HYPOTHESIS TESTING.

ref { 1. Sigmund Brandt
2. Glen Cowan

We have seen from the disease test example (application of Bayes theorem) that given a test and two hypotheses two types of errors are possible

Type - I A false positive (error rate α)
Type - II A false negative. (error rate β)

In the case of a signal - background analysis the two hypothesis are

H_0 → signal not present : Null hypothesis
H_1 → signal present : Alternative hypothesis
(can be simple or composite)

Our goal is to devise tests that minimize both the error rates.

Not possible simultaneously → If I want to reject more background events I will reduce type I rate but I will also reject more signal events, ie increase the type - II error rate.

⇒ For a given α design a test that minimizes β

$\boxed{1 - \beta \; : \; \text{power of the test}}$

designing a test means : choosing a test statistic (t) and difining a critical region S_c

• If t falls in the S_c we will reject H_0.

Examples: Covid test : t = CT value

critical region $S_c \Rightarrow t \leq 35$

H_0 : Covid not present.

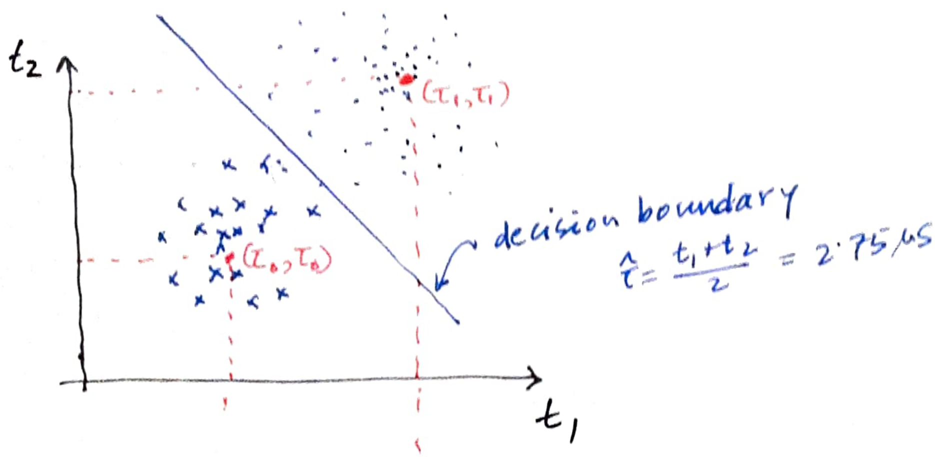
Another example :

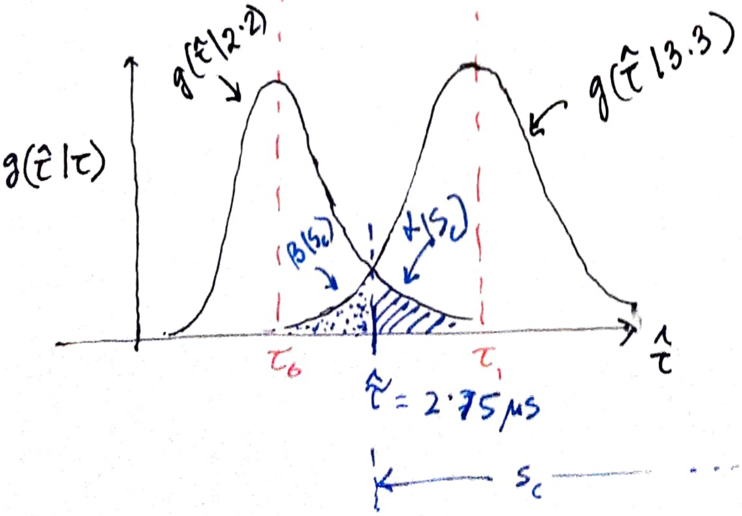
H_0 = μ lifetime τ_0 is $2.2\ \mu s$.

H_1 = τ_1 is $3.3\ \mu s$.

$$t \equiv \hat{\tau} = \frac{1}{n}\sum t_i$$

E space : $\vec{x} = (t_1, t_2 \cdots t_n)$





if $\hat{\tau} \geq 2.2\mu s$

H_0 is rejected.

Note: if we try to reduce α, β increases

We want to find t s.t. $g(\hat{\tau}|\tau_0)$ and $g(\hat{\tau}|\tau_1)$ are as far apart as possible.

** Neural networks are often used to find most optimal decision boundaries $\rightarrow$ t = NN output.

We see that each hypothesis is defined by a distribution.

$H_0 : \prod_i \frac{1}{\tau_0} e^{-t_i/\tau_0}$ $\qquad$ $H_1 = \prod_i \frac{1}{\tau_1} e^{-t_i/\tau_1}$

$\Rightarrow$ $H_0 : \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-(\hat{\tau}-\tau_0)^2/2\sigma_0^2}$ $\qquad$ $H_1 = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-(\hat{\tau}-\tau_1)^2/2\sigma_1^2}$

$g(\hat{\tau}|\tau_0)$ $\qquad$ $g(\hat{\tau}|\tau_1)$

$\int_{S_c} g(\hat{\tau}|\tau_0)\,d\hat{\tau} = \alpha$ $\qquad$ $\int_{\bar{S}_c} g(\hat{\tau}|\tau_1)\,d\hat{\tau} = \beta$

$\boxed{\text{UMP}}$: Uniformly most powerful test is one which maximizes $(1-\beta)$ for all possible H_1.

* UMP may or may not exist