

$$\begin{pmatrix} |K_{ud}| & |K_{us}| & |K_{ub}| \\ |K_{cd}| & |K_{cs}| & |K_{cb}| \\ |K_{td}| & |K_{ts}| & |K_{tb}| \end{pmatrix} \simeq \begin{pmatrix} \underline{0.9745} & \underline{0.2245} & 0.0004 \\ \underline{0.2244} & \underline{0.9736} & \underline{0.0421} \\ 0.0089 & \underline{0.0413} & \underline{0.9991} \end{pmatrix}$$

Cries out for an explanation!

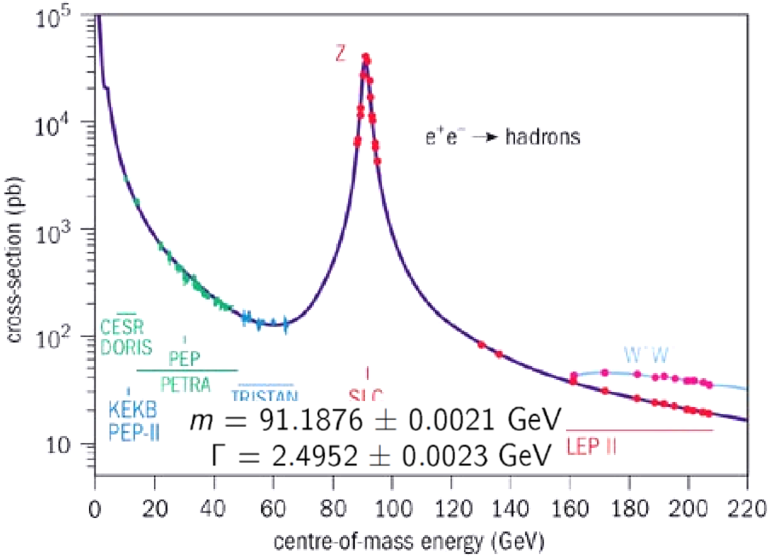
What about leptons? The lepton c.c. terms are

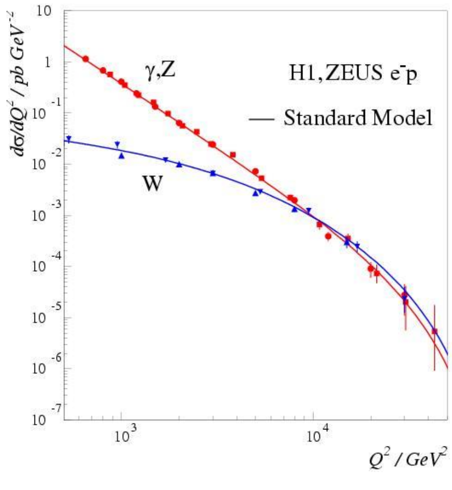
$$
\begin{aligned}
\mathcal{L}_{\mathrm{cc}}^{(e)} &= \frac{g}{2\sqrt{2}} \quad \bar{\mathcal{E}}_L \quad \gamma^\mu \quad \mathcal{N}_L \quad W_\mu^+ + \mathrm{H.c.} \\
&= \frac{g}{2\sqrt{2}} \left(\bar{\hat{\mathcal{E}}}_L \mathbb{V}_L^e\right) \gamma^\mu \quad \mathcal{N}_L \quad W_\mu^+ + \mathrm{H.c.} \\
&= \frac{g}{2\sqrt{2}} \quad \bar{\hat{\mathcal{E}}}_L \quad \gamma^\mu \left(\mathbb{V}_L^e \mathcal{N}_L\right) \quad W_\mu^+ + \mathrm{H.c.} \\
&= \frac{g}{2\sqrt{2}} \quad \bar{\hat{\mathcal{E}}}_L \quad \gamma^\mu \quad \widehat{\mathcal{N}}_L \quad W_\mu^+ + \mathrm{H.c.}
\end{aligned}
$$

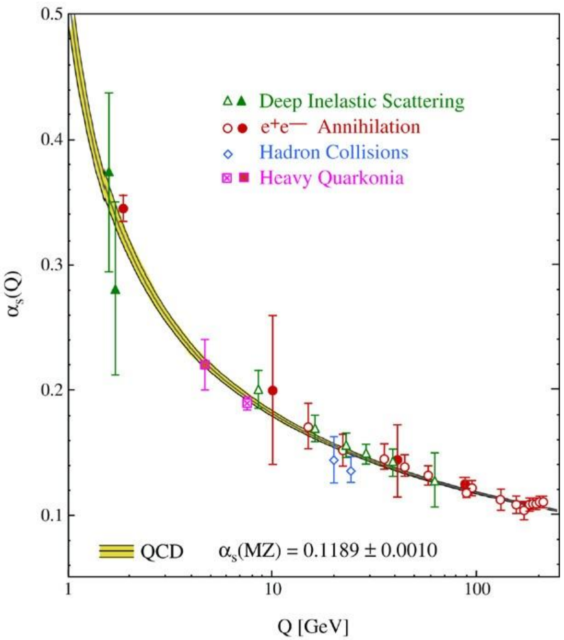
where $\widehat{\mathcal{N}}_L = \mathbb{V}_L^e \mathcal{N}_L$.

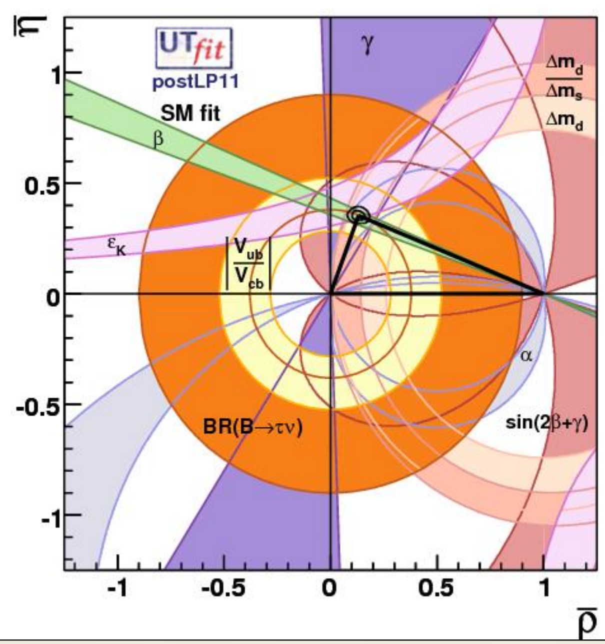
Thus there is no mixing in the lepton sector. The cause of this is that neutrinos are massless.

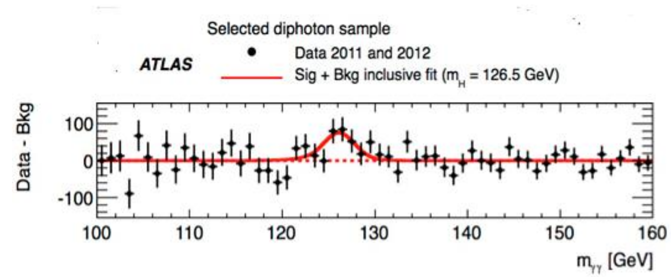
Success story of the Standard Model









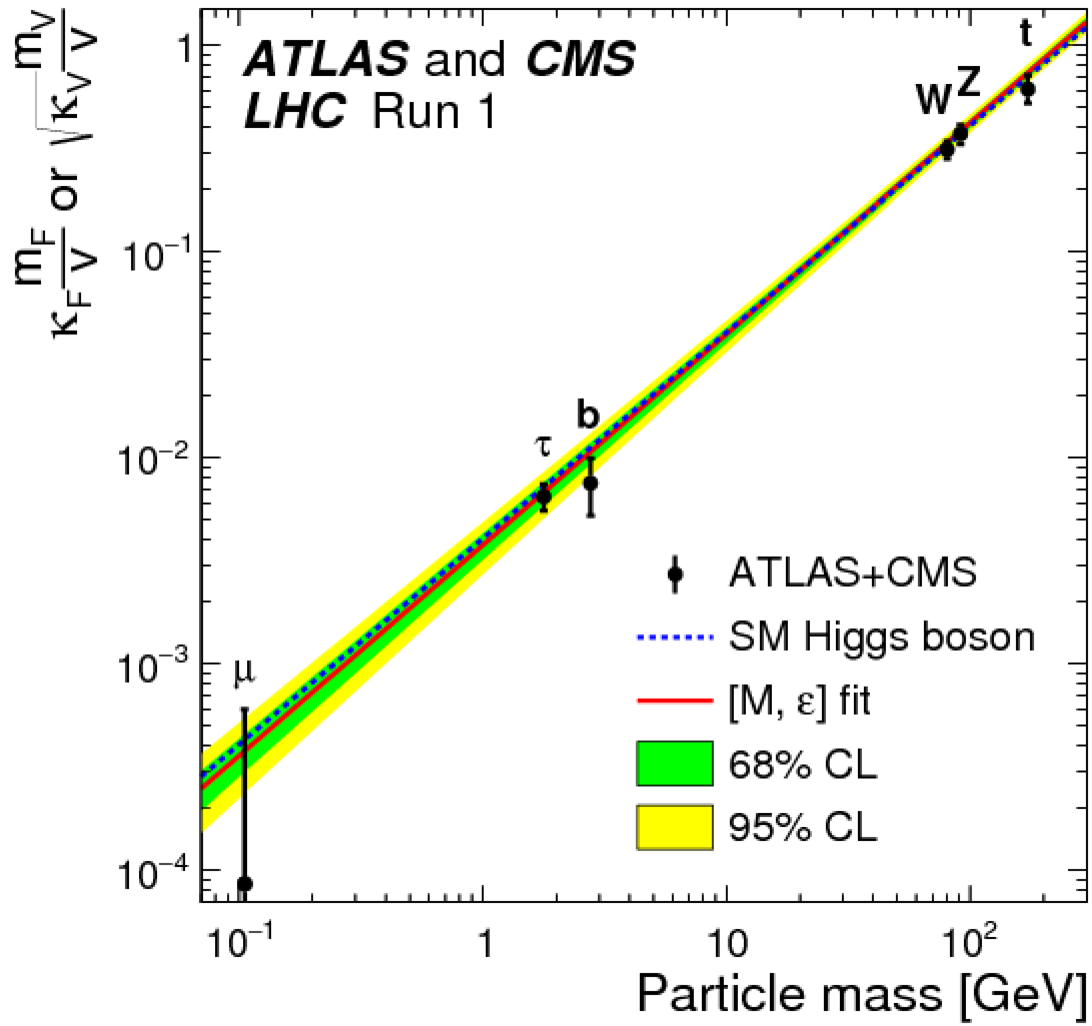


Mass matrices are proportional to Yukawa coupling matrices…

$$M^{(ij)}_{\ell,u,d} = \frac{v}{\sqrt{2}}\, f^{(ij)}_{\ell,u,d}$$

Diagonalisation of the one leads to diagonalisation of the other…

$$m_{\ell,u,d} = \frac{v}{\sqrt{2}}\, f_{\ell,u,d}$$



Plot of mass versus coupling should be a straight line…

Summary of Standard Model

- Nature at the sub-nuclear scale seems to prefer local gauge symmetries, leading to fundamental interactions
- These are of non-Abelian nature, involving multiple fields with degenerate masses
- The electroweak symmetry is spontaneously broken – or hidden – and this corresponds to a phase transition which happened in the early Universe
- The phase transition was driven by a scalar field, which is the Higgs boson and it has been found
- All particles acquired their (rest) masses from this phase transition, for this need Yukawa and self-interaction forces
- The Standard Model is complete and (almost) all the parameters are measured. Only issue is the RH neutrino.

The Seven Great Questions

clueless

1. Dark matter
2. Dark energy

unexpected

3. Baryogenesis
4. Neutrino Mass

5. Strong CP problem
6. Vacuum Stability
7. Hierarchy Problem

embarrassing

The Seven Great Questions

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embarrassing

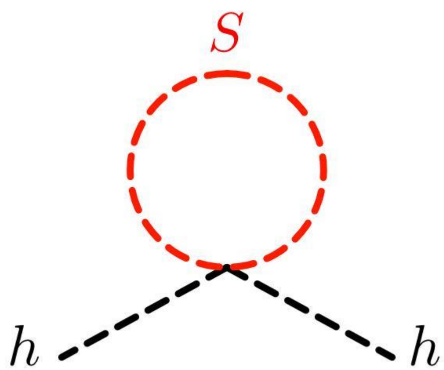
$$(M_H)^2 = 3.273{,}459{,}429{,}634{,}290{,}543{,}867{,}496{,}473{,}159{,}645 \\ - 3.273{,}459{,}429{,}634{,}290{,}543{,}867{,}496{,}473{,}159{,}643$$

It is hard to believe that such an extreme cancellation can be an accident...

The Hierarchy Problem

$$\mathcal{L} \supset \lambda_S |H|^2 S^2$$

this is **not** the hierarchy problem
the regulator is not physical



$$= \frac{\lambda_S}{16\pi^2}\left[\Lambda_{\mathrm{UV}}^2 - 2m_S^2 \ln\left(\frac{\Lambda_{\mathrm{UV}}}{m_S}\right) + \cdots\right]$$

From Prof. Flip Tanedo's Twitter 2019

this is the hierarchy problem

the Higgs mass is quadratically sensitive to the mass of any new particles that couple to it

Even if we cancel the m_S^2 term at some energy scale, it will reappear at a different energy scale, because m_S will run…

Most of well-known BSM models created to solve this problem:
Technicolor, Supersymmetry, Extra Dimensions, Little Higgs Models

Thank You